A factorization model of V1 complex cell activity: amplitude and phase

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animated basis functions

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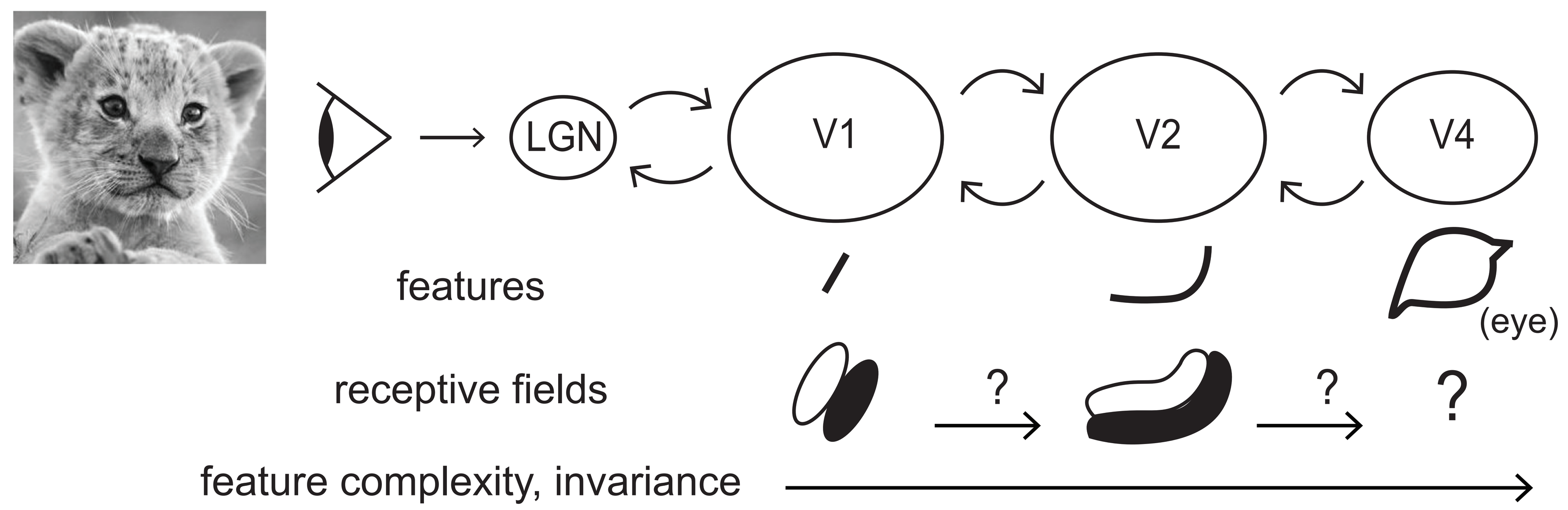
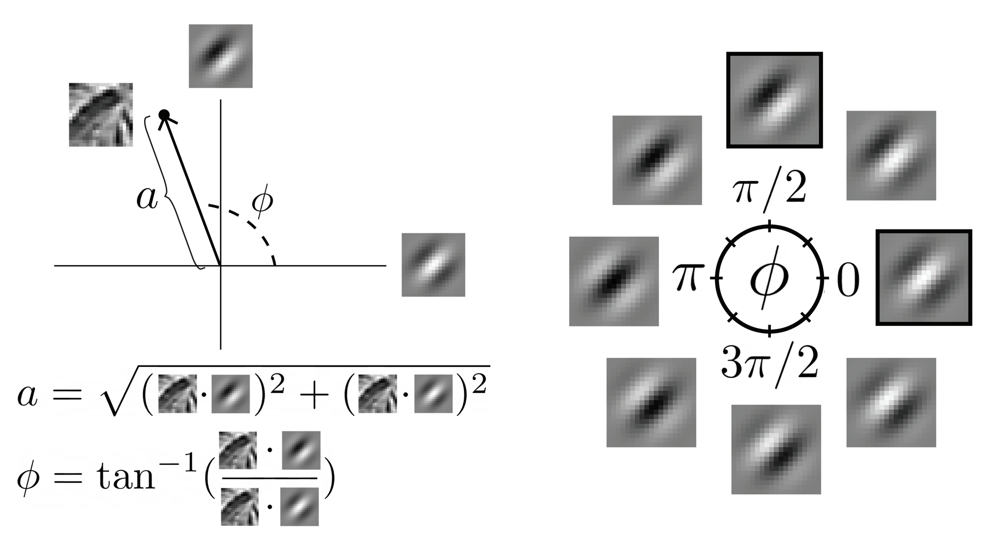
Ventral stream feature abstraction

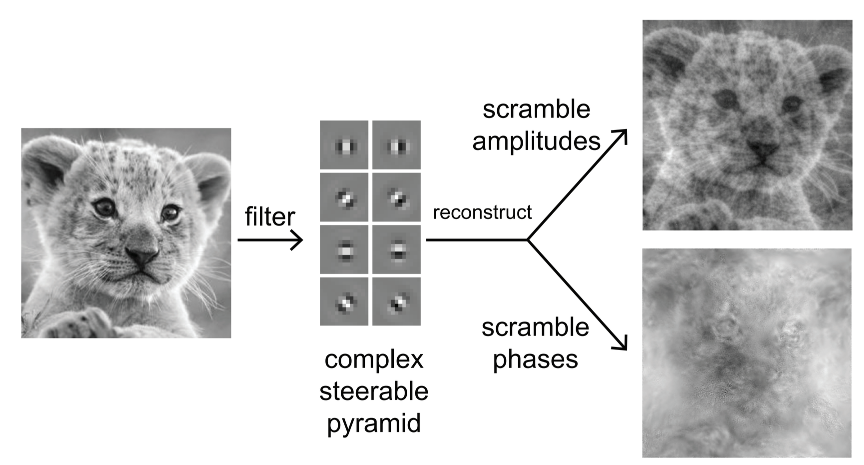


How is visual cortex selective and invariant to higher level features?

Relative phase contains natural image structure



Each complex filter pair response consists of amplitude and phase.

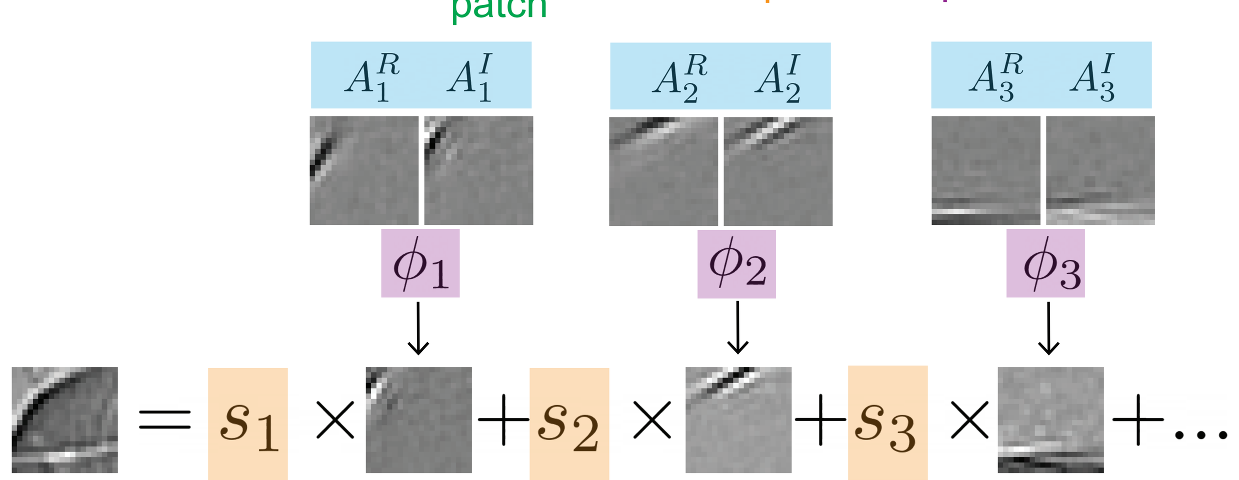


Scrambling phase results in loss of structure and form.

Model

Generative form $$\hat{x} = \sum_i s_i \left[\cos\phi_i A_i^R + \sin\phi_i A_i^I\right]$$

reconstructed patch, amplitude, phase real basis, imaginary basis



Inferring phase **steers** basis functions and parameterizes local transformations.

Energy function

$$E = \sum_{\text{pixels}} (x - \hat{x})^2 + \lambda \sum_i C(s_i)$$

reconstruction penalty; sparsity cost

To learn complex basis functions and infer amplitude and phase responses of each unit, we minimize the energy function.

Learning rule

$$\Delta A_i \propto (x - \hat{x}) s_i e^{j\phi_i}$$

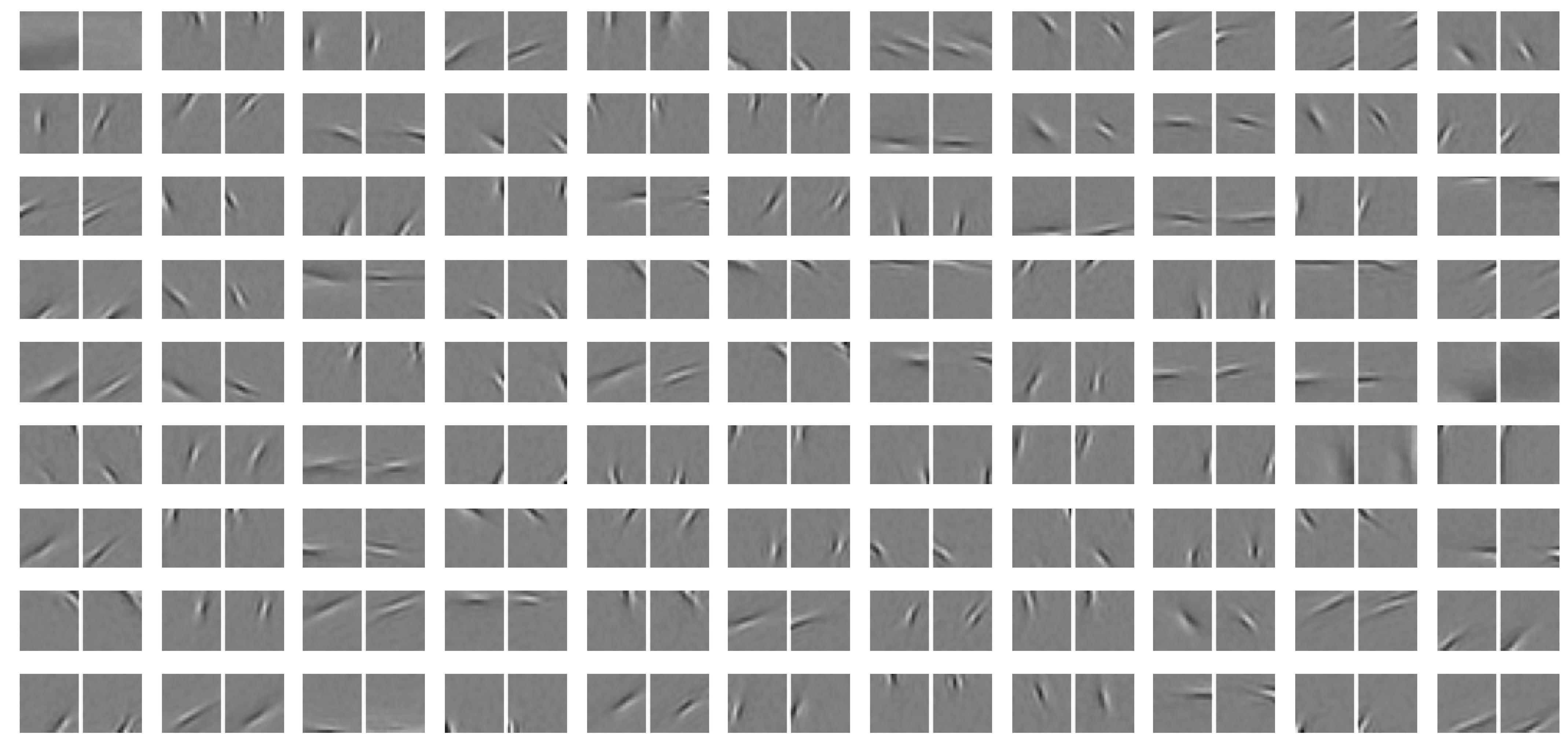
Inference dynamics

$$\dot{s}_i \propto \Re\{b_i\} - \lambda C'(s_i)$$

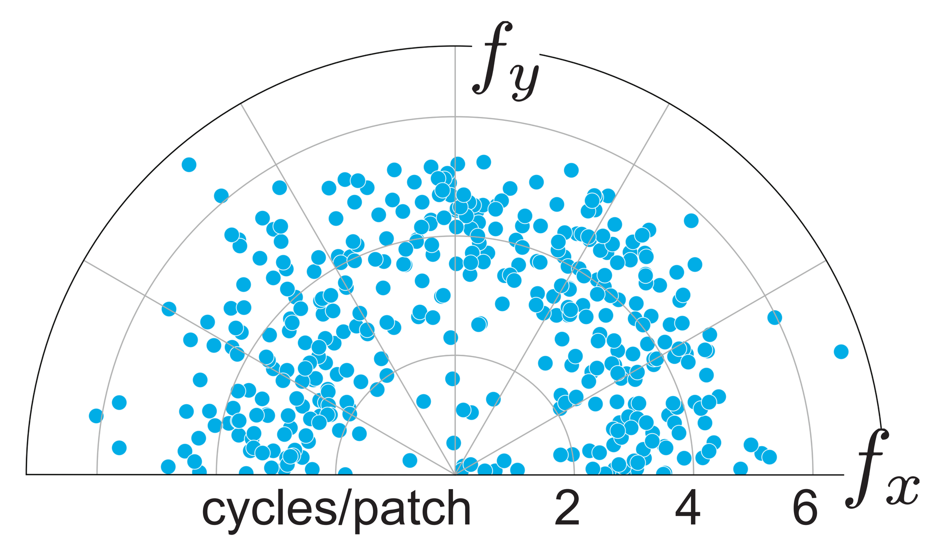
$$\dot{\phi}_i \propto \Im\{b_i\}$$

$$b_i = e^{-j\phi_i} A_i^\top (x - \hat{x})$$

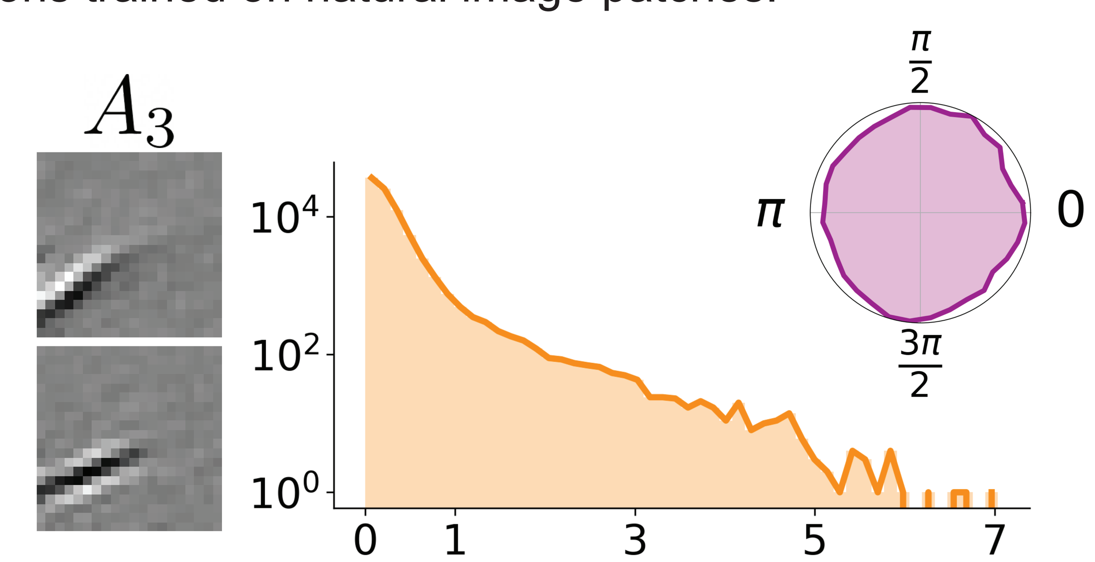
Learned complex basis functions



Random subset of learned complex basis functions trained on natural image patches.

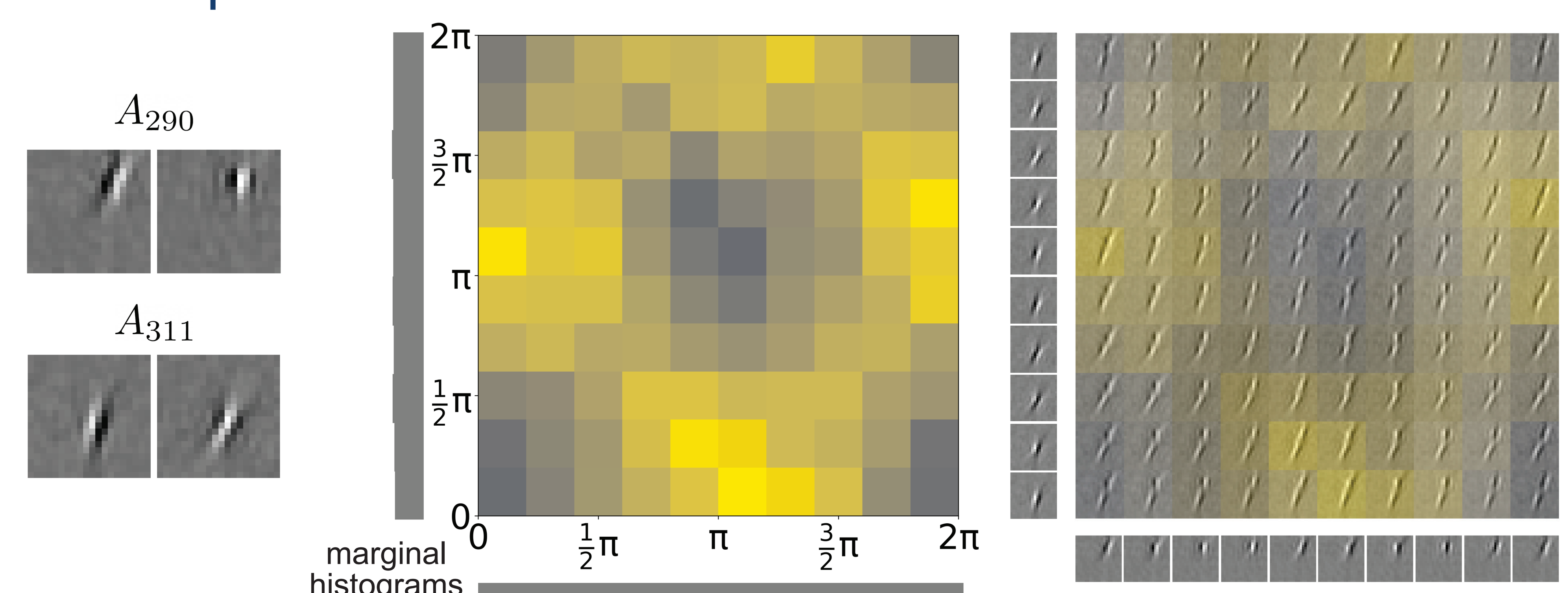


Basis functions tile the spatial frequency domain (real components shown).



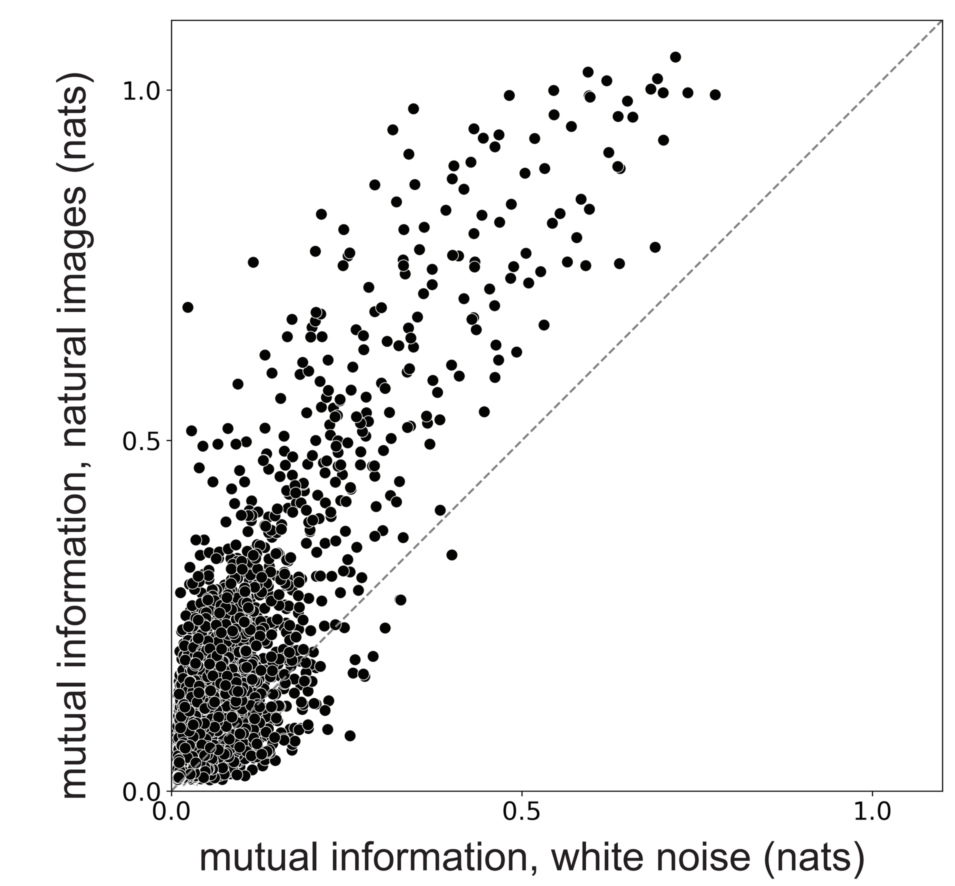
For one complex unit, typical histograms of inferred amplitudes and phases across the dataset.

Relative phase statistics

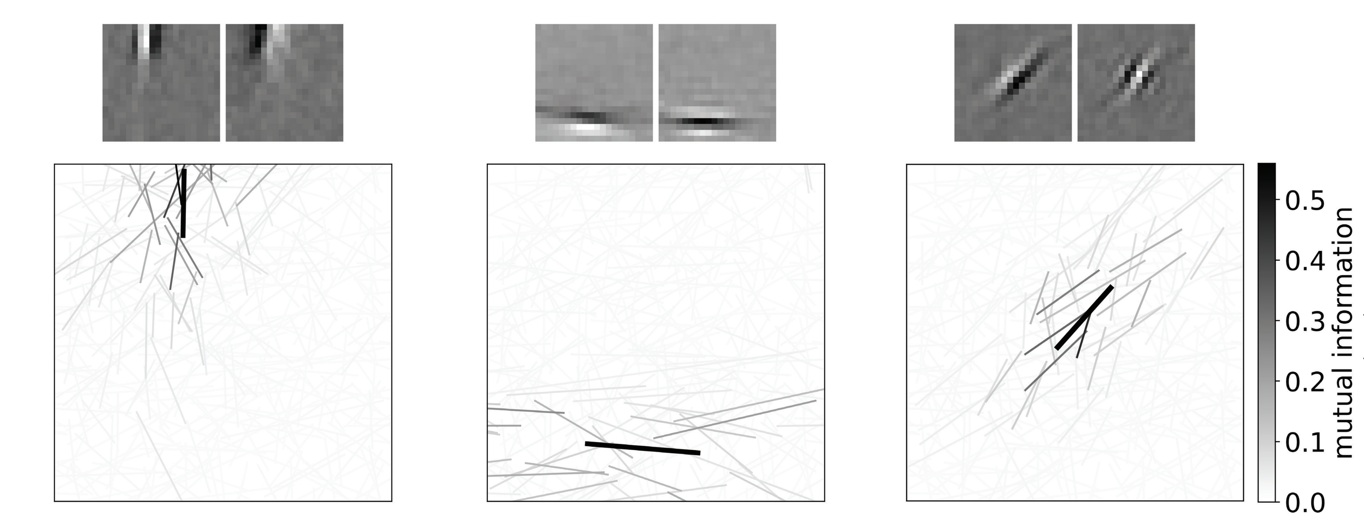


Dependent joint phase distribution of two units. Left: basis functions of two units. Center: joint histogram of inferred phase. Right: visualization of phase.

Population phase mutual information



Each point indicates MI between two complex unit phase distributions, calculated from inference on white noise vs natural images.



Units shaded by mutual information with target unit.

Summary

Our model's steerable basis functions parameterize local transformations explicitly with phase.

Dependencies in first-order phase statistics may provide clues for higher-level feature selectivity and invariance in the visual system.

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References

[1] Portilla, J., & Simoncelli, E.P. (2000). A Parametric Texture Model Based on Joint Statistics of Complex Wavelet Coefficients. International Journal of Computer Vision.
[2] Hyvärinen, A., & Hoyer, P. (2000). Emergence of phase-and shift-invariant features by decomposition of natural images into independent feature subspaces. Neural Computation.
[3] Cadieu, C., & Olshausen, B.A. (2012). Learning Intermediate-Level Representations of Form and Motion from Natural Movies. Neural Computation.
[4] Paiton, D. M., Shepard, S., Chan, K. H. R., & Olshausen, B. A. (2020). Subspace locally competitive algorithms. In Proceedings of the 2020 Annual Neuro-Inspired Computational Elements Workshop.