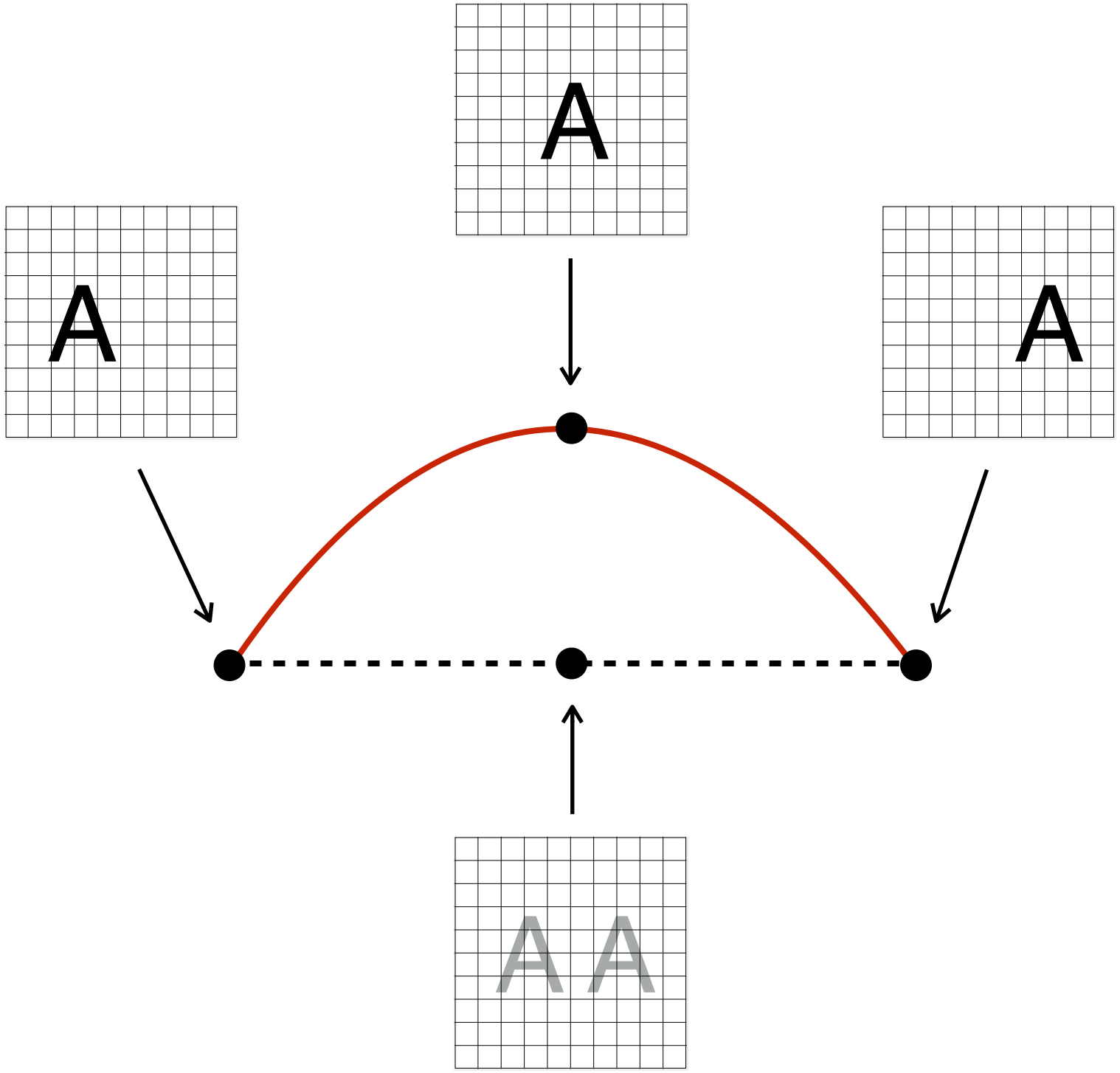
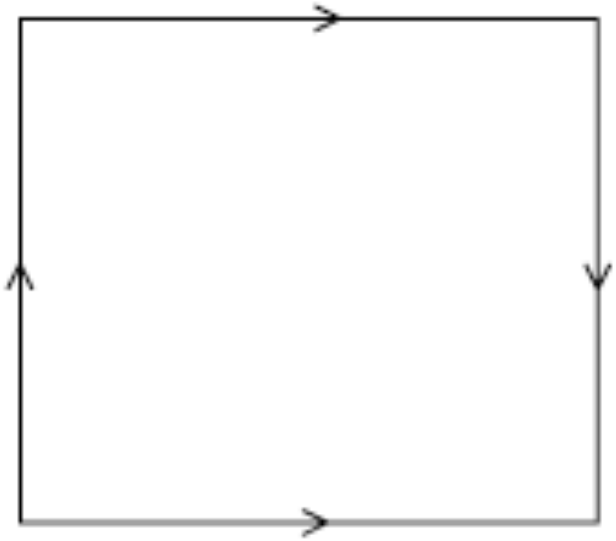
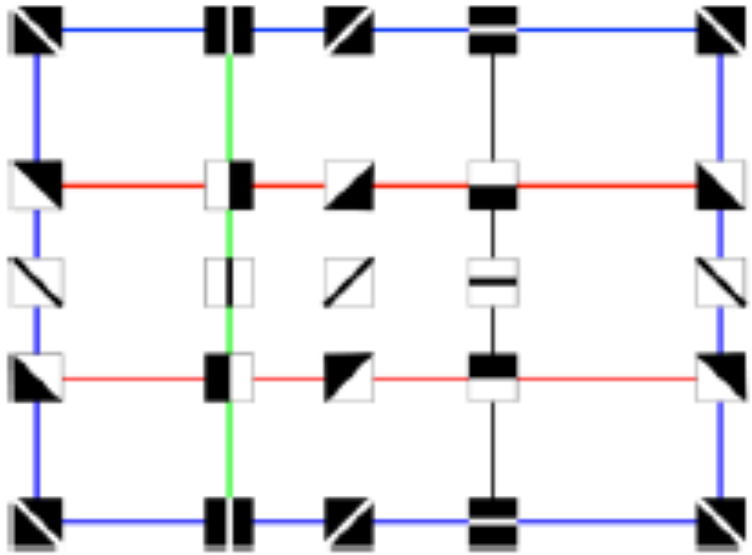
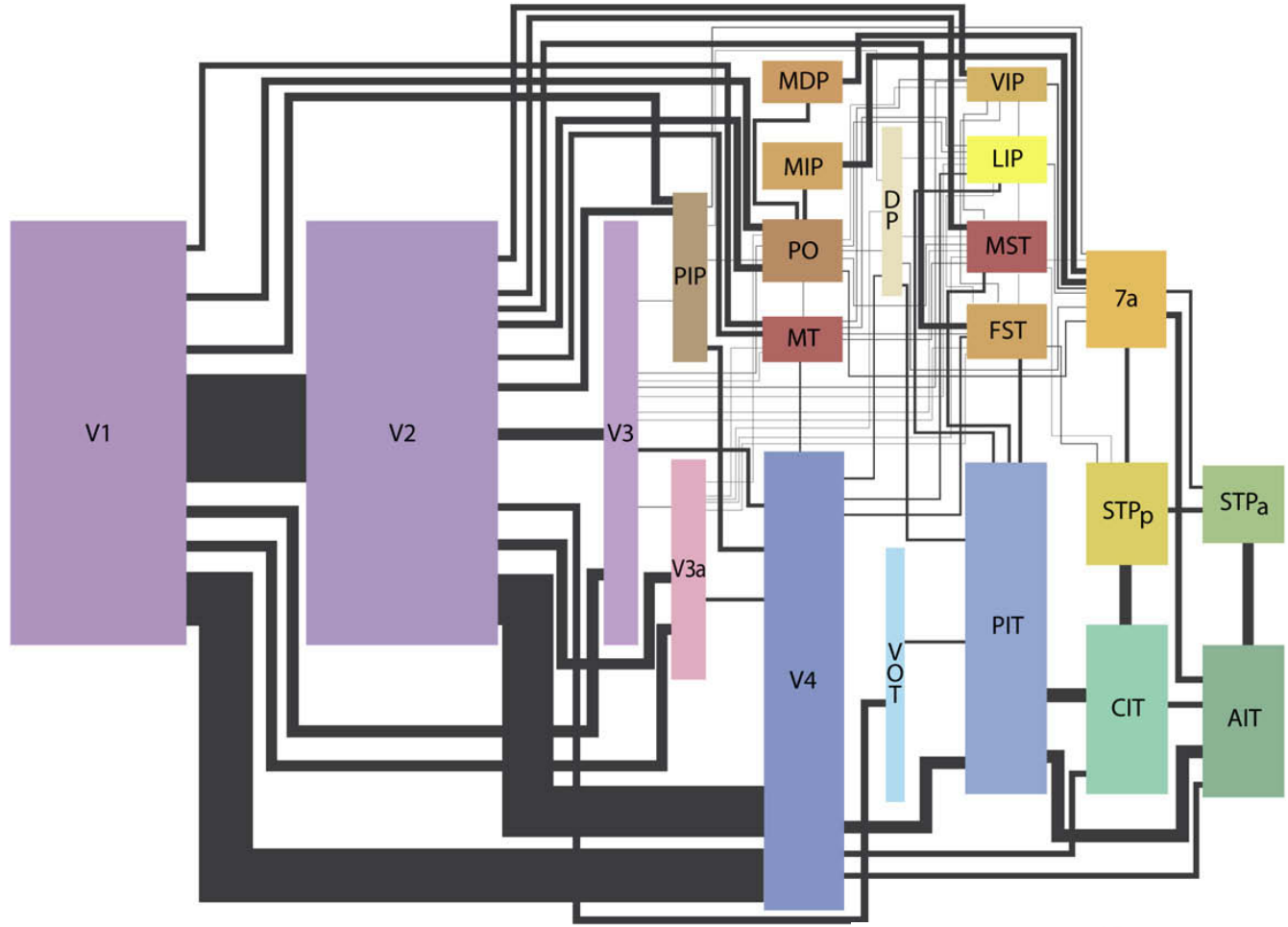
Manifold learning

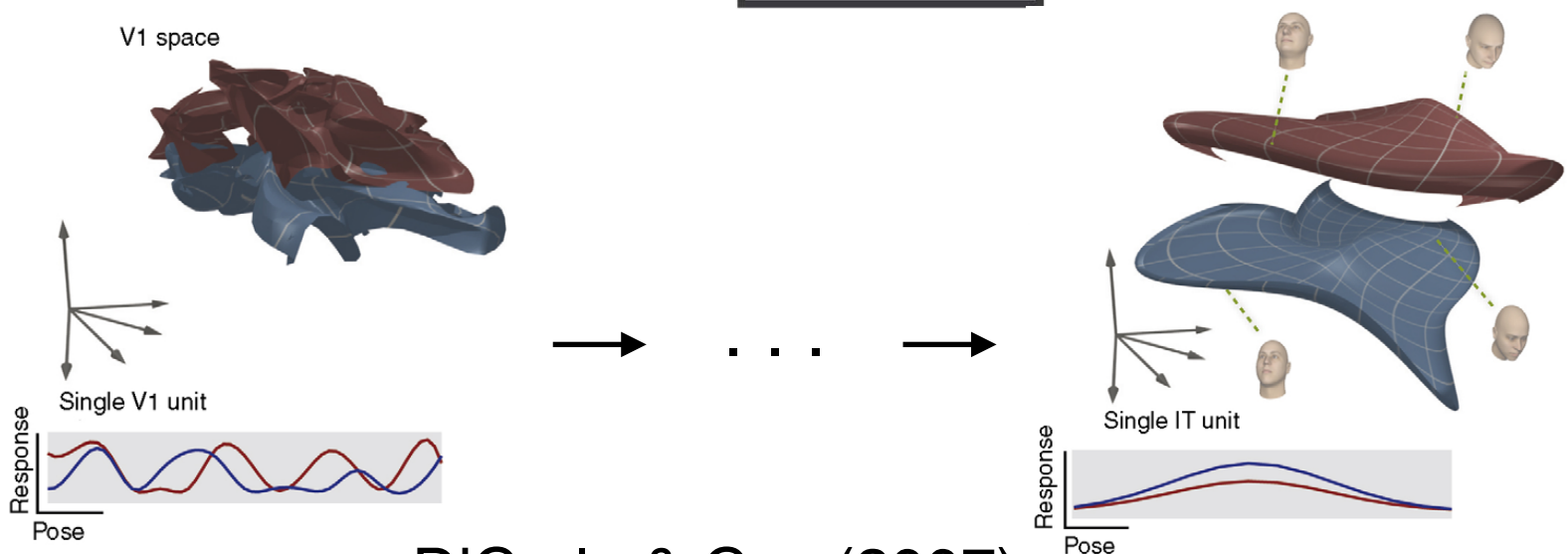

A
A
A
A A

Distribution of 3x3 pixel image patches drawn from natural scenes forms a Klein bottle (Carlsson et al., 2008)









DiCarlo & Cox (2007)

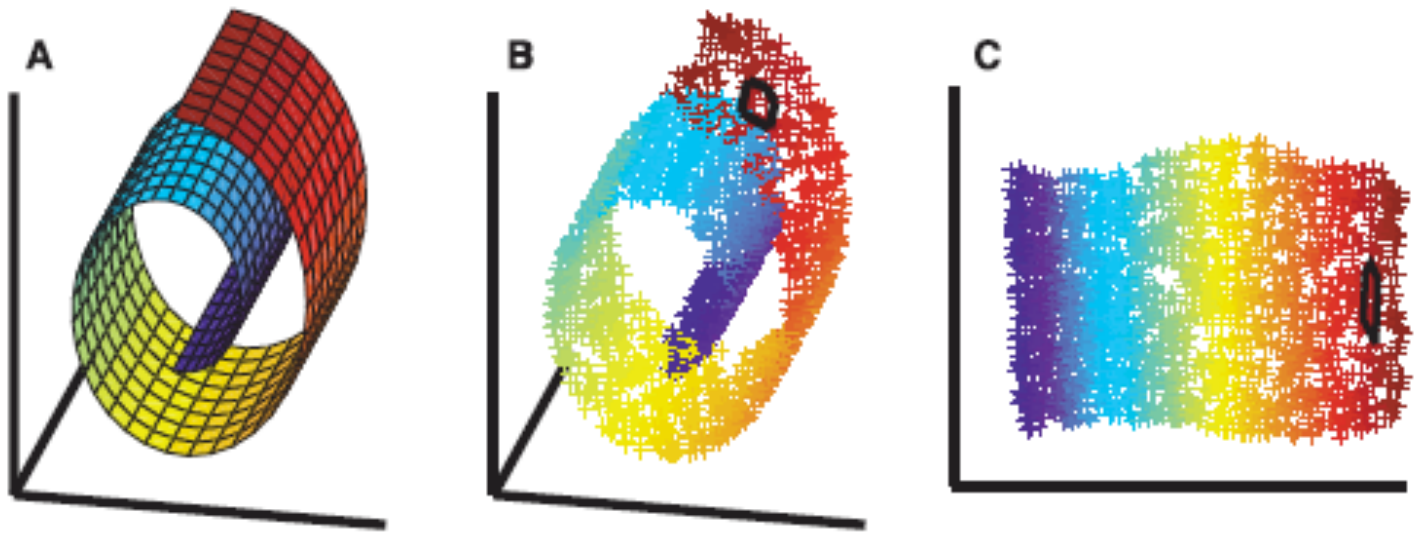
Nonlinear Dimensionality Reduction by Locally Linear Embedding

Sam T. Roweis[1] and Lawrence K. Saul[2]

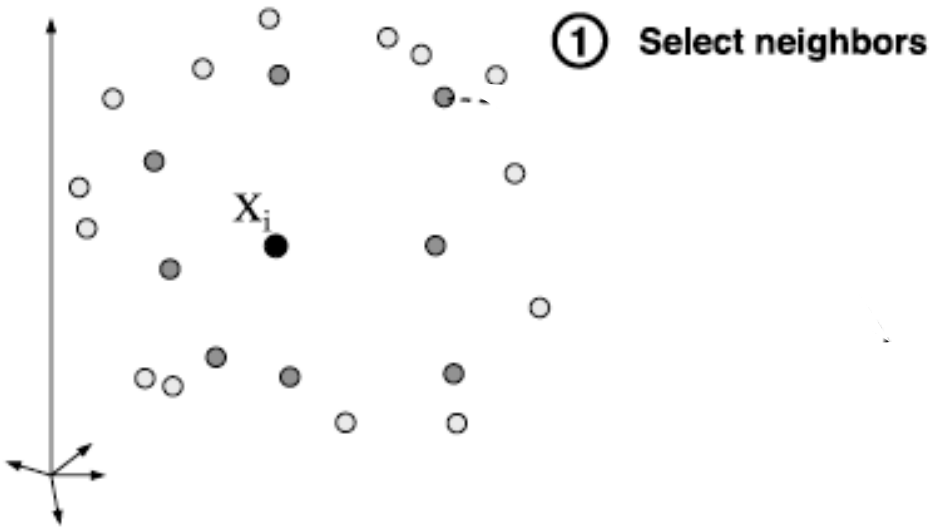
A Global Geometric Framework for Nonlinear Dimensionality Reduction

Joshua B. Tenenbaum,[1]* Vin de Silva,[2] John C. Langford[3]

Science, 22 Dec. 2000



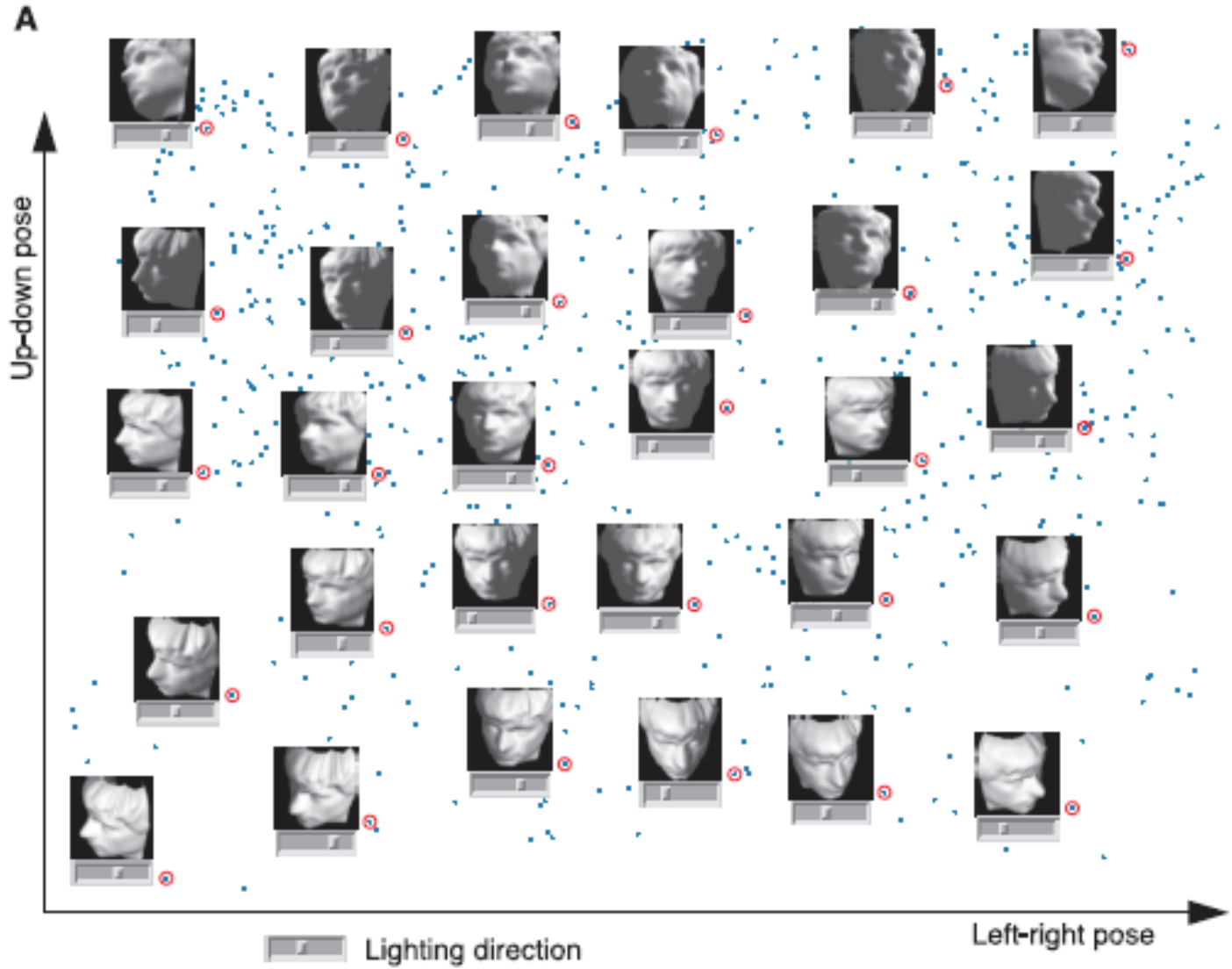
Local Linear Embedding (LLE)



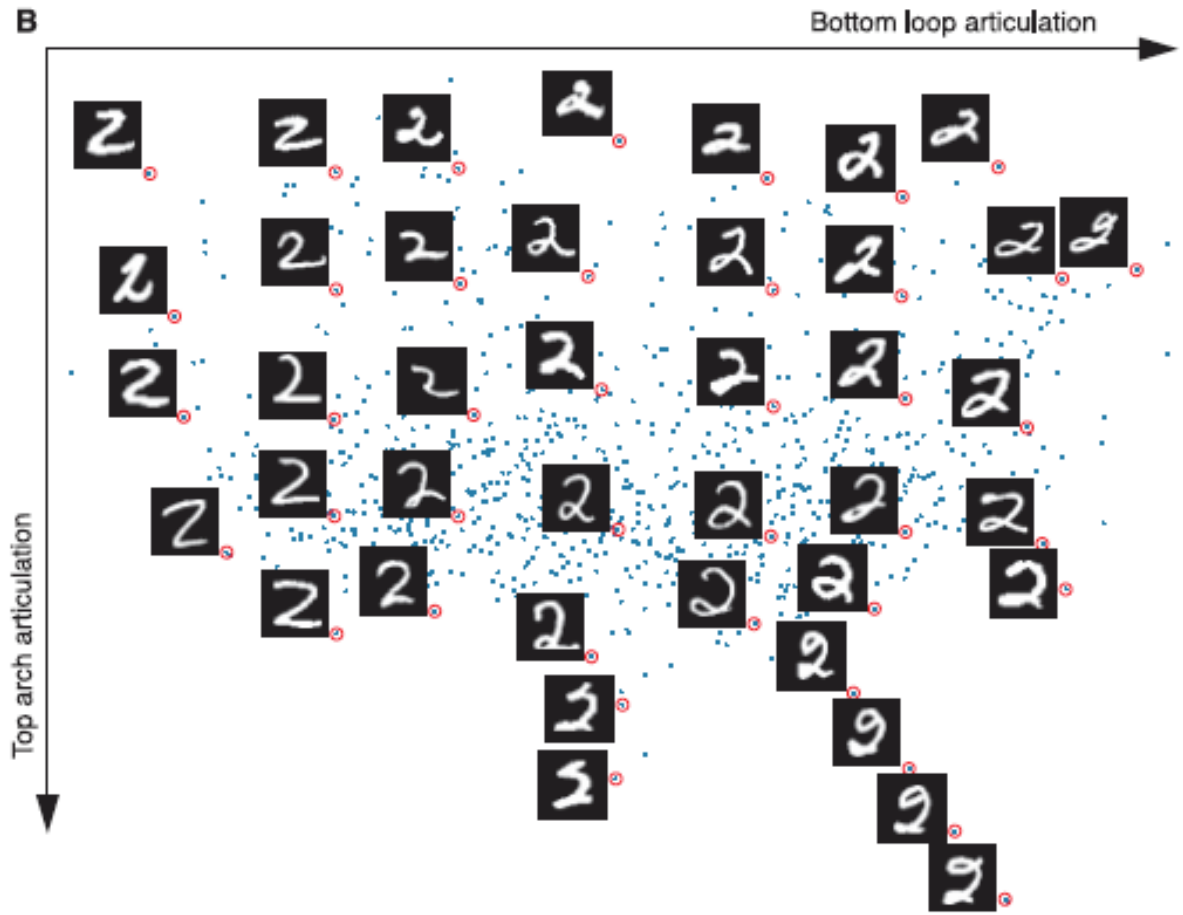
$$\varepsilon(W) = \sum_i \left| \vec{X}_i - \Sigma_j W_{ij} \vec{X}_j \right|^2$$

$$\Phi(Y) = \sum_i \left| \vec{Y}_i - \Sigma_j W_{ij} \vec{Y}_j \right|^2$$

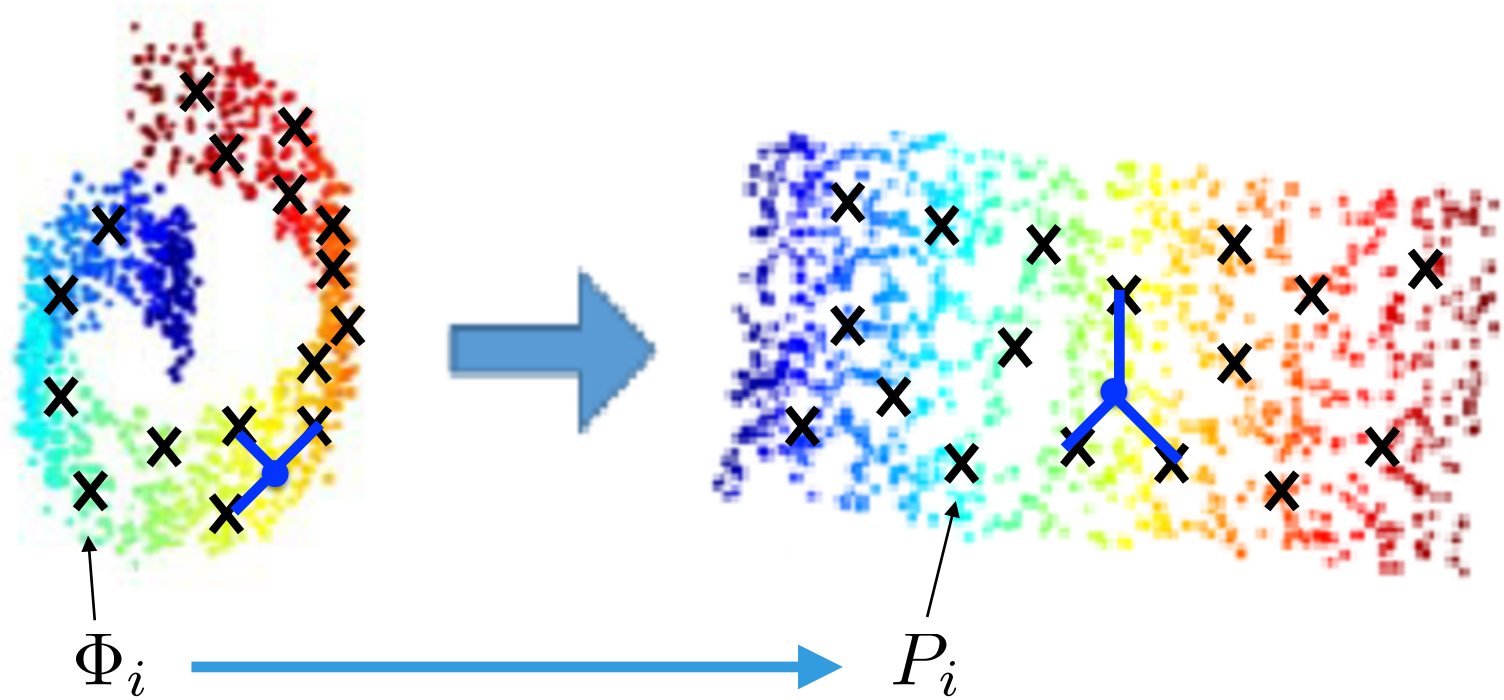
Manifold of facial pose and lighting



Hand-written digits



Local Linear Landmarks (LLL)
(Vladymyrov & Carreira-Perpinán, 2013)

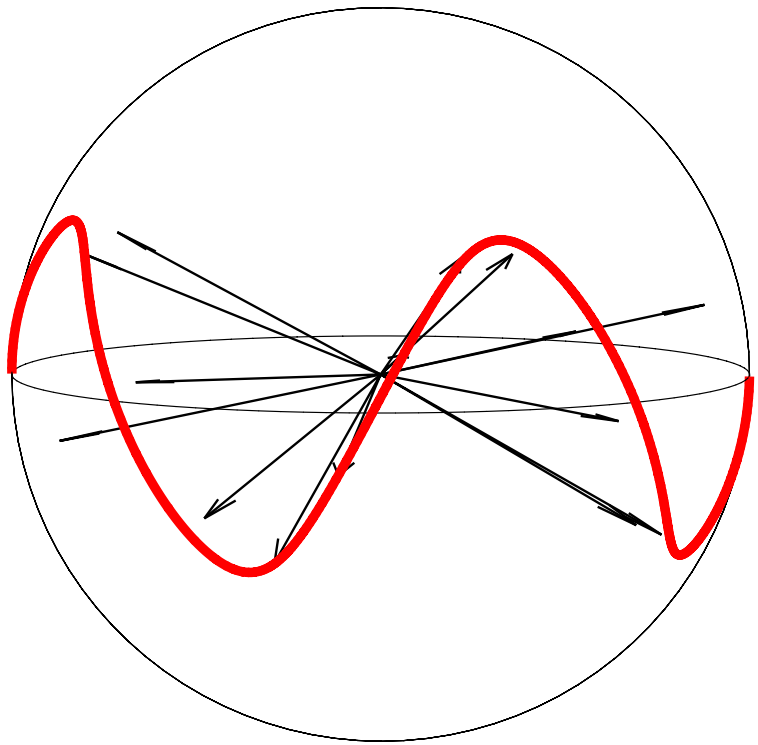


$$x = \Phi\,\alpha + n \longrightarrow y = P\,\alpha$$

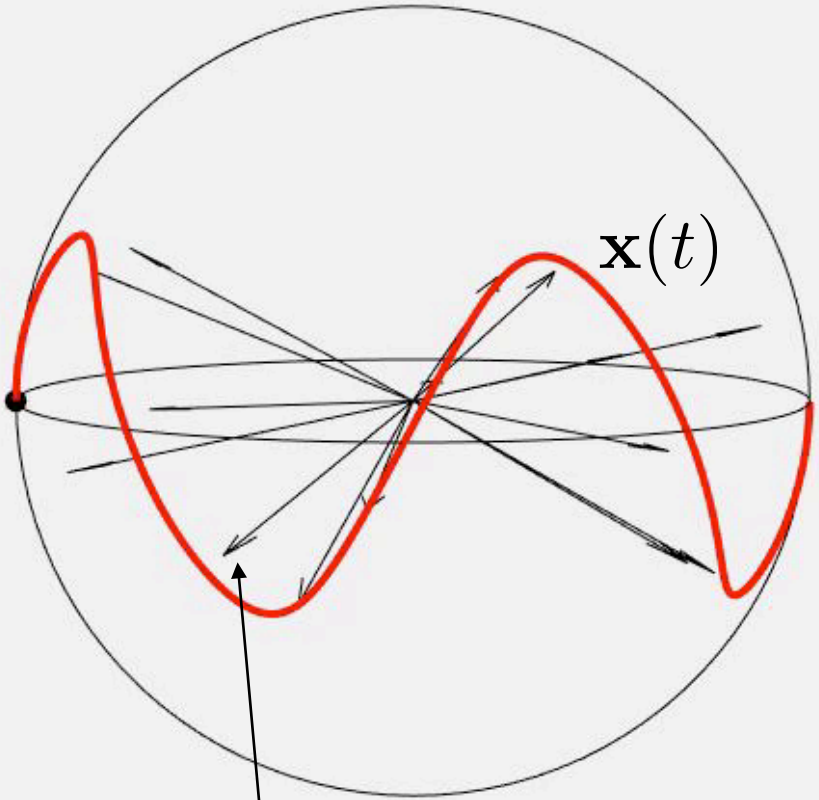
local linear approximation

embedding

Basis functions learned by sparse coding form a locally linear approximation to the manifold of natural images



$$\mathbf{x}(t) = \Phi\,\alpha(t) + \mathbf{n}(t)$$



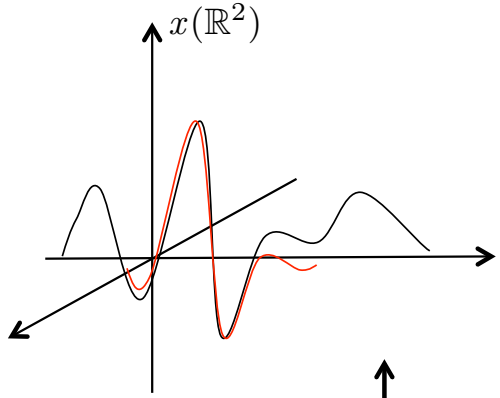
Φ_i → P_i

$$\beta(t) = P\,\alpha(t)$$

Sparse Manifold Transform

(Yubei Chen, Ph.D. thesis; Neurips 2018)

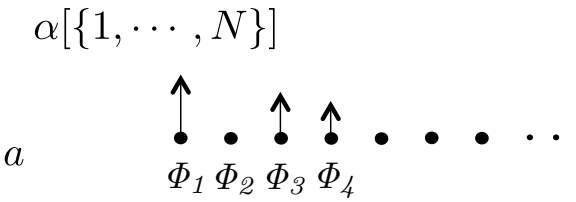
A)



sparse coding

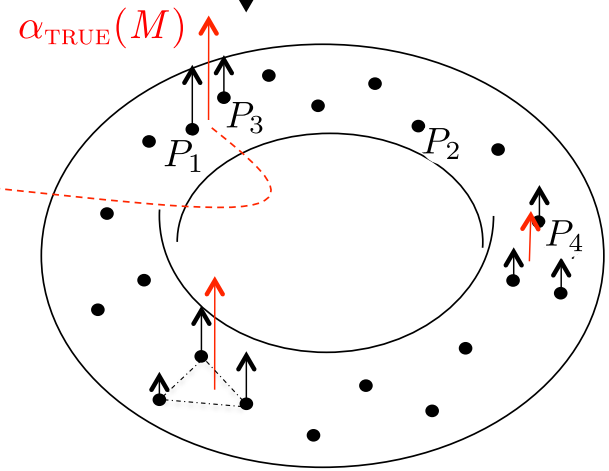
$$\alpha = \arg\min_a \ ||x - \Phi\, a||^2 + \lambda\, a$$

B)



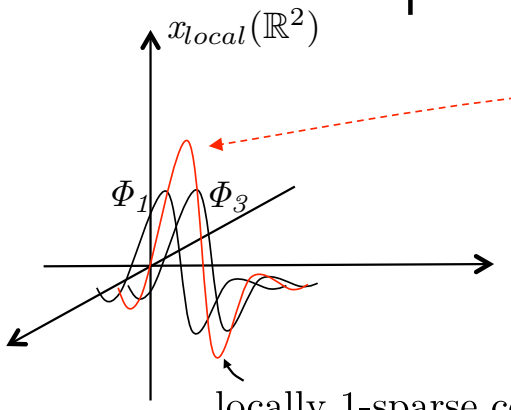
manifold embedding

C)



dictionary local linear interpolation, or "steering"

D)



locally 1-sparse component

k'-sparse composition

Unsupervised learning principles

- Linear Hebbian learning → PCA
- Competitive Hebbian learning → clustering
- Sparse coding → feature learning
- Self-organizing map → topographic maps
- Sparse manifold transform → manifold learning
- Slow feature analysis → invariance