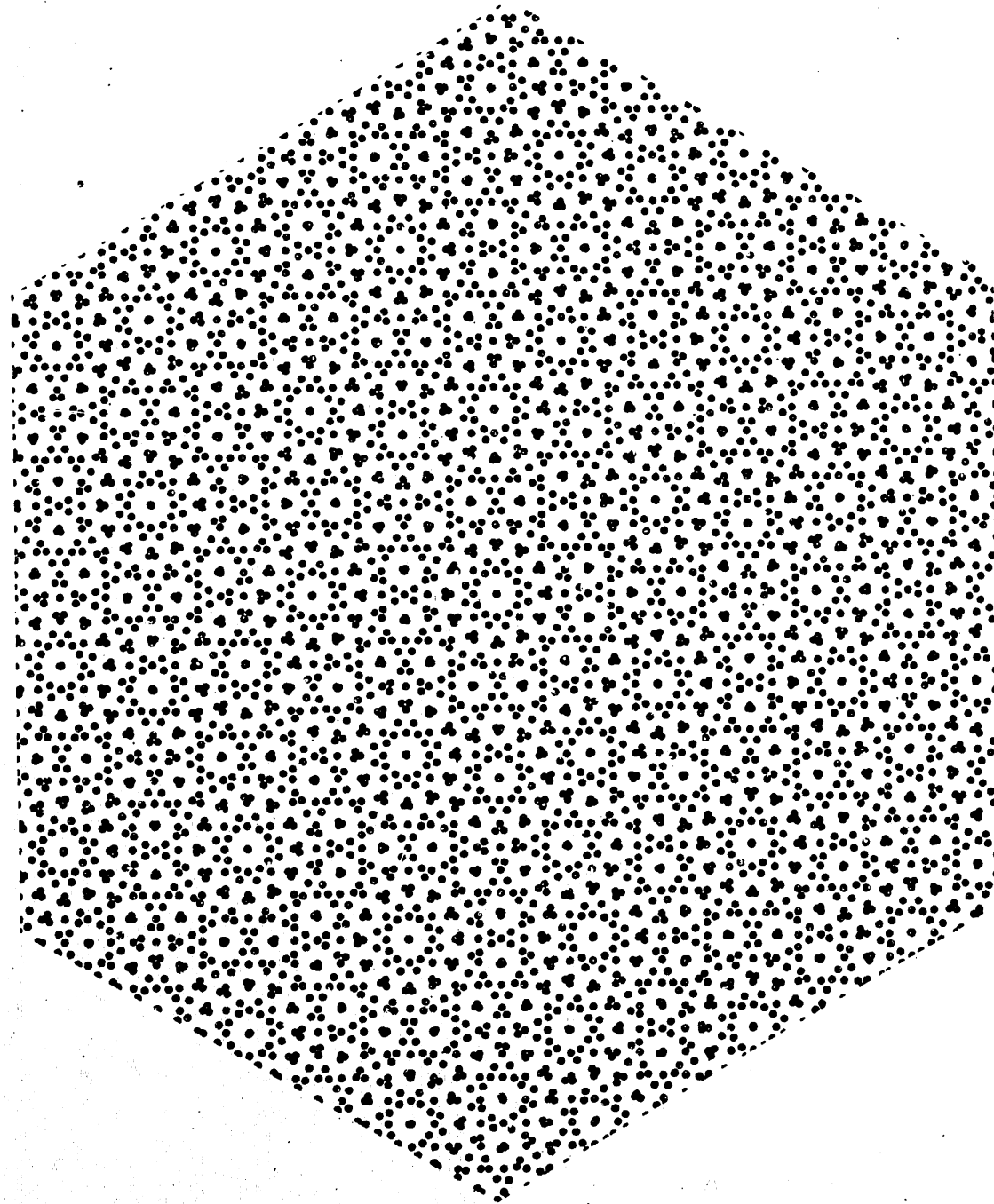


# Attractor neural networks

A 'puzzle picture'  
Dallenbach (1951)

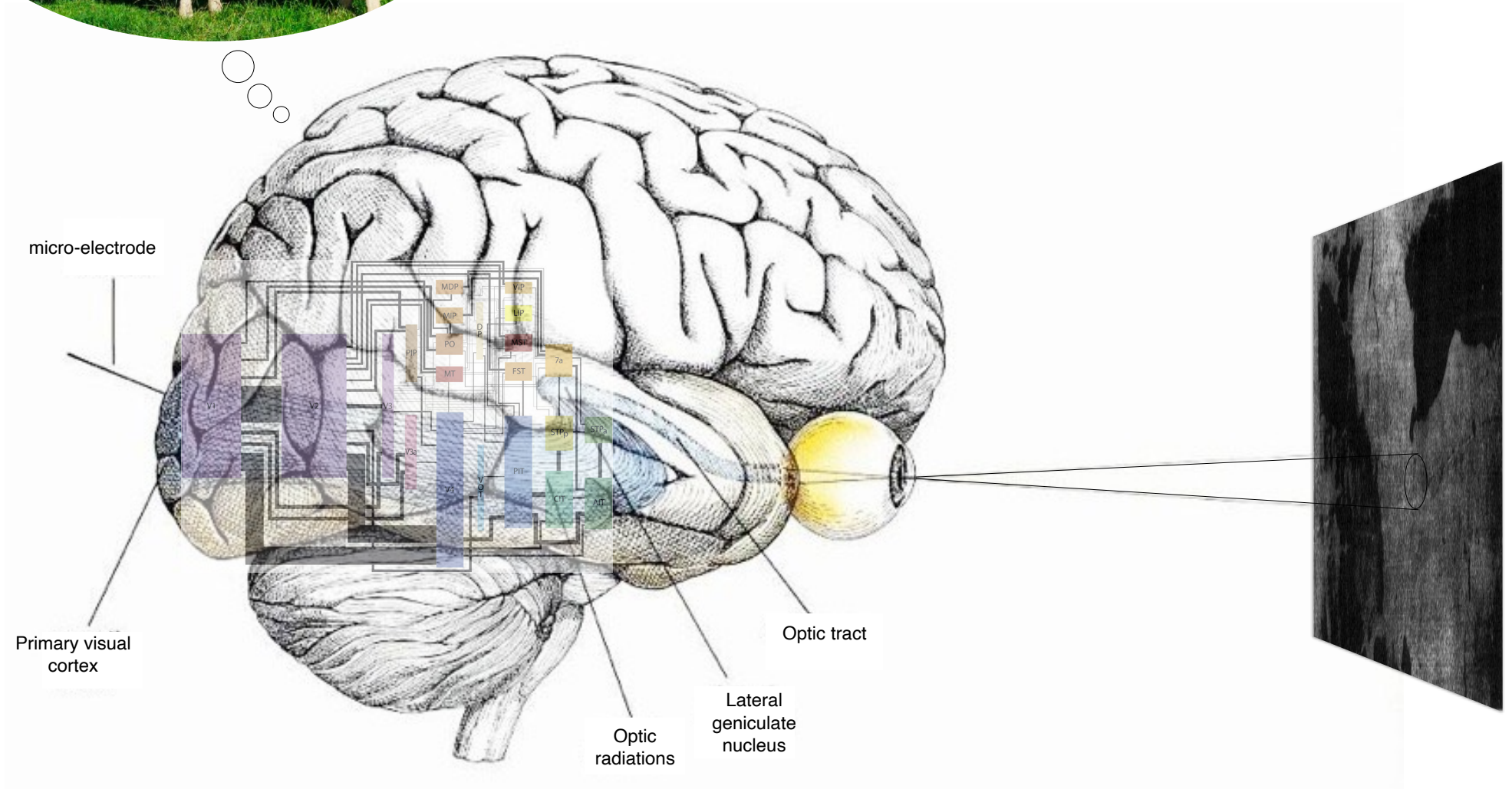
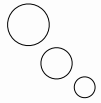


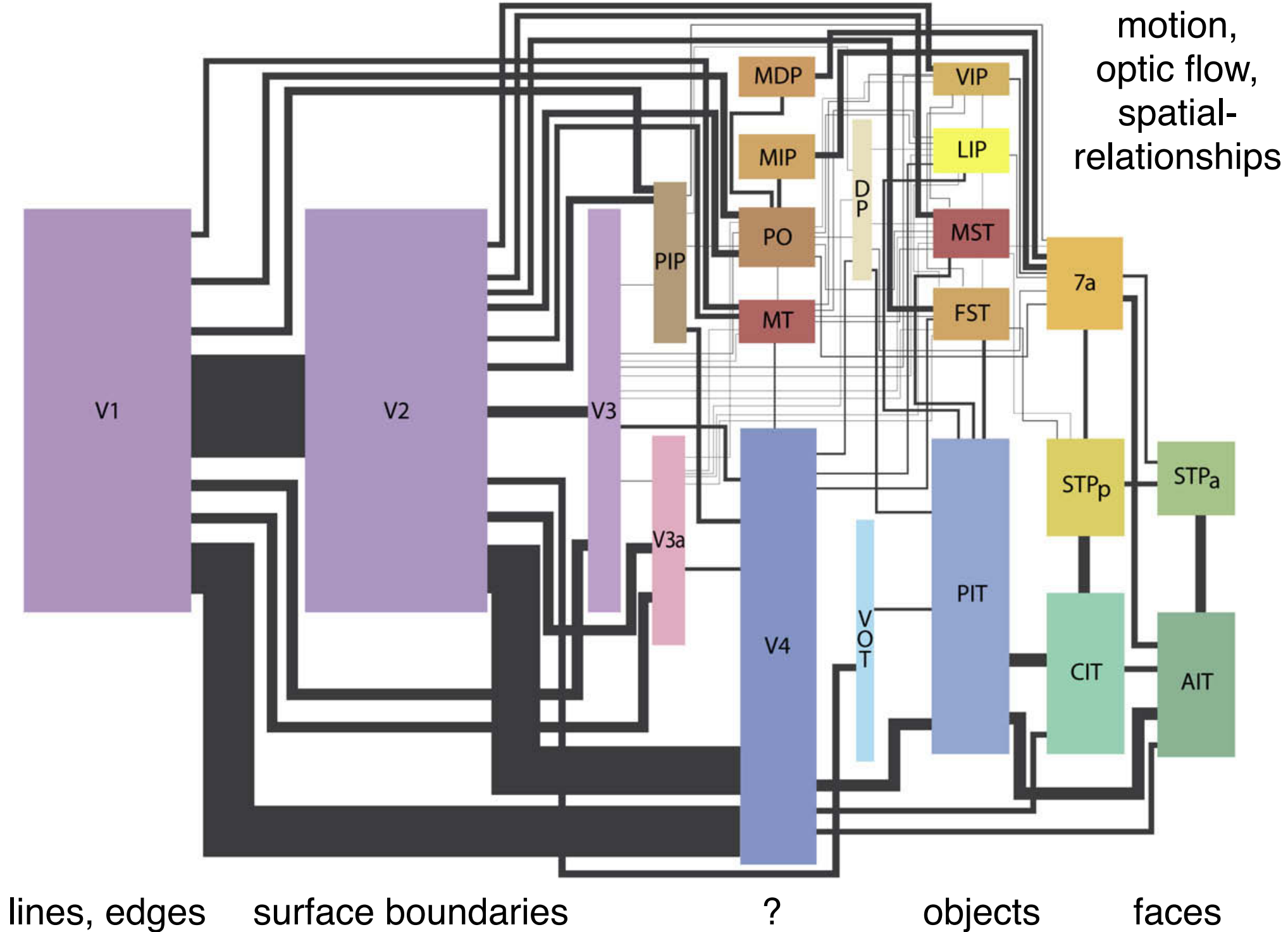


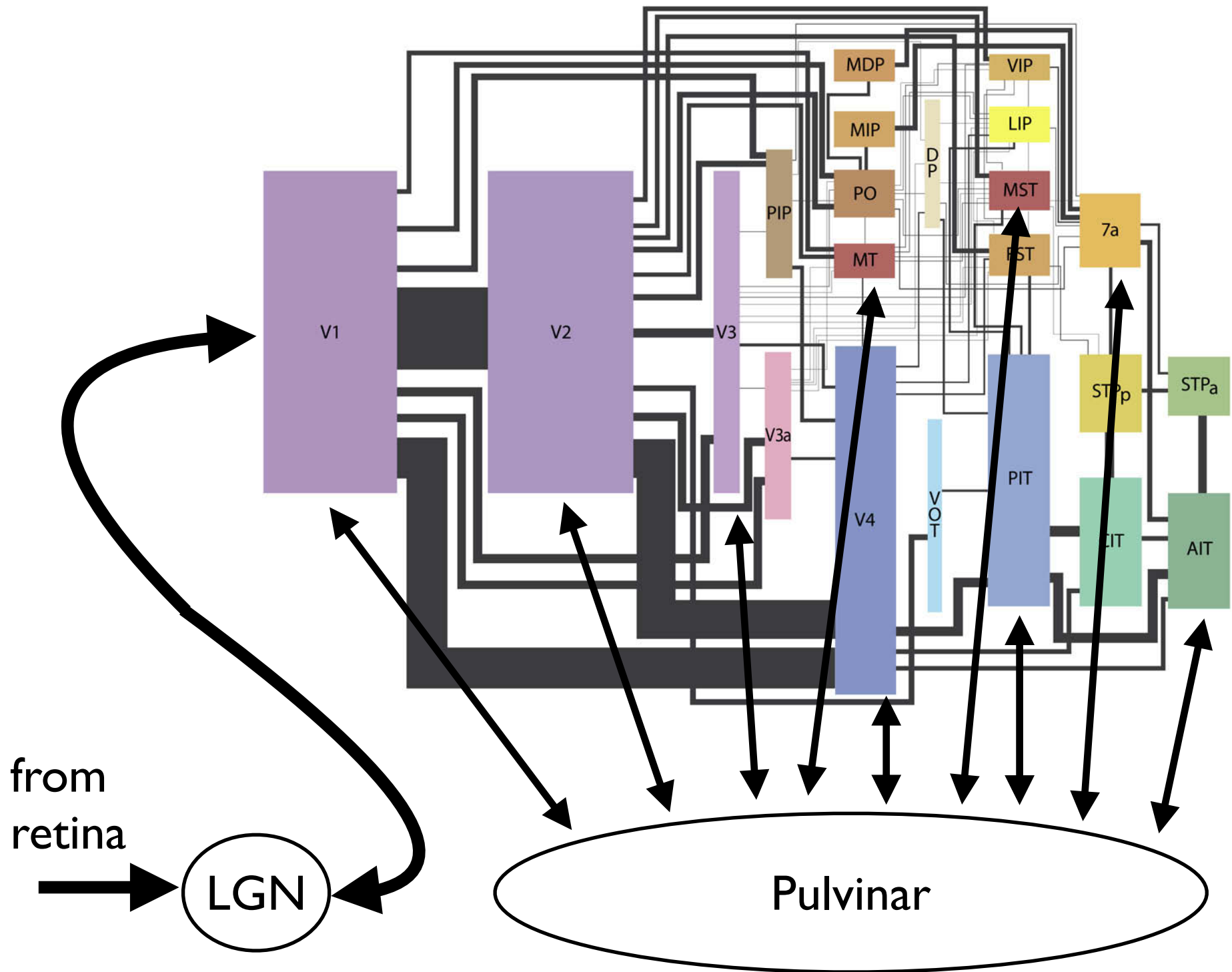
from Marroquin (1976)

# Andy Goldsworthy artwork

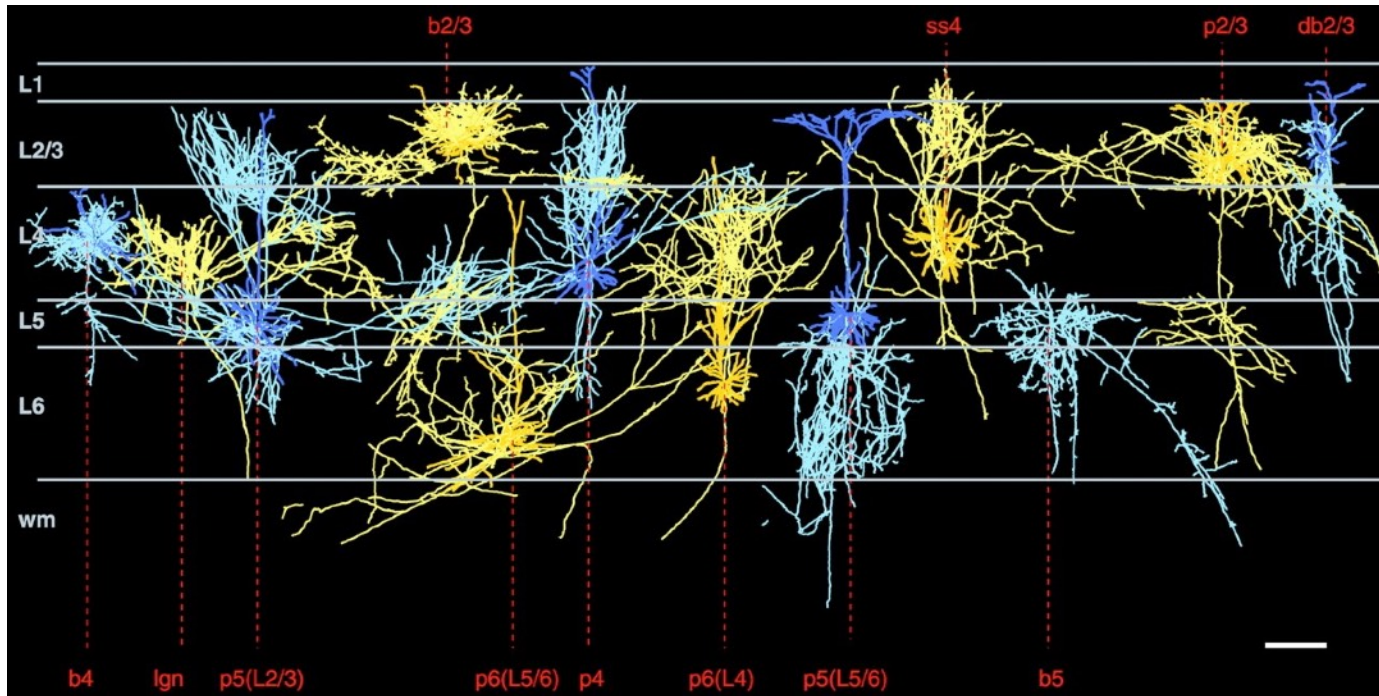




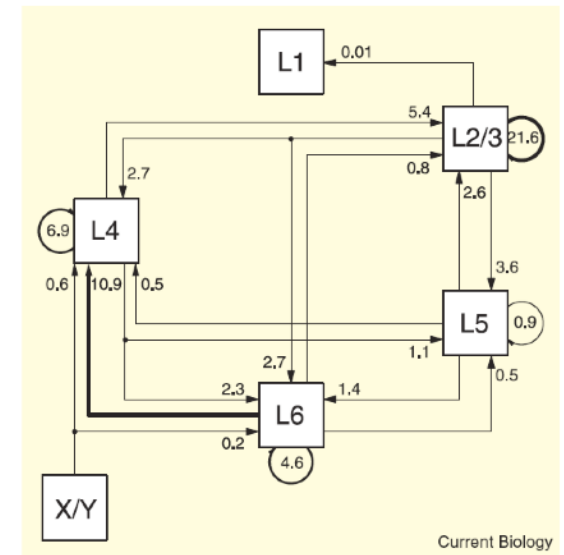




# Recurrent neuronal circuits in the neocortex (Douglas & Martin 2007)

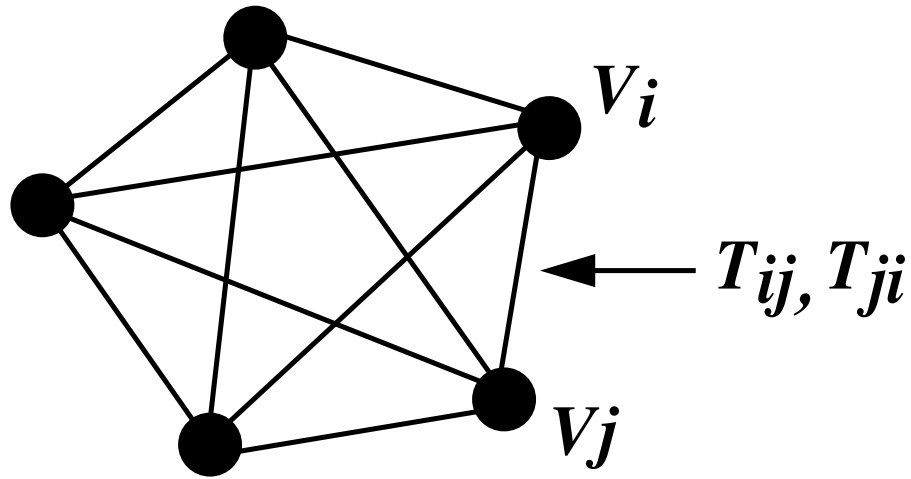


(Binzegger, Douglas & Martin, 2004)



(Douglas and Martin, 2007)





Dynamics:

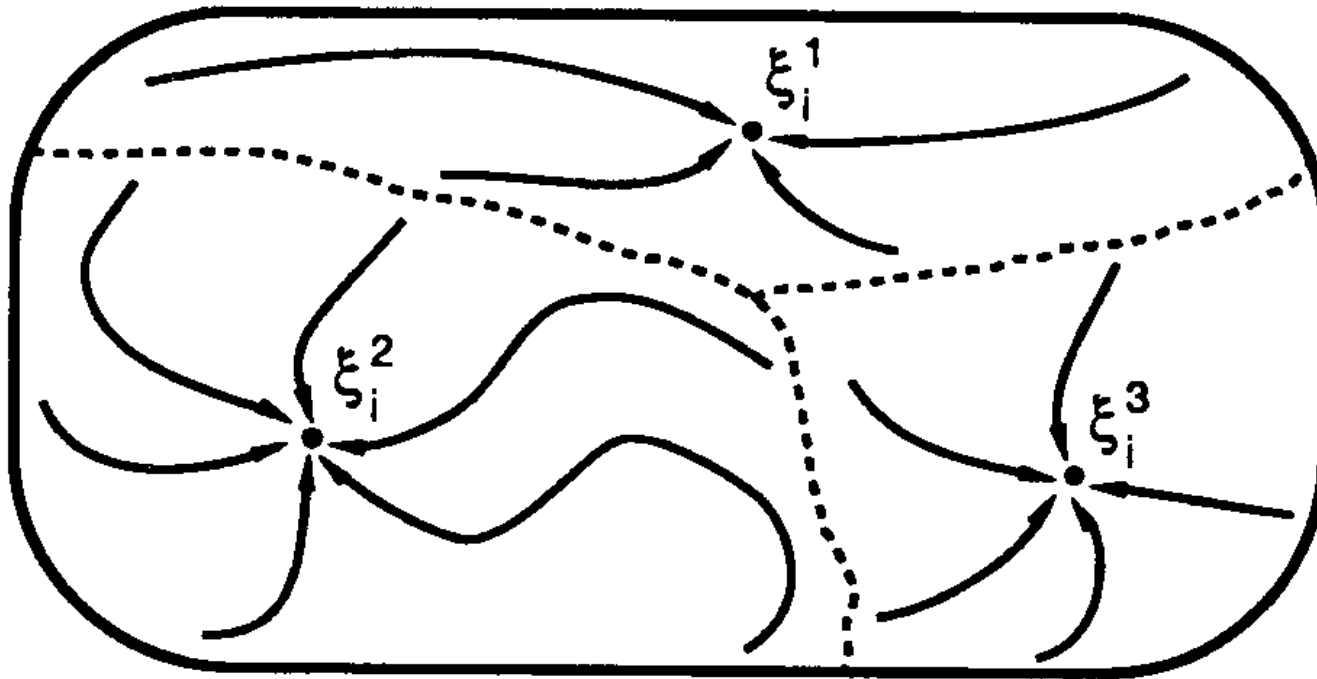
$$U_i = \sum_j T_{ij} V_j$$

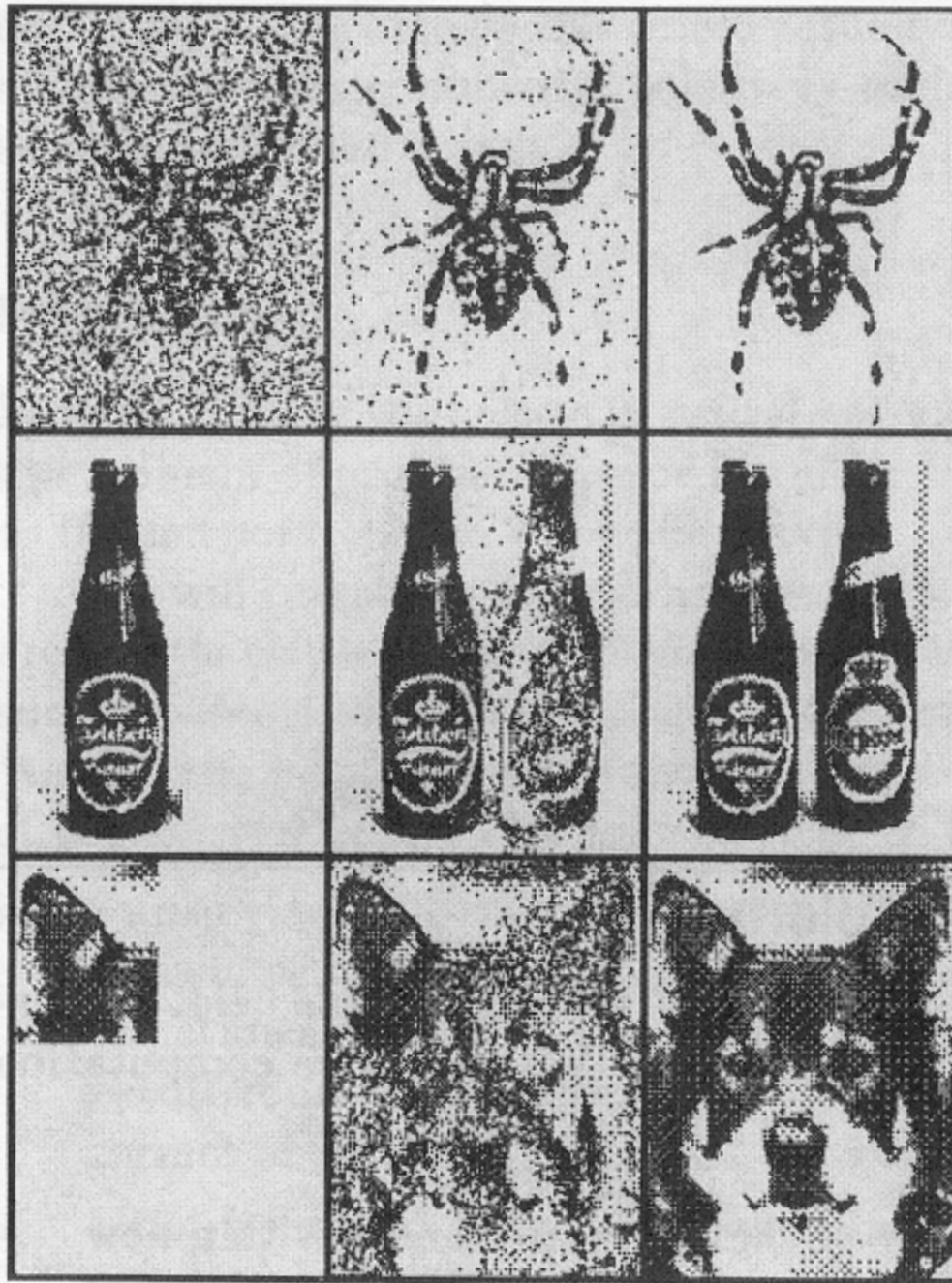
$$V_i = \text{sign}(U_i)$$

Energy function:

$$E = -\frac{1}{2} \sum_{i,j \neq i} T_{ij} V_i V_j$$

# Basins of attraction





input  $\longrightarrow$  recall

# Outer-product (Hebb) rule

$$\begin{aligned} T_{ij} &= \sum_{\alpha} P_i^{(\alpha)} P_j^{(\alpha)} \\ &= P_i^{(1)} P_j^{(1)} + P_i^{(2)} P_j^{(2)} + P_i^{(3)} P_j^{(3)} + \dots \end{aligned}$$

or 
$$\mathbf{T} = \mathbf{P}^{(1)} \mathbf{P}^{(1)T} + \mathbf{P}^{(2)} \mathbf{P}^{(2)T} + \mathbf{P}^{(3)} \mathbf{P}^{(3)T} + \dots$$

Thus

$$\begin{aligned} \mathbf{U} &\cong (\mathbf{P}^{(1)} \mathbf{P}^{(1)T} + \mathbf{P}^{(2)} \mathbf{P}^{(2)T} + \mathbf{P}^{(3)} \mathbf{P}^{(3)T} + \dots) \mathbf{V} \\ &= \mathbf{P}^{(1)} (\mathbf{P}^{(1)} \cdot \mathbf{V}) + \mathbf{P}^{(2)} (\mathbf{P}^{(2)} \cdot \mathbf{V}) + \mathbf{P}^{(3)} (\mathbf{P}^{(3)} \cdot \mathbf{V}) + \dots \end{aligned}$$

$$\mathbf{V} = \text{sgn}(\mathbf{U})$$

# Capacity vs. error rate

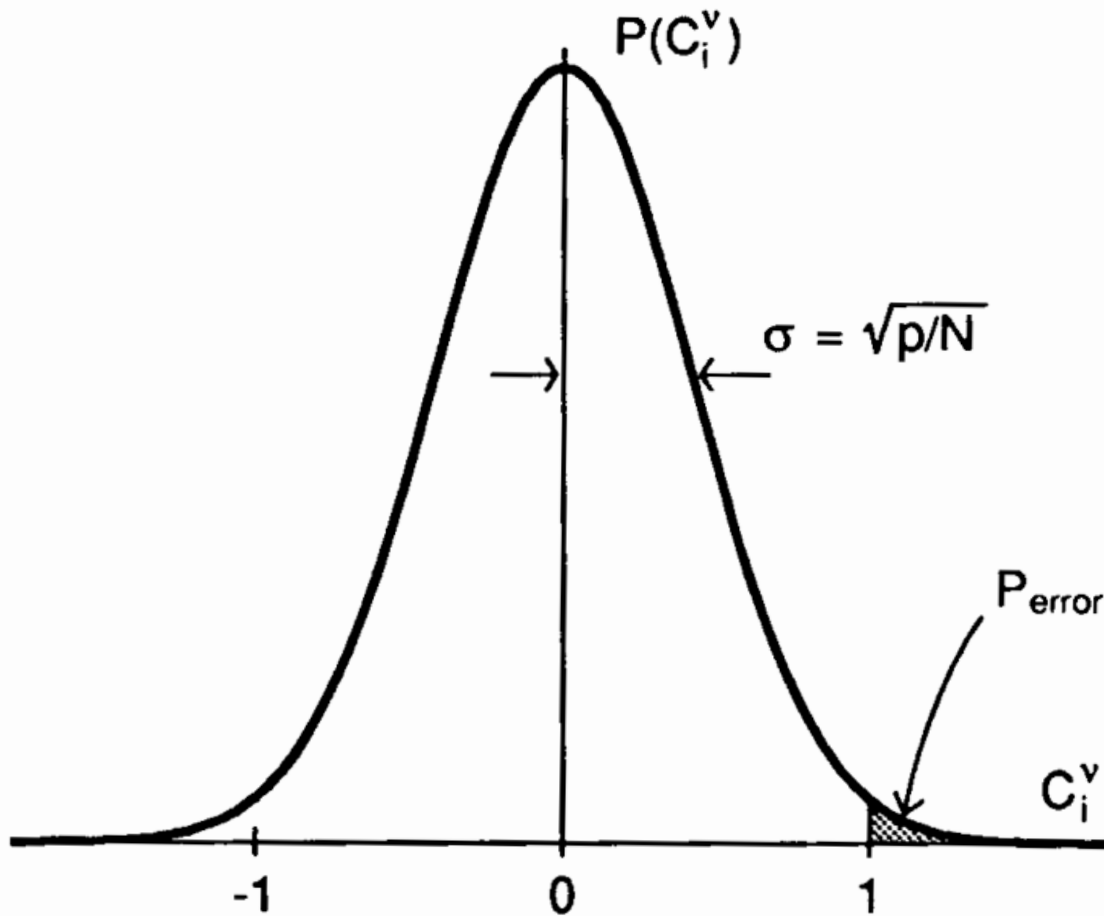
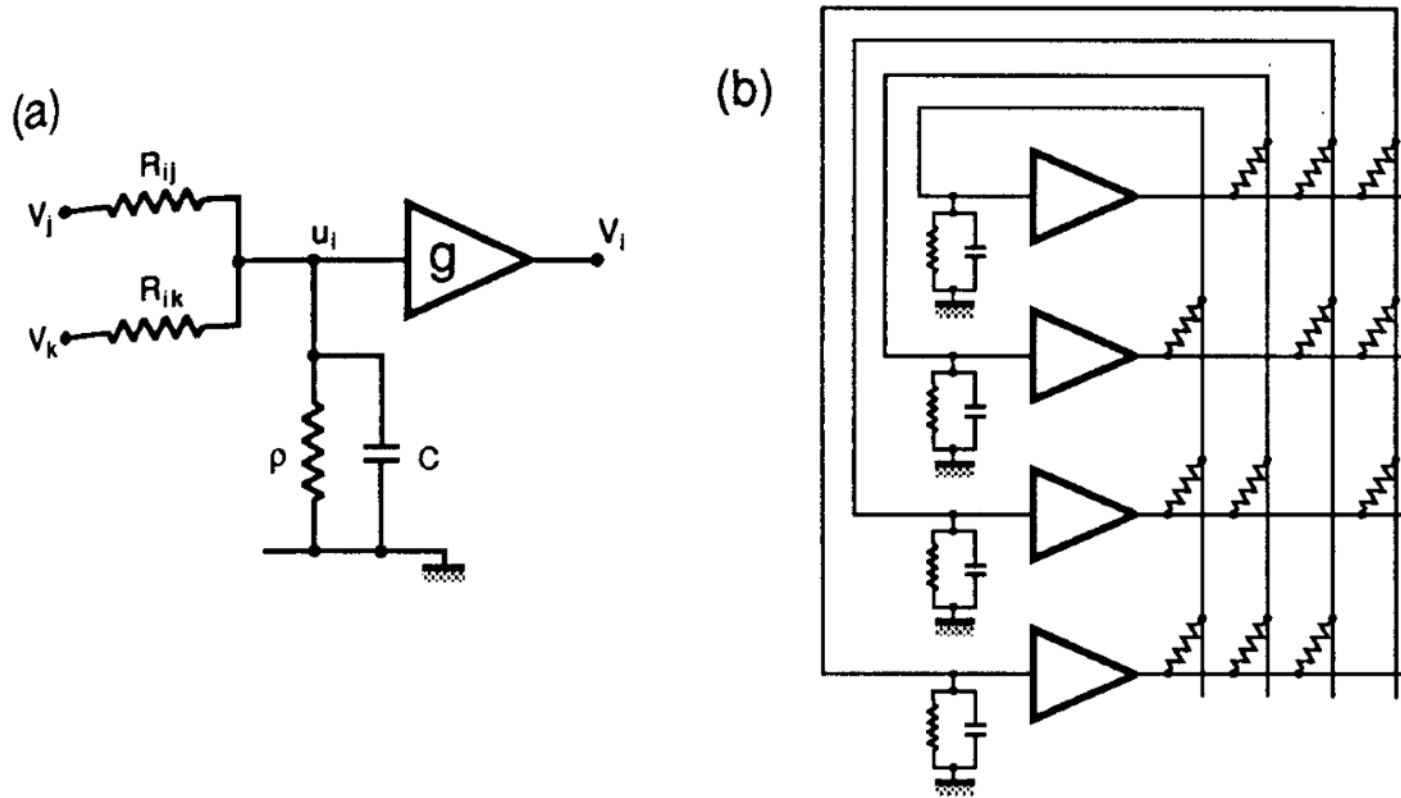


TABLE 2.1 Capacities

$P_{\text{error}}$	$p_{\text{max}}/N$
0.001	0.105
0.0036	0.138
0.01	0.185
0.05	0.37
0.1	0.61

# Hopfield network with analog units

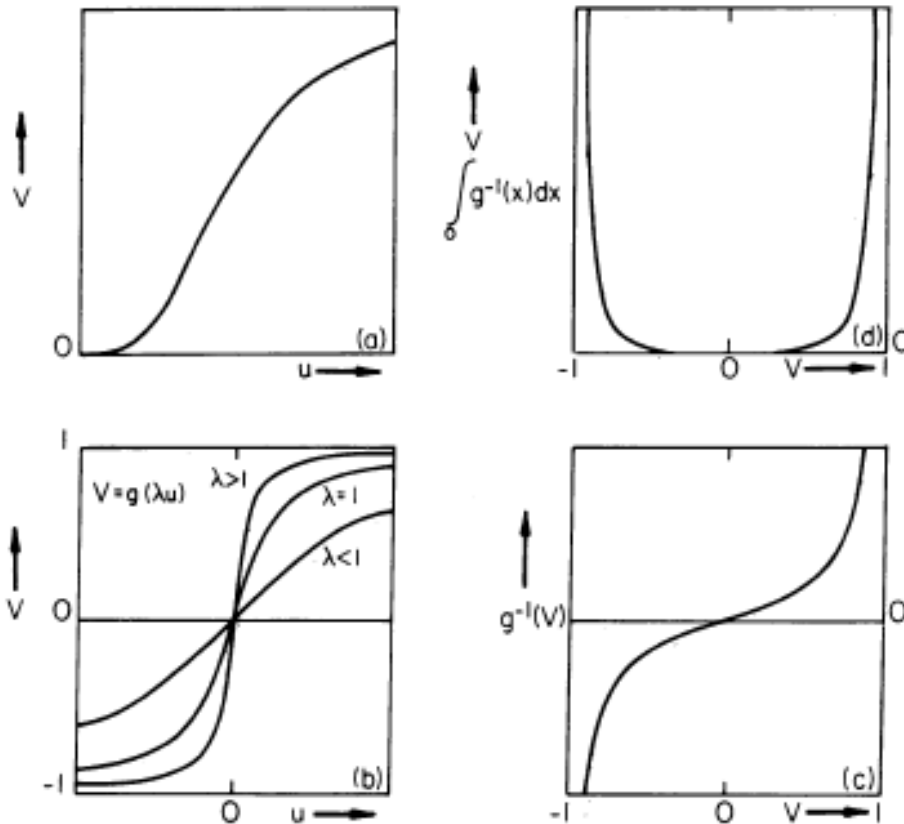


$$\tau \dot{u}_i + u_i = \sum_{j \neq i} T_{ij} V_j + I_i$$

$$V_i = g(u_i)$$

# Lyapunov function

$$E = -\frac{1}{2} \sum_i \sum_{j \neq i} T_{ij} V_i V_j + \sum_i \int_0^{V_i} g^{-1}(V) dV - \sum_i V_i I_i$$



# From Lyapunov function to dynamics

$$E = -\frac{1}{2} \sum_i \sum_{j \neq i} T_{ij} V_i V_j + \sum_i \int_0^{V_i} g^{-1}(V) dV - \sum_i V_i I_i$$

$$\frac{\partial E}{\partial V_k} = - \sum_{j \neq k} T_{kj} V_j + g^{-1}(V_k) - I_k$$

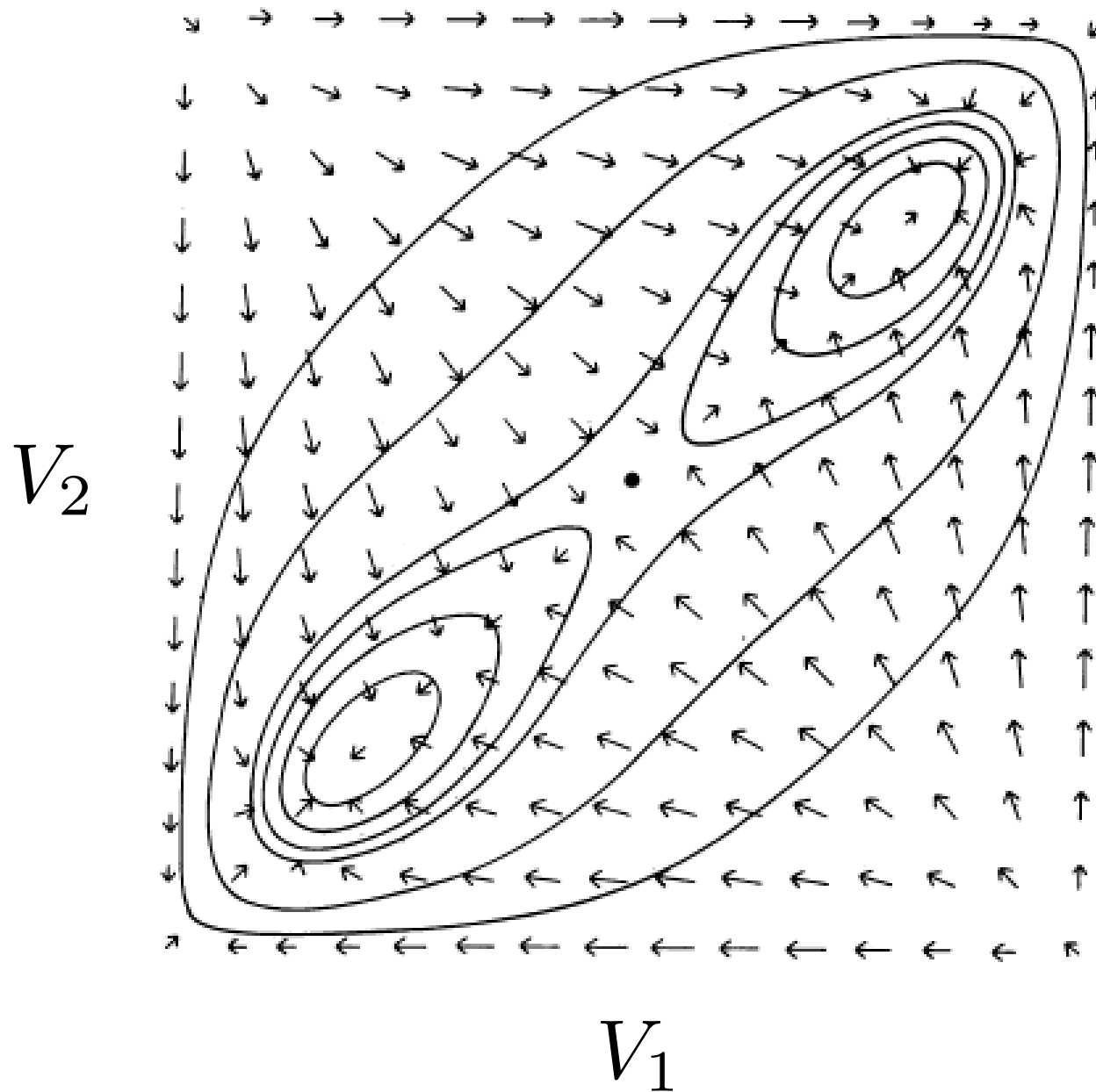
**Let**  $u_i = g^{-1}(V_i) \Rightarrow V_i = g(u_i)$

$$\dot{u}_i \propto -\frac{\partial E}{\partial V_i} = \sum_{j \neq i} T_{ij} V_j + I_i - u_i$$

**Thus**  $\dot{E} = \frac{\partial E}{\partial V} \frac{\partial V}{\partial u} \dot{u} < 0$

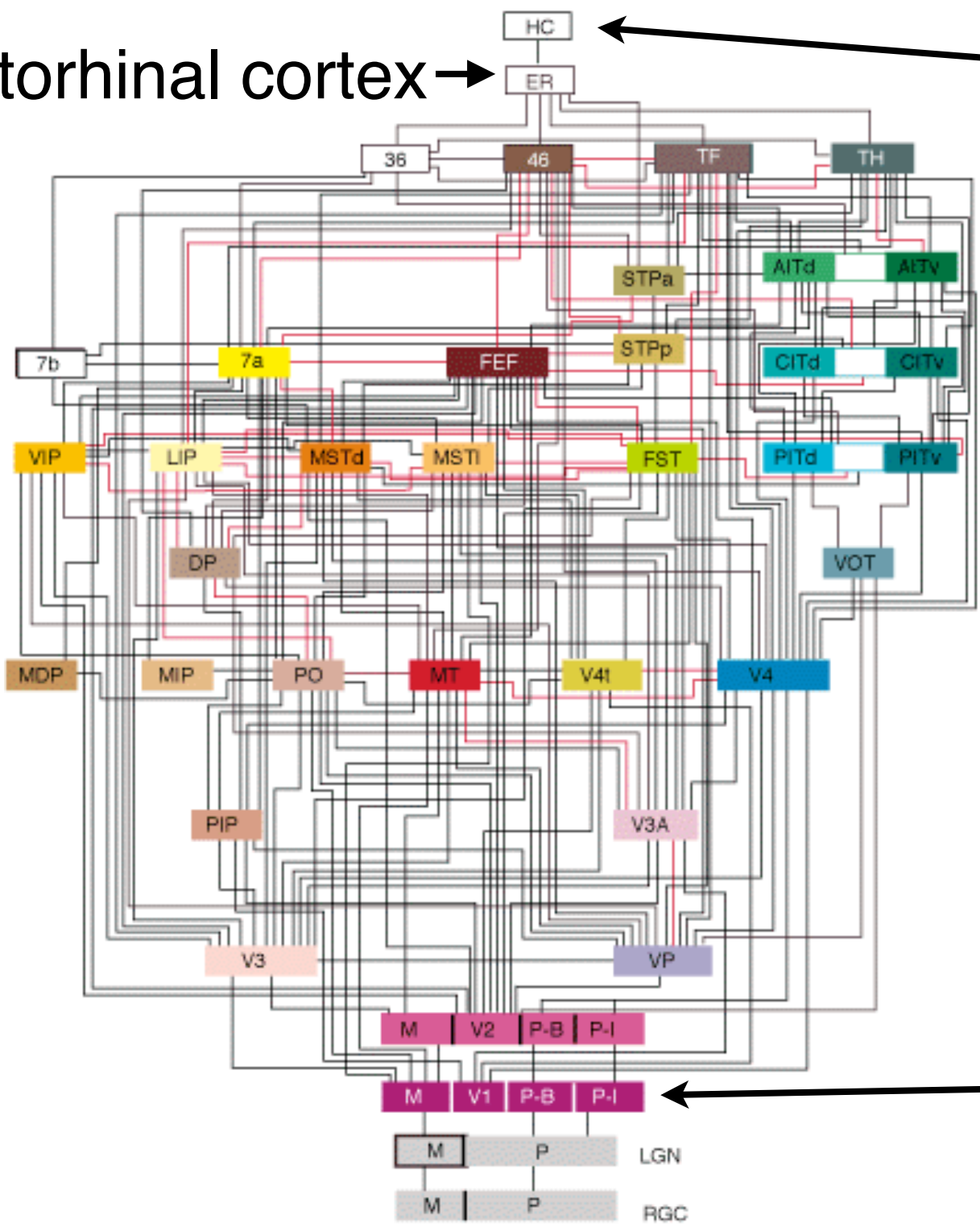


# State space



Entorhinal cortex →

Hippocampus ←



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 ?  
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 .  
 .  
 .  
 VI

←

# Place cells, grid cells, head-direction cells and continuous attractor neural networks

