Computing with high-dimensional vectors

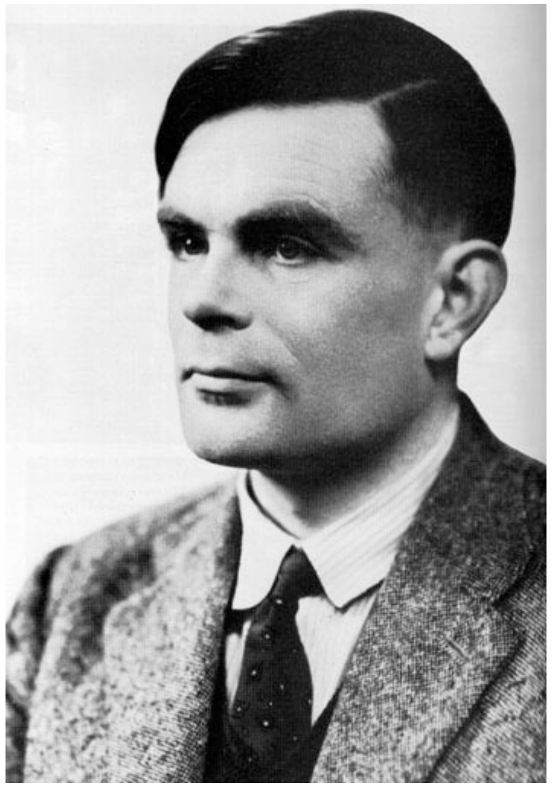
aka

Holographic Reduced Representation (HRR)

Vector Symbolic Architecture (VSA)

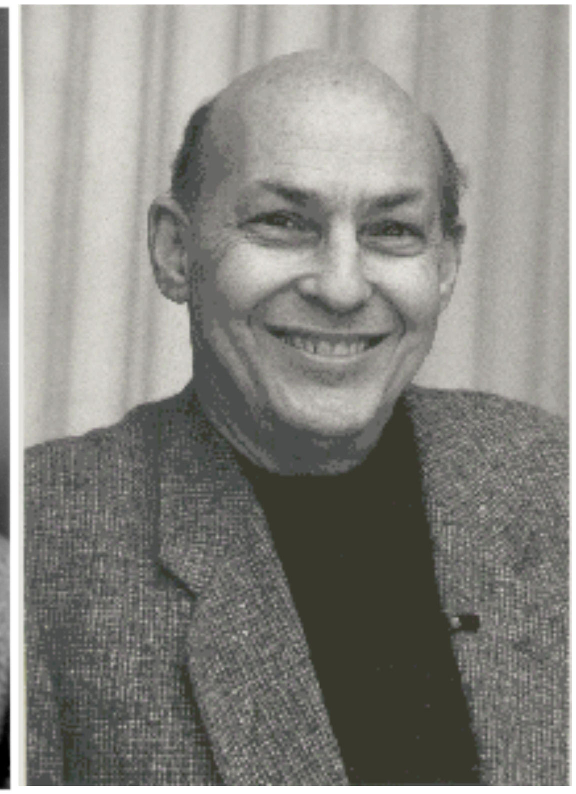
Hyperdimensional Computing (HDC)

Artificial Intelligence

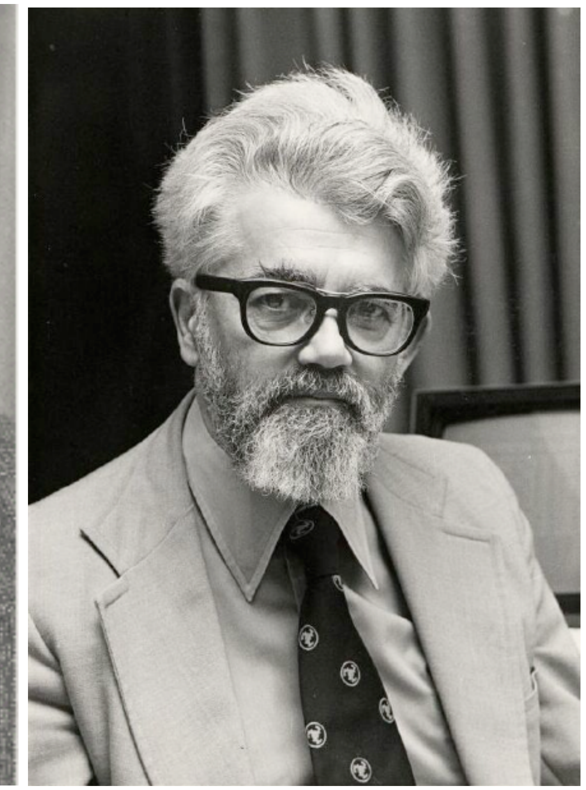


Alan Turing

John von Neumann



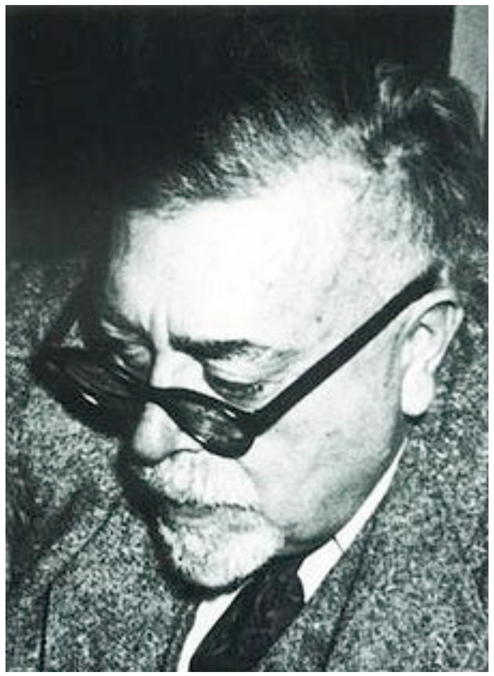
Marvin Minsky



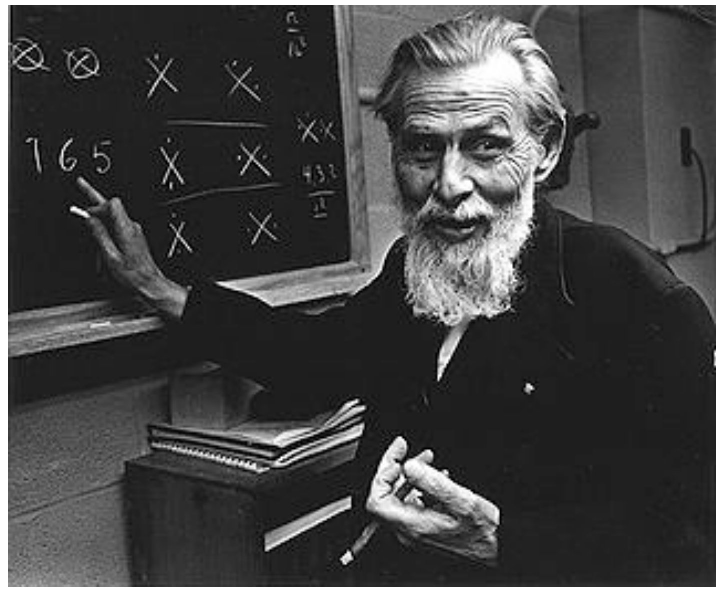
John McCarthy

Among the most challenging scientific questions of our time are the corresponding analytic and synthetic problems: How does the brain function? Can we design a machine which will simulate a brain?
-- *Automata Studies*, 1956

Cybernetics/neural networks



Norbert Wiener



Warren McCulloch & Walter Pitts

Frank Rosenblatt

“The theory reported here clearly demonstrates the feasibility and fruitfulness of a quantitative statistical approach to the organization of cognitive systems. By the study of systems such as the perceptron, it is hoped that those fundamental laws of organization which are common to all information handling systems, machines and men included, may eventually be understood.” -- Frank Rosenblatt

The Perceptron: A Probabilistic Model for Information Storage and Organization in the Brain. In, *Psychological Review*, Vol. 65, No. 6, pp. 386-408, November, 1958.

Single neuron recording ⇒ Single neuron thinking

What the Frog's Eye Tells the Frog's Brain*

J. Y. LETTVIN†, H. R. MATURANA‡, W. S. McCULLOCH||, SENIOR MEMBER, IRE, AND W. H. PITTS||

Perception, 1972, volume 1, pages 371–394

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Single units and sensation: A neuron doctrine for perceptual psychology?

H B Barlow
Department of Physiology–Anatomy, University of California, Berkeley, California 94720
Received 6 December 1972

Abstract. The problem discussed is the relationship between the firing of single neurons in sensory pathways and subjectively experienced sensations. The conclusions are formulated as the following five dogmas:

1. To understand nervous function one needs to look at interactions at a cellular level, rather than either a more macroscopic or microscopic level, because behaviour depends upon the organized pattern of these intercellular interactions.
2. The sensory system is organized to achieve as complete a representation of the sensory stimulus as possible with the minimum number of active neurons.
3. Trigger features of sensory neurons are matched to redundant patterns of stimulation by experience as well as by developmental processes.
4. Perception corresponds to the activity of a small selection from the very numerous high-level neurons, each of which corresponds to a pattern of external events of the order of complexity of the events symbolized by a word.
5. High impulse frequency in such neurons corresponds to high certainty that the trigger feature is present.

The development of the concepts leading up to these speculative dogmas, their experimental basis, and some of their limitations are discussed.

Holographic Reduced Representations	Vector Symbolic Architectures	Hyperdimensional Computing
		
Tony Plate	Ross Gayler	Pentti Kanerva

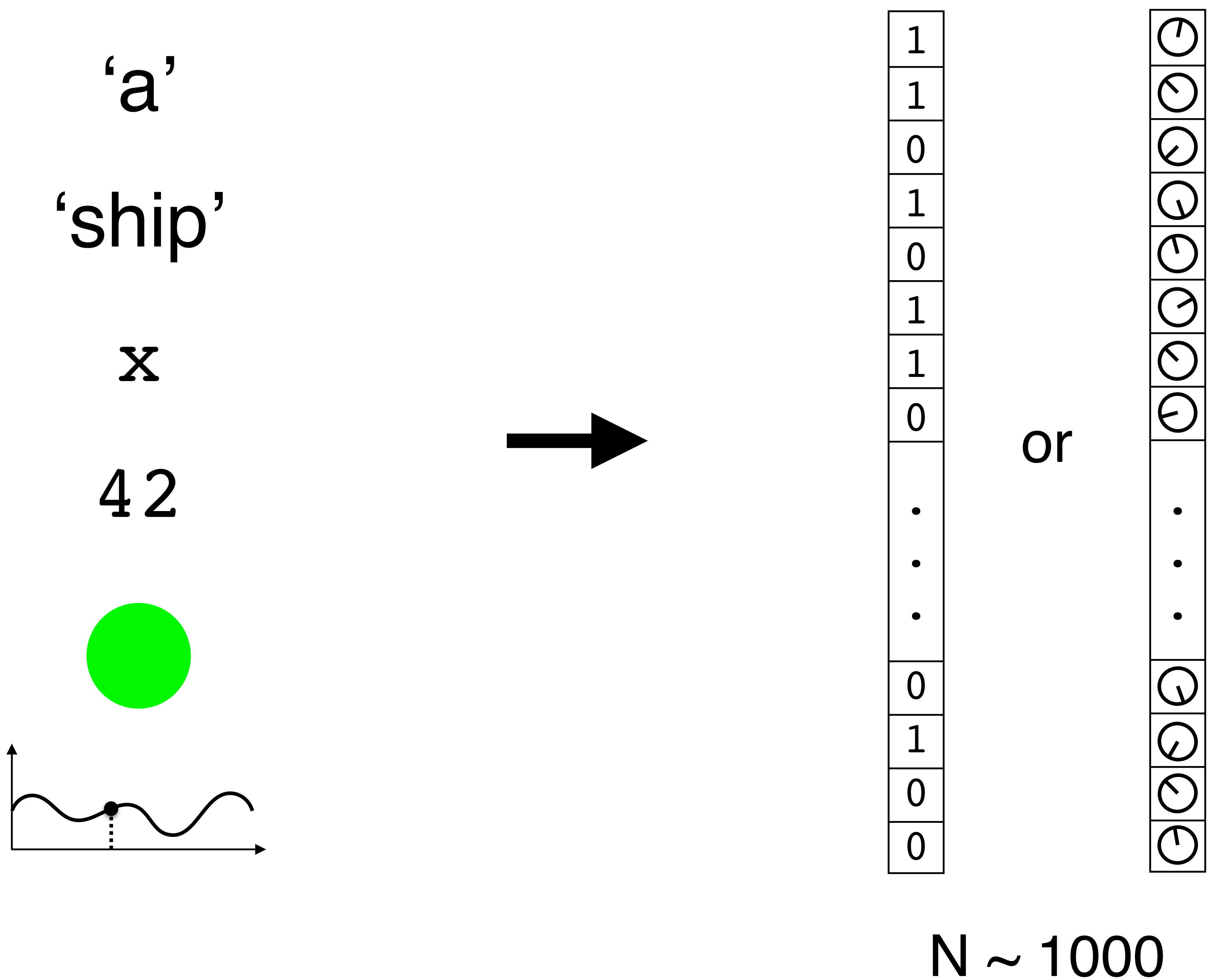
Plate, T. A. (1995). Holographic reduced representations. *IEEE Transactions on Neural networks, 6(3)*, 623-641.

Gayler, R. W. (2004). Vector symbolic architectures answer Jackendoff's challenges for cognitive neuroscience. arXiv:cs/0412059.

Kanerva P (2009) Hyperdimensional Computing: An Introduction to Computing in Distributed Representation with High-Dimensional Random Vectors. *Cognitive Computing, 1*: 139-159.

- Everything represented as a high-dimensional vector.
- Algebra over vectors (instead of numbers).

Computing with High-Dimensional Vectors
(aka ‘HD computing’)



HDC Algebra

Set or **Bundling**	{ a, b, c, … }	**Z**[a] + **Z**[b] + **Z**[c] + ….
Key-Value binding	x ← a	**K**[x] ⊙ **V**[a]
Spatial relations, transformation	'object a' at 'position x' shift by y	**S** = **O**[a] ⊙ **Z**[x] **S_new** = **Z**[y] ⊙ **S**
Sequencing	[a b c …]	**Z**[a] + ρ(**Z**[b]) + ρ^2(**Z**[c]) + ….

Kanerva P (2009) Hyperdimensional Computing: An Introduction to Computing in Distributed Representation with High-Dimensional Random Vectors. *Cognitive Computing, 1*.

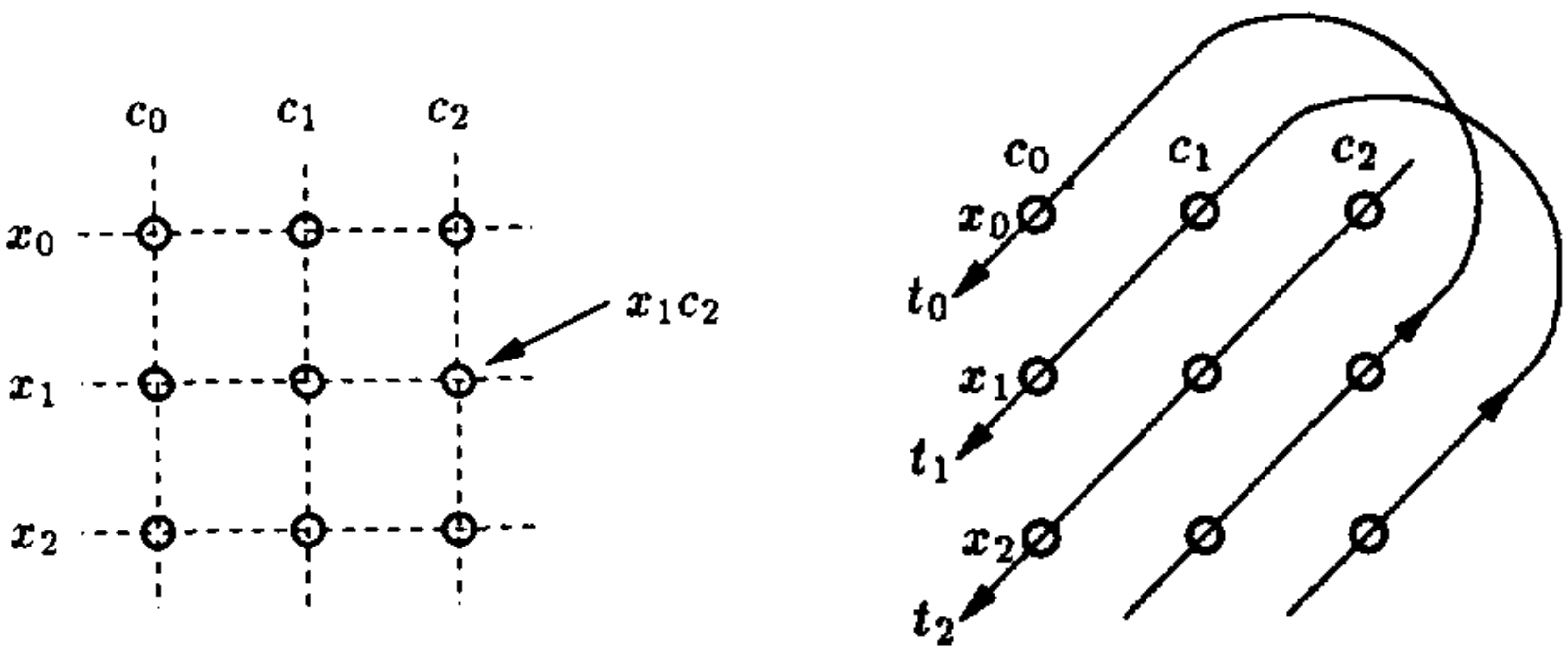
Kleyko, D., Davies, M., Frady, E. P., Kanerva, P., Kent, S. J., Olshausen, B. A., ... & Sommer, F. T. (2022). Vector symbolic architectures as a computing framework for emerging hardware. *Proceedings of the IEEE, 110*.

Menon, A … Rabaey, J (2022) On the Role of Hyperdimensional Computing for Behavioral Prioritization in Reactive Robot Navigation Tasks. *ICRA 2022*.

	Traditional computing/AI	Neural nets	HD computing
Symbolic computing with variables and binding	✓	✗	✓
Distributed representation	✗	✓	✓
Learn from data	✗	✓	✓
Robust (error-correcting)	✗	?	✓
Transparent	✓	✗	✓

Holographic Reduced Representations

Tony A. Plate



$$\mathbf{t} = \mathbf{c} \circledast \mathbf{x}$$

$$\begin{aligned} t_0 &= c_0x_0 + c_2x_1 + c_1x_2 \\ t_1 &= c_1x_0 + c_0x_1 + c_2x_2 \\ t_2 &= c_2x_0 + c_1x_1 + c_0x_2 \end{aligned}$$

$$t_j = \sum_{k=0}^{n-1} c_k x_{j-k}$$

for $j = 0$ to $n-1$
(Subscripts are modulo-n)

Binding via circular convolution

$$\tilde{\mathbf{t}} = \tilde{\mathbf{c}} \circledast \tilde{\mathbf{x}}.$$

Unbinding via circular correlation

$$\tilde{\mathbf{y}} = \tilde{\mathbf{c}} \oplus \tilde{\mathbf{t}}$$

$$\tilde{\mathbf{y}} \approx \tilde{\mathbf{x}}$$

Composition via superposition

$$\tilde{\mathbf{t}} = \tilde{\mathbf{c}}_1 \circledast \tilde{\mathbf{x}}_1 + \tilde{\mathbf{c}}_2 \circledast \tilde{\mathbf{x}}_2$$

$$\tilde{\mathbf{c}}_1 \oplus \tilde{\mathbf{t}} = \tilde{\mathbf{c}}_1 \oplus \tilde{\mathbf{c}}_1 \circledast \tilde{\mathbf{x}}_1 + \tilde{\mathbf{c}}_1 \oplus \tilde{\mathbf{c}}_2 \circledast \tilde{\mathbf{x}}_2$$

$$\approx \tilde{\mathbf{x}}_1 + \text{noise}$$

Variable binding

'X=a', 'Y=b'

$$\tilde{\mathbf{t}} = \tilde{\mathbf{x}} \circledast \tilde{\mathbf{a}} + \tilde{\mathbf{y}} \circledast \tilde{\mathbf{b}}.$$

Language

"Mark ate the fish."

$$\tilde{\mathbf{s}}_1 = \mathbf{eat} + \mathbf{agt}_{\mathrm{eat}} \circledast \mathbf{mark} + \mathbf{obj}_{\mathrm{eat}} \circledast \mathbf{the\_fish}.$$

Four examples

- Analogical reasoning
- Language identification via trigram statistics
- Sequence memory
- Visual scene analysis

Reasoning

What is the dollar of Mexico?

Analogical Mapping with Multiplication by Hypervector

What is the Dollar of Mexico?

Encoding of **USA** and **MEX**ico: **Na**me of country, **Ca**pital city, **Mon**etary unit

USA = Nam*Us + Cap*Dc + Mon*$
MEX = Nam*Mx + Cap*Mc + Mon*P

Pairing up the two--binding

Pair = USA*MEX

Analyzing the pair

Pair = Us*Mx + Dc*Mc + $*P + noise

Literal interpretation of *Dollar of Mexico* produces nonsense:

```
$*MEX = $ * (Nam*Mx + Cap*Mc + Mon*P)
      = $*Nam*Mx + $*Cap*Mc + $*Mon*P
      =  noise   +  noise   +  noise
          (nothing cancels out)
```

However, what in Mexico corresponds to Dollar in USA?

```
$*Pair = $ * (USA*MEX)
       = $ * (Us*Mx + Dc*Mc + $*P + noise)
       = $*Us*Mx + $*Dc*Mc + $*$*P + $*noise
       =  noise  +  noise  +     P +  noise
       = P + noise
       ≈ P
```

Language identification from trigram statistics
(Joshi, Halseth, Kanerva 2017)

Encode a trigram vector for each three-letter sequence A, B, C as

$$ABC = \rho(\rho(A)) * \rho(B) * C = \rho\rho A * \rho B * C$$

Add all trigram vectors of a text into a 10,000-D Profile Vector. For example, the text segment

“the quick brown fox jumped over ...”

gives rise to the following trigram vectors, which are added into the profile for English

Engl += THE + HE# + E#Q + #QU + QUI + UIC + ...

	ell	eng	ita	ces	est	spa	nld	por	lav	lit	ron	pol	fra	bul	deu	dan	fin	hun	swe	slk	slv
ell	987	1	.	.	.	.	3	3	.	.	.	1	.	4	.	.	1	.	.	.	.
eng	2	982	.	4	.	.	1	.	2	.	.	.	6	.	.	1	.	2	.	.	.
ita	.	.	992	.	1	2	.	.	.	.	2	3	.	.	.	.	.	.	.	.	.
ces	1	1	.	940	1	.	.	.	1	1	1	1	.	5	1	.	.	.	.	35	12
est	1	.	.	1	983	.	.	.	3	.	.	.	3	.	1	1	5	1	1	.	.
spa	.	.	6	.	.	946	2	30	8	1	2	.	5	.	.	.	.	.	.	.	.
nld	.	1	.	.	.	.	980	1	.	.	2	1	.	.	5	9	.	.	1	.	.
por	.	1	2	.	.	1	1	991	.	.	.	.	3	1	.	.	.	.	.	.	.
lav	2	.	.	1	.	.	.	2	963	26	.	2	.	2	.	1	.	.	.	1	.
lit	2	.	1	2	1	1	.	2	18	969	.	.	1	.	.	.	.	.	.	1	2
ron	.	.	1	.	.	1	.	2	.	1	987	2	4	2	.	.	.	.	.	.	.
pol	2	1	.	3	1	.	.	.	.	.	.	984	.	4	.	.	.	.	.	4	1
fra	3	.	2	.	.	4	2	1	1	2	1	.	982	.	.	1	.	.	.	1	.
bul	1	.	.	7	.	.	4	.	.	.	.	.	.	984	.	.	.	.	.	3	1
deu	.	2	1	1	.	.	3	.	.	.	.	.	3	.	985	4	.	.	1	.	.
dan	.	2	.	.	.	.	9	.	.	.	.	.	2	.	.	974	.	.	13	.	.
fin	.	.	.	.	4	.	2	.	1	.	.	.	.	.	.	.	993	.	.	.	.
hun	.	.	.	.	.	.	6	1	1	1	.	.	.	.	.	2	.	989	.	.	.
swe	.	1	.	.	.	1	5	.	.	.	4	.	1	.	4	10	.	.	974	.	.
slk	2	.	.	72	.	.	1	.	2	1	4	18	.	6	1	.	.	.	.	881	12
slv	1	.	.	5	2	.	.	1	.	.	1	.	.	6	1	1	.	.	.	.	982

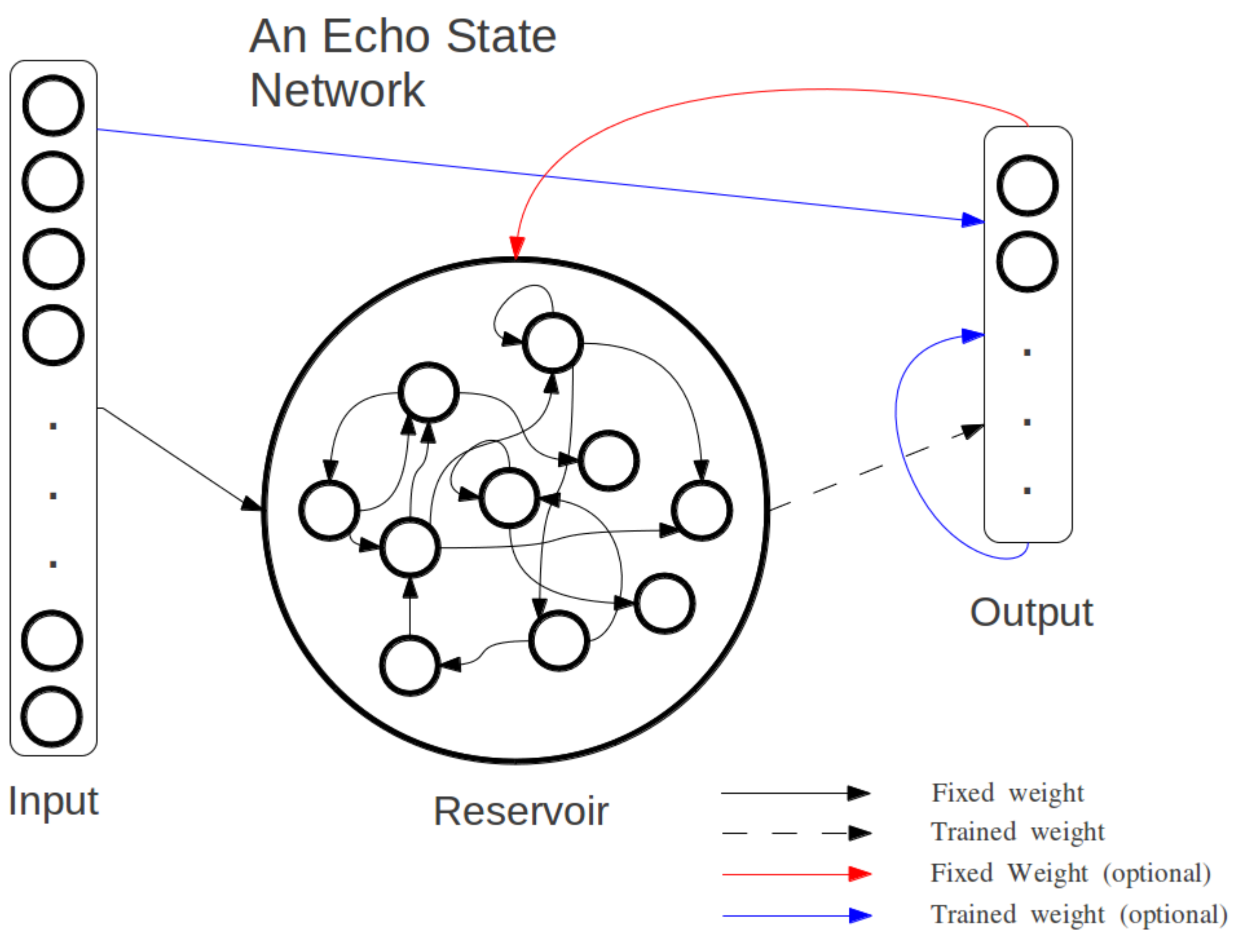
LEGEND: bul = Bulgarian, ces = Czech, dan = Danish, deu = German, ell = Greek, eng = English, est = Estonian, fin = Finnish, fra = French, hun = Hungarian, ita = Italian, lav = Latvian, lit = Lithuanian, nld = Dutch, pol = Polish, por = Portuguese, ron = Romanian, slk = Slovak, slv = Slovene, spa = Spanish, swe = Swedish.

1. Language Vectors: We made 10,000-D language vectors for 21 EU languages from seed vectors representing letters. Projected onto a plane, the languages cluster according to known families:

```
                                        Italian
                                         *    *Romanian
                                        Portuguese
                                         *    *Spanish
    *Slovene                                      *French
*Bulgari *Czech
     *Slovak                                       *English
                           *Greek
   *Polish                      *Lithuanian
                                 *Latvian
                           *Estonian
                     *       *Finnish
                    Hungarian


                                                  *Dutch
                                          *Danish  *German
                                             *Swedish
```

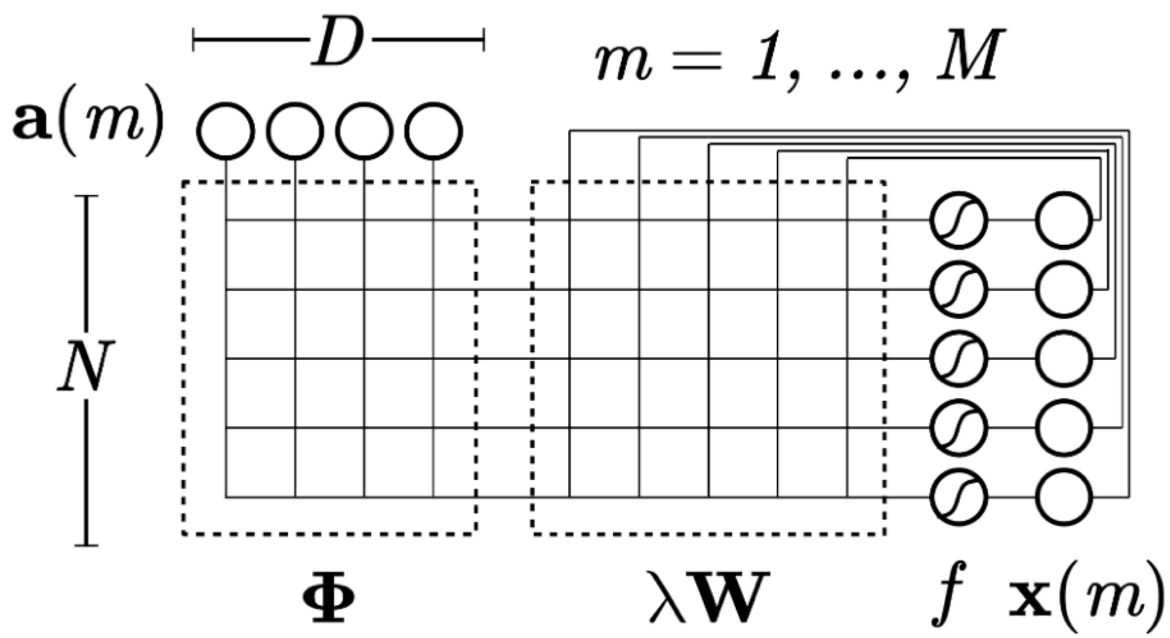
Reservoir computing and recurrent neural networks



Jaeger (2001), *GMD Report 148*

Maass, Natshlager & Markram (2002), *Neural Computation*

A simple working memory



$$\mathbf{x}(M) = \sum_{m=1}^{M} \mathbf{W}^{M-m}\mathbf{\Phi}\mathbf{a}(m)$$

$$\boldsymbol{x}(m) = \boldsymbol{W}\boldsymbol{x}(m-1) + \boldsymbol{\Phi}\boldsymbol{a}(m)$$

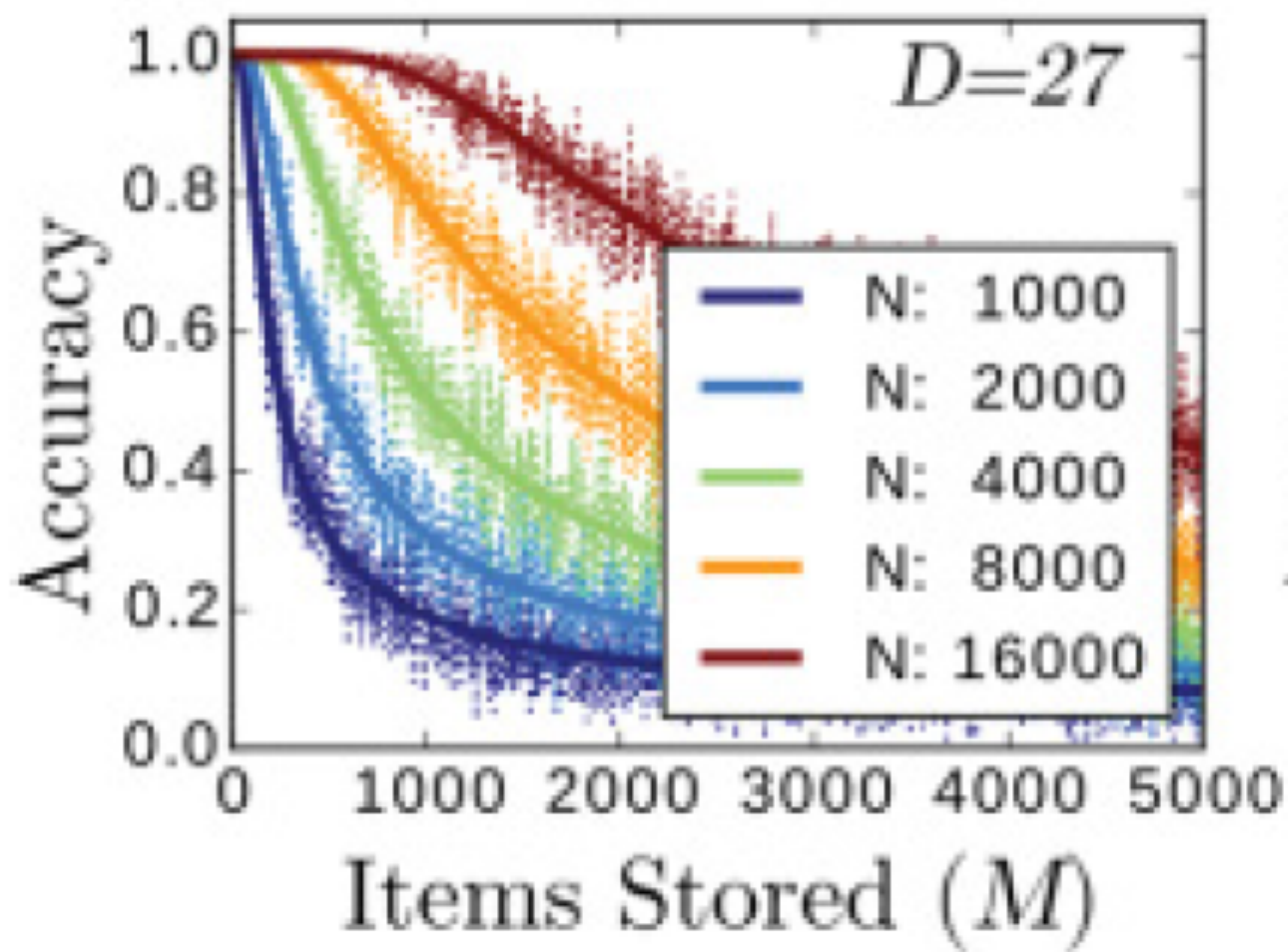
Each input gets a time tag and is added to the memory

$\boldsymbol{W}$: unitary, mixing properties

$\boldsymbol{\Phi}$: random

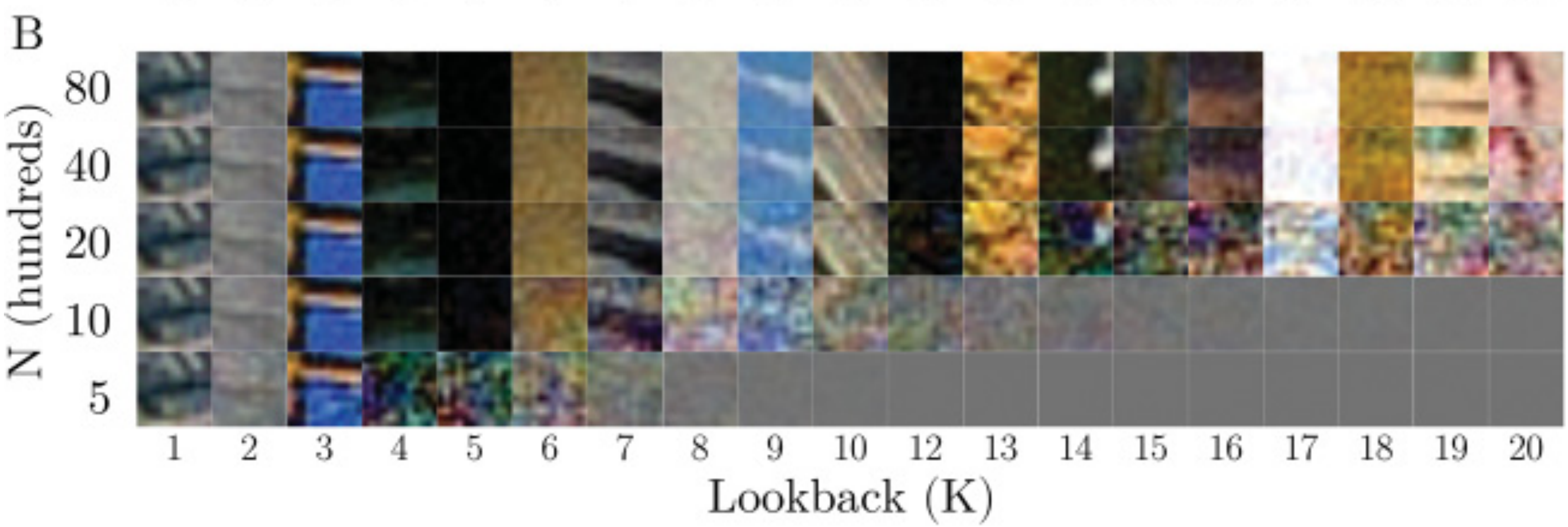
Frady, Kleyko & Sommer (2018). ***Neural Computation***

A theory for information readout



$$p_{corr}(s(K)) = \int_{-\infty}^{\infty} \frac{dh}{\sqrt{2\pi}} e^{-\frac{1}{2}h^2} \left[\Phi\left(h + s(K)\right)\right]^{D-1}$$

Image sequence storage and retrieval



Other efforts

- Berkeley/Stanford EE (Rabaey, Salahuddin, Mitra, Wong) - hardware implementation, cnFET's, PCM/RRAM
- Waterloo (Eliasmith) - SPAUN
- U Maryland (Fernmuller, Aloimonos) - event-based camera robot navigation
- BMW (Mirus, Blouw, Stewart, Conradt) - vehicle position monitoring and prediction.
- VSA online seminar series: https://sites.google.com/ltu.se/vsaonline/winter-2021
- Website: https://www.hd-computing.com