# Self-organizing maps, cortical maps and plasticity

## Cortical maps



#### (from Penfield & Rasmussen, 1950)

# Somatosensory cortex







VI topgraphic map

## Orientation selectivity changes in a regular manner across the surface of VI



Hubel & Wiesel's "ice cube tray model"



## Orientation maps in VI (Blasdel 1992)



#### Direction selective cells in cat area 18 (Ca++ imaging - Ohki, Chung, Ch'ng, Kara, Reid, 2005)



## Cortical maps are Plastic

(monkey auditory cortex - Recanzone, Schreiner & Merzenich, 1993)





### Kohonen's algorithm

$$\Delta \mathbf{w}_i = \eta \,\Lambda(i, i^*) \,(\mathbf{x} - \mathbf{w}_i)$$

$$\Lambda(i,i^*) = e^{-\frac{|\mathbf{r}_i - \mathbf{r}_{i^*}|^2}{2\sigma^2}}$$

#### Kohonen's algorithm applied to a 2D input array



#### Map conforms to topology of input data



### Kohonen's algorithm

Energy function:

$$E(\{\mathbf{w}_i\}) = \frac{1}{2} \sum_{i,k,\mu} M_k^{\mu} \Lambda(i,k) |\mathbf{x}^{(\mu)} - \mathbf{w}_i|^2$$
$$= \frac{1}{2} \sum_{i,\mu} \Lambda(i,i^*) |\mathbf{x}^{(\mu)} - \mathbf{w}_i|^2$$

Gradient descent:

$$\Delta \mathbf{w}_{i} = -\eta \frac{\partial E}{\partial \mathbf{w}_{i}}$$
$$= \eta \sum_{\mu} \Lambda(i, i^{*}) (\mathbf{x}^{(\mu)} - \mathbf{w}_{i})$$

#### Traveling salesman problem ('Elastic net' - Durbin & Willshaw 1987)

$$\Delta \mathbf{w}_{i} = \eta \left( \sum_{\mu} \Lambda^{\mu}(i) \left( \xi^{\mu} - \mathbf{w}_{i} \right) + \kappa \left( \mathbf{w}_{i+1} - 2\mathbf{w}_{i} + \mathbf{w}_{i-1} \right) \right) \qquad \Lambda^{\mu}(i) = \frac{\exp(-|\xi^{\mu} - \mathbf{w}_{i}|^{2}/2\sigma^{2})}{\sum_{j} \exp(-|\xi^{\mu} - \mathbf{w}_{j}|^{2}/2\sigma^{2})}$$
$$E\{\mathbf{w}_{i}\} = -\sigma \sum_{\mu} \log \left[ \sum_{i} \exp(-|\xi^{\mu} - \mathbf{w}_{i}|^{2}/2\sigma^{2}) \right] + \frac{\kappa}{2} \sum_{i} |\mathbf{w}_{i+1} - \mathbf{w}_{i}|^{2}$$





#### Model of ocular dominance column development (Miller, Keller & Stryker, 1989)



$$\frac{dS^{L}(x,\alpha,t)}{dt} = \lambda A(x-\alpha) \sum_{\gamma,\beta} I(x-\gamma) [C^{LL}(\alpha-\beta)S^{L}(\gamma,\beta,t) + C^{LR}(\alpha-\beta)S^{R}(\gamma,\beta,t)] - \gamma S^{L}(x,\alpha,t) - \varepsilon A(x-\alpha)$$







## Results

#### cortical map

#### receptive fields

synaptic strength max

0



#### Simulation of monocular deprivation



#### Elastic net model of position and orientation map (Durbin & Mitchison, 1990)

position map





Joint map of orientation and ocular dominance demonstrates tradeoff between feature diversity vs. smoothness within each feature dimension (from Blasdel 1992)



orientation columns



ocular dominance



## The cortical column: a structure without a function

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This year, the field of neuroscience celebrates the 50th anniversary of Mountcastle's discovery of the cortical column. In this review, we summarize half a century of research and come to the disappointing realization that the column may have no function. Originally, it was described as a discrete structure, spanning the layers of the somatosensory cortex, which contains cells responsive to only a single modality, such as deep joint receptors or cutaneous receptors. Subsequently, examples of columns have been uncovered in numerous cortical areas, expanding the original concept to embrace a variety of different structures and principles. A 'column' now refers to cells in any vertical cluster that share the same tuning for any given receptive field attribute. In striate cortex, for example, cells with the same eye preference are grouped into ocular dominance columns. Unaccountably, ocular dominance

Although the column is an attractive concept, it has failed as a unifying principle for understanding cortical function. Unravelling the organization of the cerebral cortex will require a painstaking description of the circuits, projections and response properties peculiar to cells in each of its various areas.

## Horizontal connections may enforce continuity among oriented elements





Field, Hayes & Hess (1993)

## Statistical dependencies

1556 J. Opt. Soc. Am. A/Vol. 16, No. 7/July 1999

Zetzsche et al.





Measured bivariate activity distribution p(e,o)



## Joint coefficient activations in response to a

moving edge



## **'Topographic ICA'** (Hyvarinen & Hoyer)



### Topographic ICA/Sparse coding (Hyvarinen & Hoyer)







# **'Google Brain'** (Quoc Le et al. 2012)





