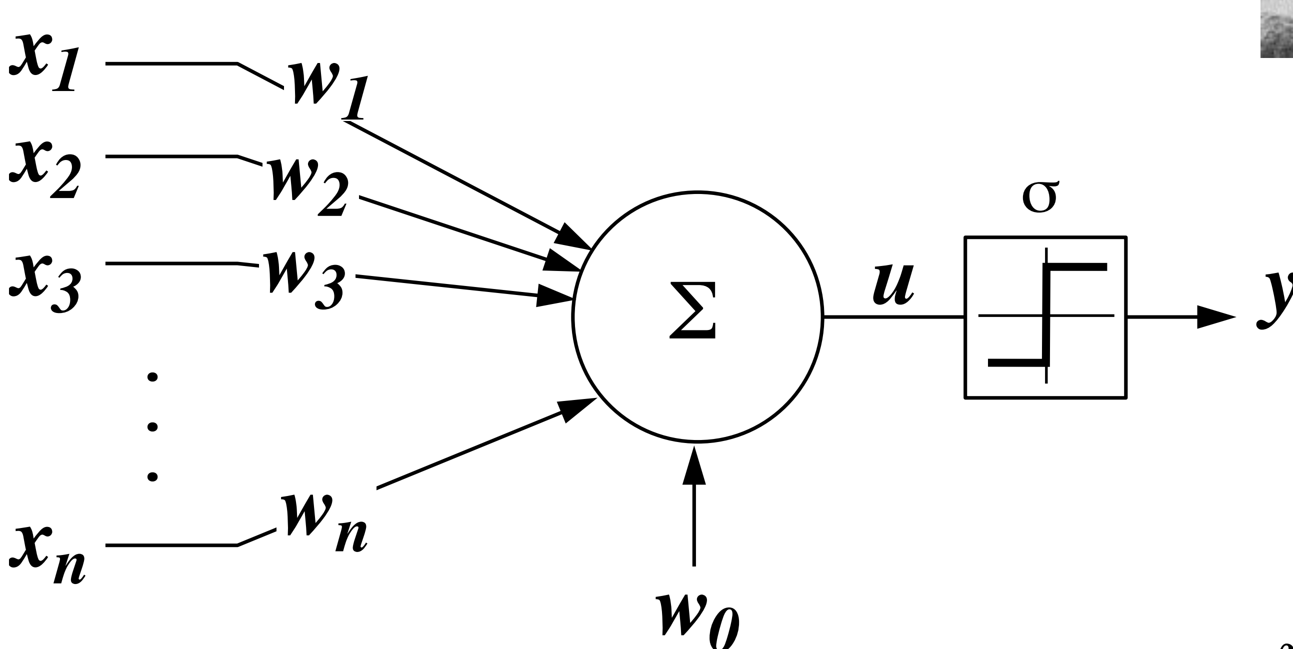
# Supervised learning

# Perceptron model

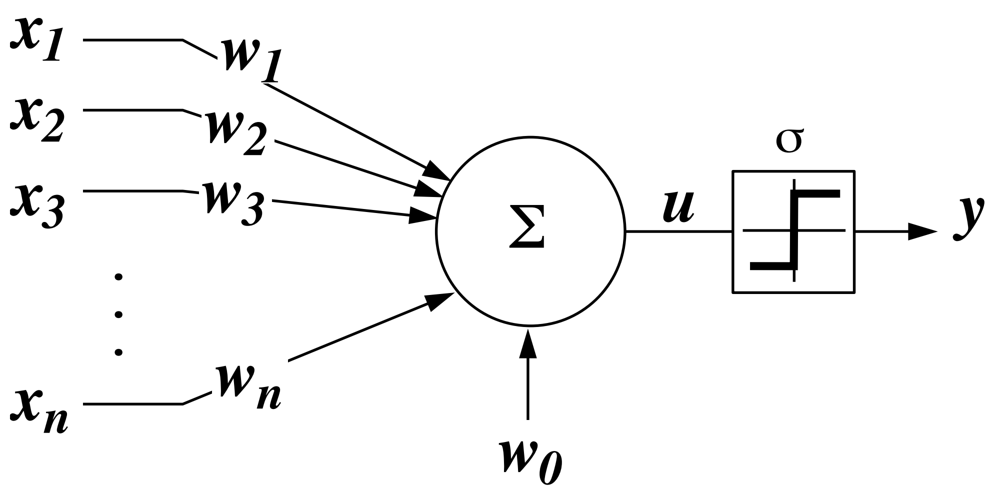
(Rosenblatt, ca. 1960)


$$u = w_0 + \sum_{i=1}^{n} w_i \, x_i$$

$$y = \sigma(u)$$

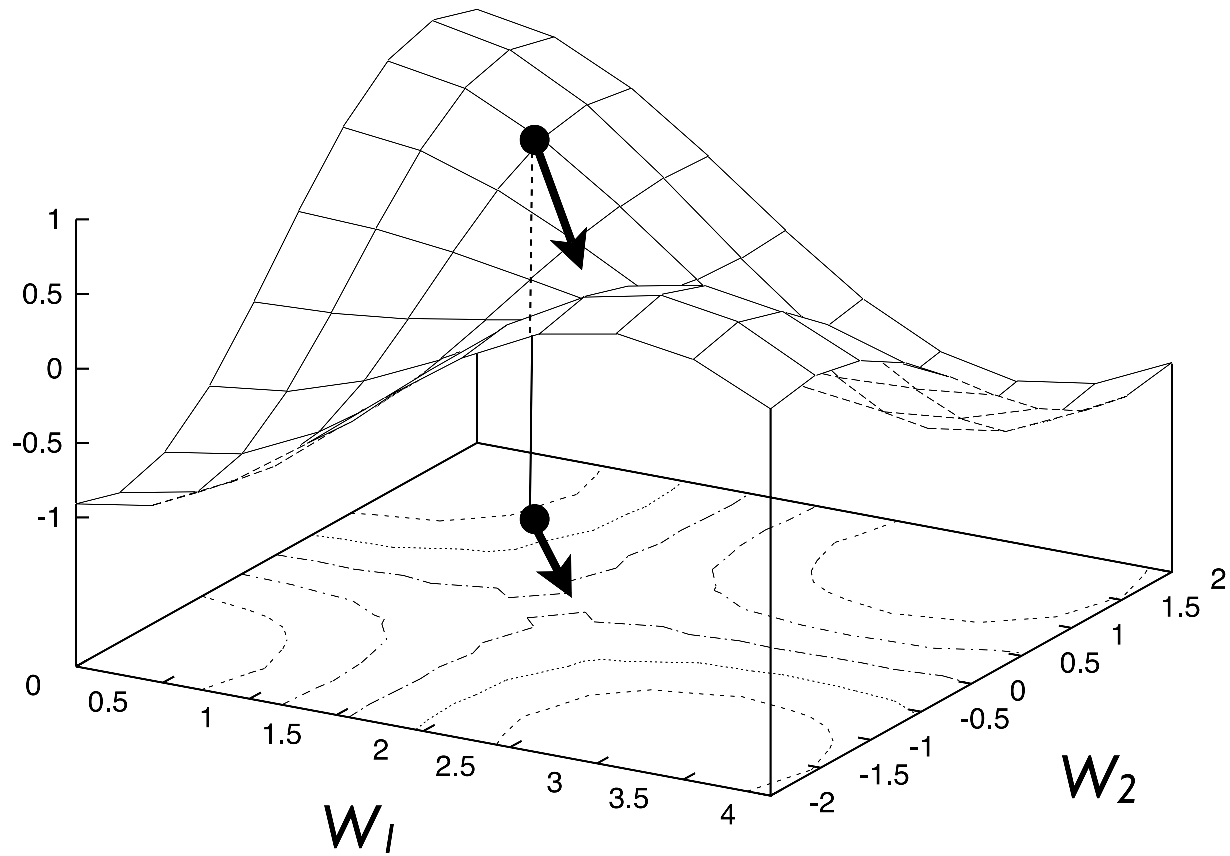
# Perceptron learning rule
## (Rosenblatt 1962)



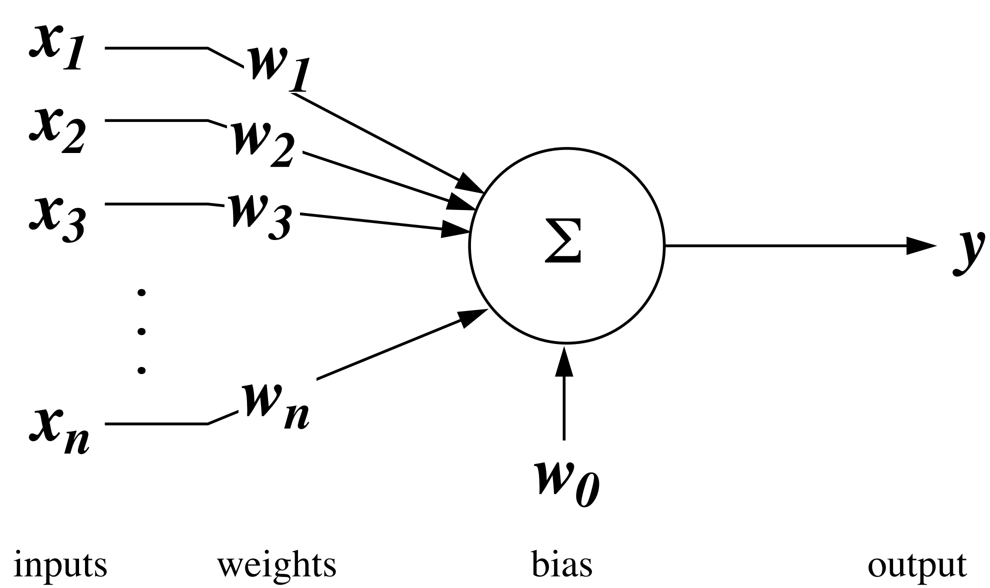

$$\Delta w_k = \begin{cases} 2\eta\, T^{(\alpha)}\, x_k^{(\alpha)} & y^{(\alpha)} \neq T^{(\alpha)} \\ 0 & \text{otherwise} \end{cases}$$

$$= \eta\left(T^{(\alpha)} - y^{(\alpha)}\right) x_k$$

# Gradient descent in weight space



# Linear neuron learning rule
## (Widrow & Hoff 1960)

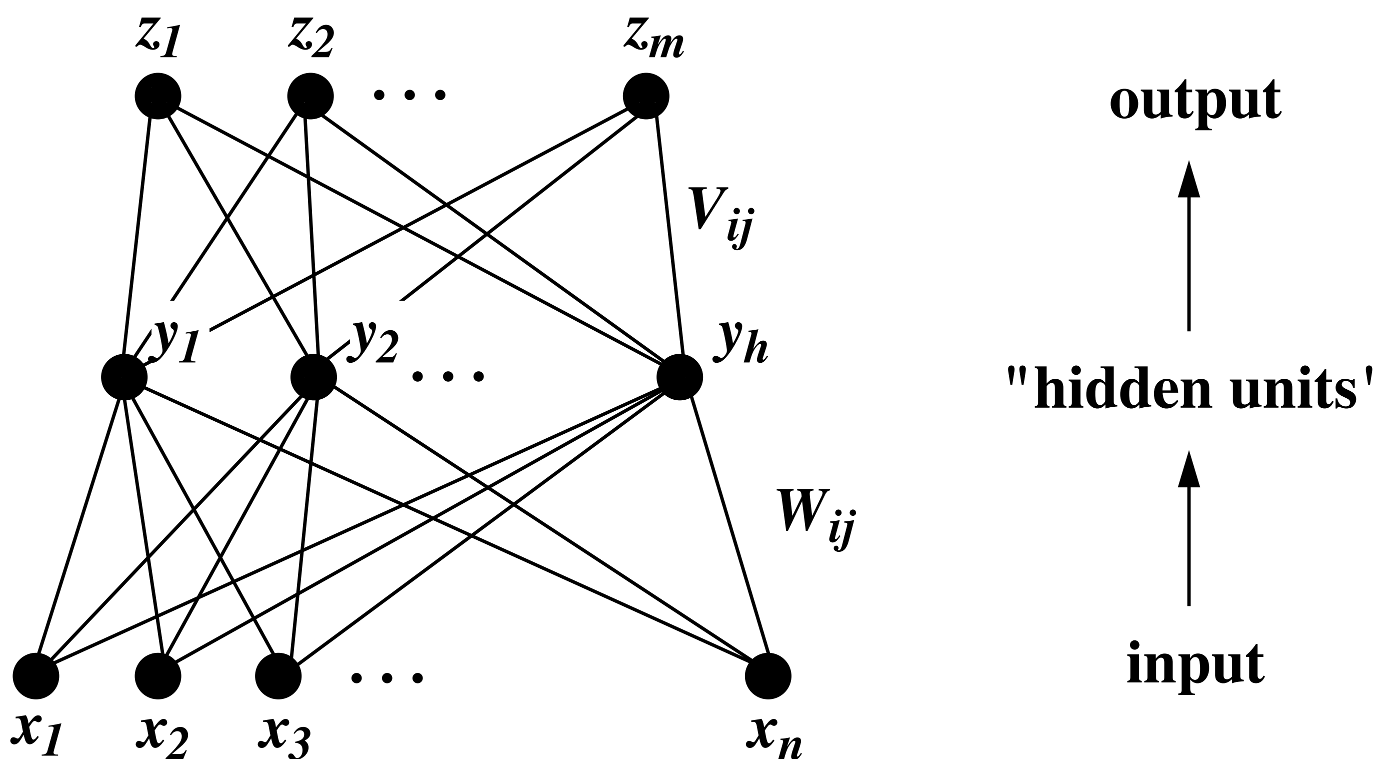



Objective function ⟶ Learning rule

$$E = \frac{1}{2} \sum_{\alpha} \left[ T^{(\alpha)} - y^{(\alpha)} \right]^2$$

$$\begin{aligned} \Delta w_k &= -\eta \frac{\partial E}{\partial w_k} \\ &= \eta \sum_{\alpha} \delta^{(\alpha)} x_k^{(\alpha)} \\ \delta^{(\alpha)} &= T^{(\alpha)} - y^{(\alpha)} \end{aligned}$$

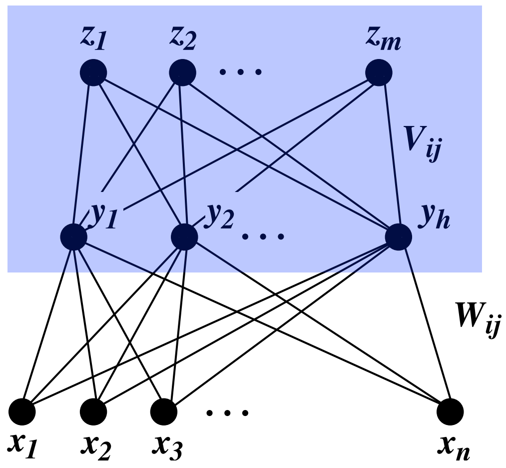
# Two-layer network




$$z_i = \sigma(\sum_j V_{ij} y_j)$$

$$y_i = \sigma(\sum_j W_{ij} x_j)$$

# Learning rule for output layer



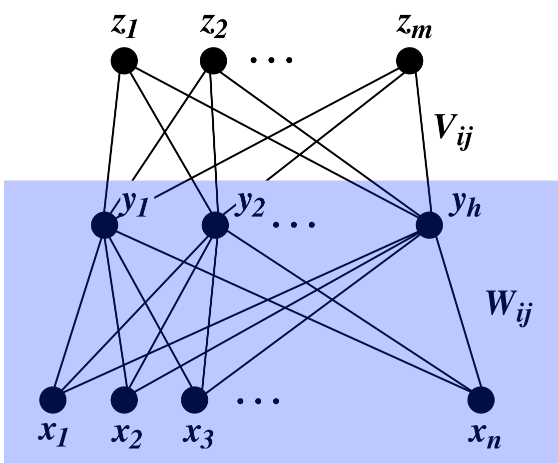

$$E^{(\alpha)} = \frac{1}{2}\sum_i \left[T_i^{(\alpha)} - z_i(\mathbf{x}^{(\alpha)})\right]^2$$

$$\begin{aligned}
\Delta V_{ij} &= -\eta \frac{\partial E}{\partial V_{ij}} \\
&= [T_i - z_i(\mathbf{x})] \frac{\partial z_i(\mathbf{x})}{\partial V_{ij}} \\
&= [T_i - z_i(\mathbf{x})]\, \sigma'(u_{z_i})\, y_j \\
&= \boxed{\delta_{z_i}\, y_j}
\end{aligned}$$

where

$$\begin{aligned}
\delta_{z_i} &= [T_i - z_i(\mathbf{x})]\, \sigma'(u_{z_i}) \\
u_{z_i} &= \sum_j V_{ij}\, y_j
\end{aligned}$$

# Learning rule for hidden layer




$$\Delta W_{kl} = -\eta \frac{\partial E}{\partial W_{kl}} = \eta \sum_i [T_i - z_i(\mathbf{x})] \frac{\partial z_i(\mathbf{x})}{\partial W_{kl}} .$$

$$\frac{\partial z_i(\mathbf{x})}{\partial W_{kl}} = \frac{\partial z_i(\mathbf{x})}{\partial y_k} \frac{\partial y_k}{\partial W_{kl}}$$

$$\Delta W_{kl} = \eta \sum_i [T_i - z_i(\mathbf{x})] \sigma'(u_{z_i}) V_{ik} \sigma'(u_{y_k}) x_l = \boxed{\eta \, \delta_{y_k} x_l}$$

where $\delta_{y_k} = \sigma'(u_{y_k}) \sum_i \delta_{z_i} V_{ik}$ ← back-propagation of error

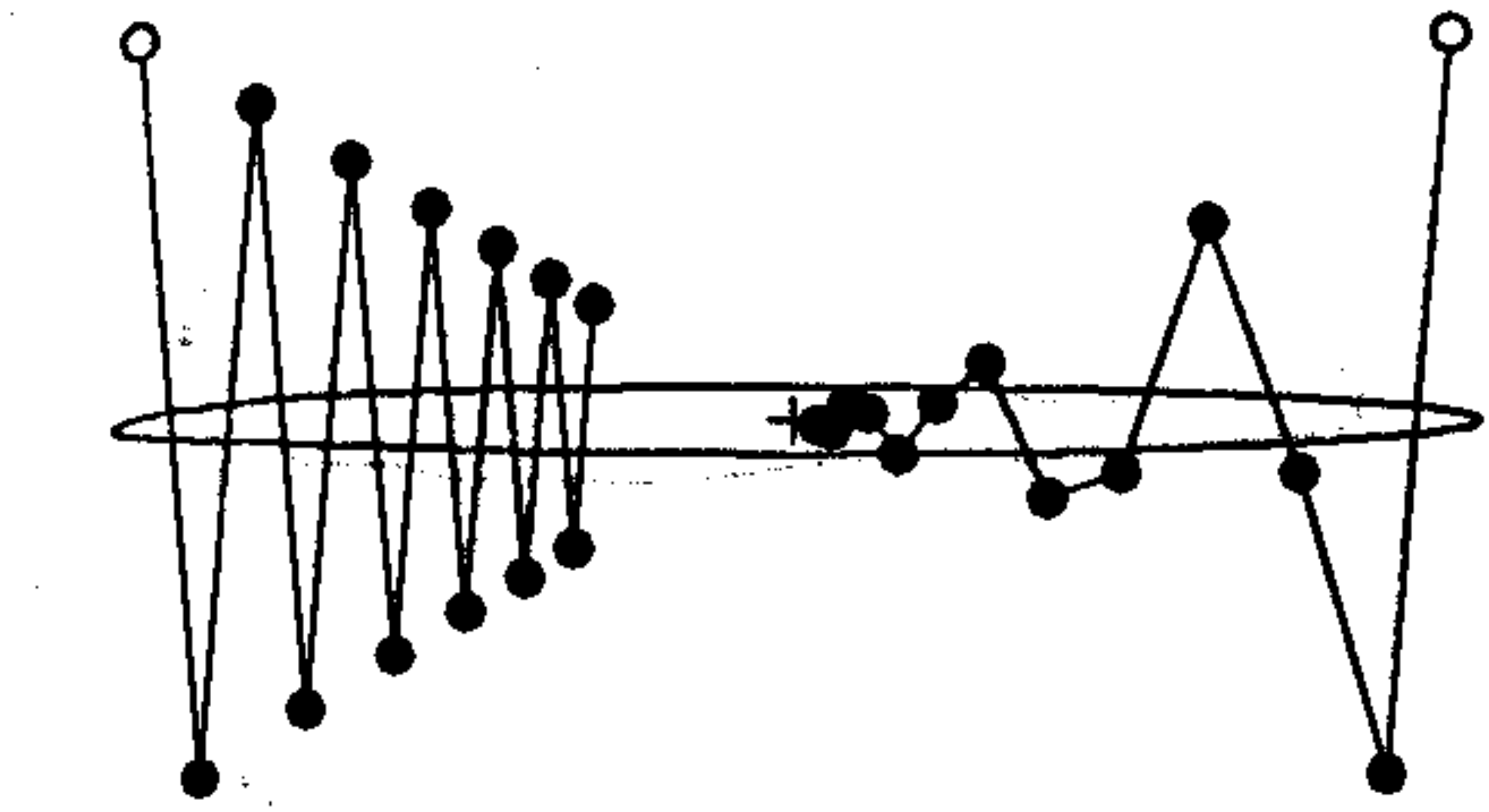
# Momentum

$$\Delta w_{kl}(t+1) = -\eta \frac{\partial E}{\partial w_{kl}} + \alpha \, \Delta w_{kl}(t)$$

Converges to

$$\Delta w_{kl} \approx -\frac{\eta}{1-\alpha} \frac{\partial E}{\partial w_{kl}}$$

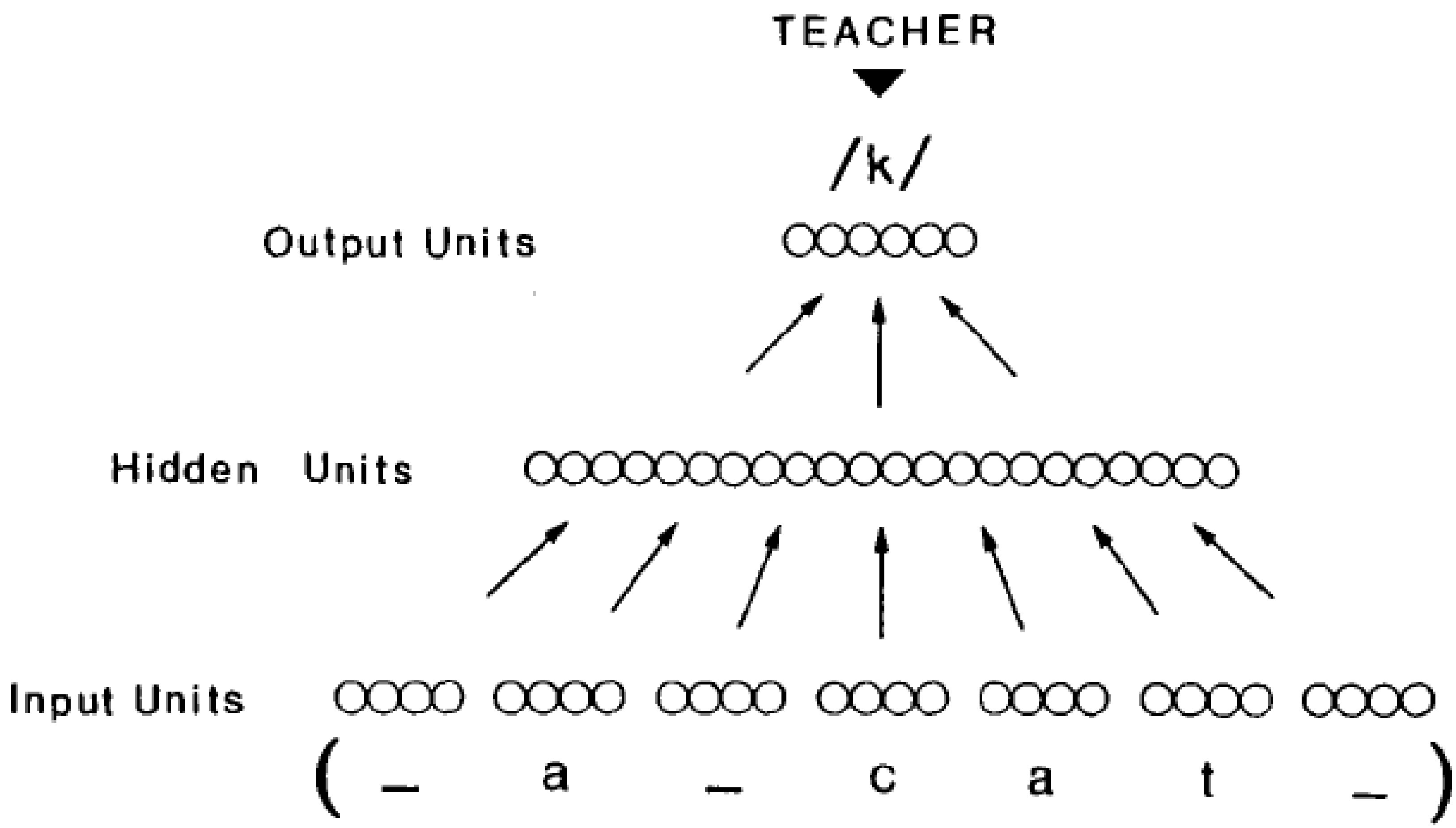
# Momentum



without momentum

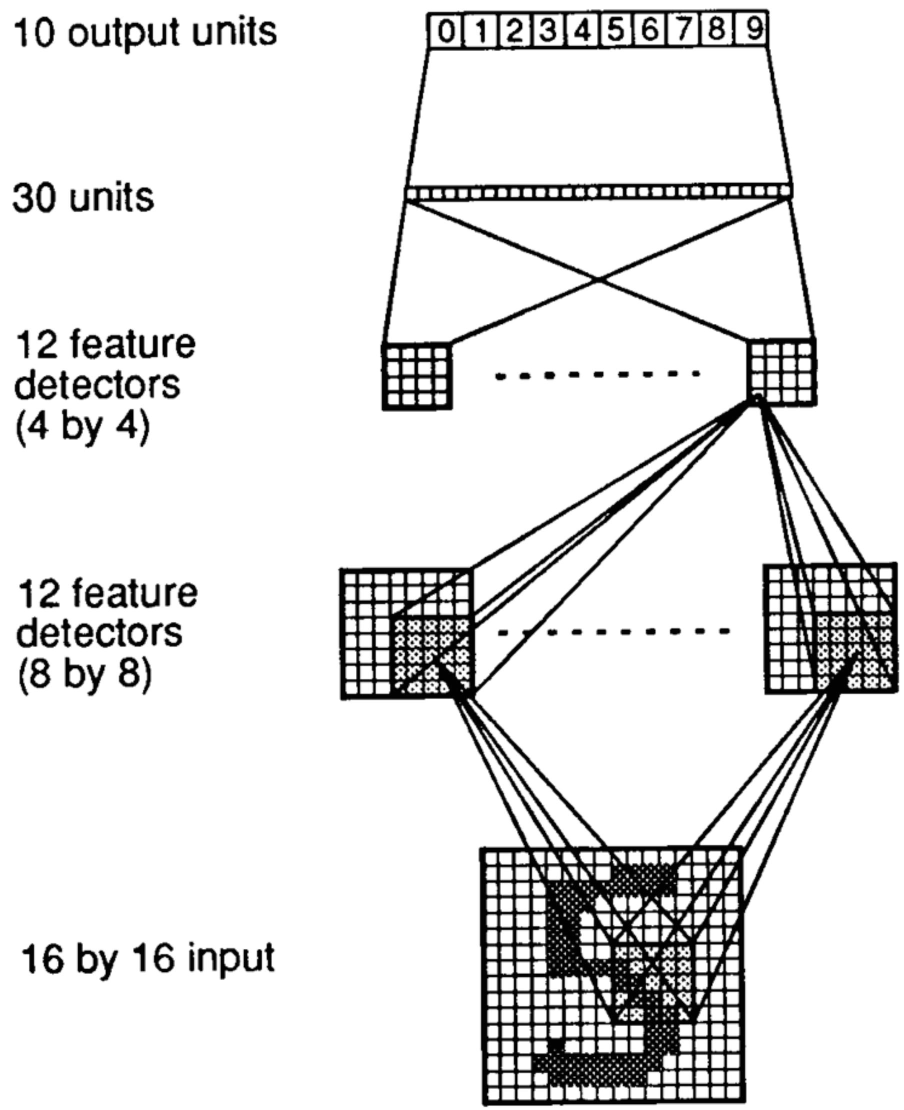
with momentum

# NetTalk
## (Sejnowski & Rosenberg 1987)



[redacted]

# "LeNet"
## (Yann LeCun et al., 1989)



1989

# ALVINN, an autonomous land vehicle in a neural network

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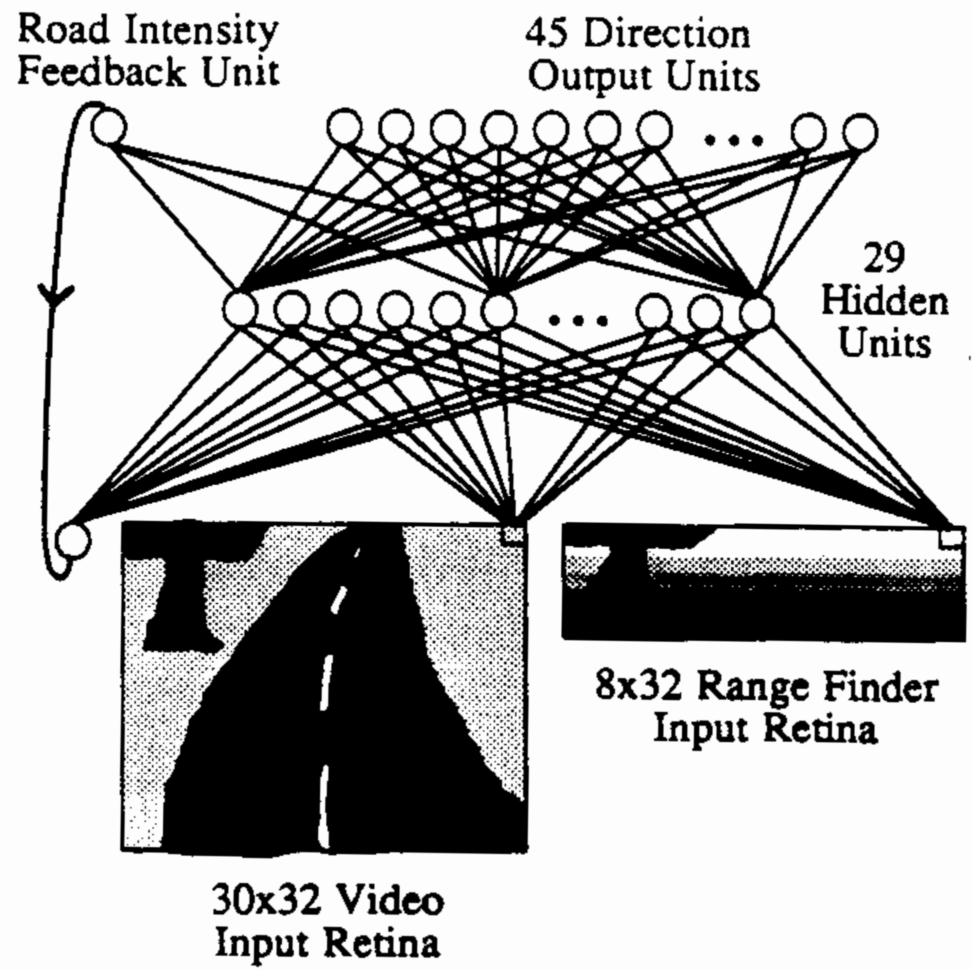
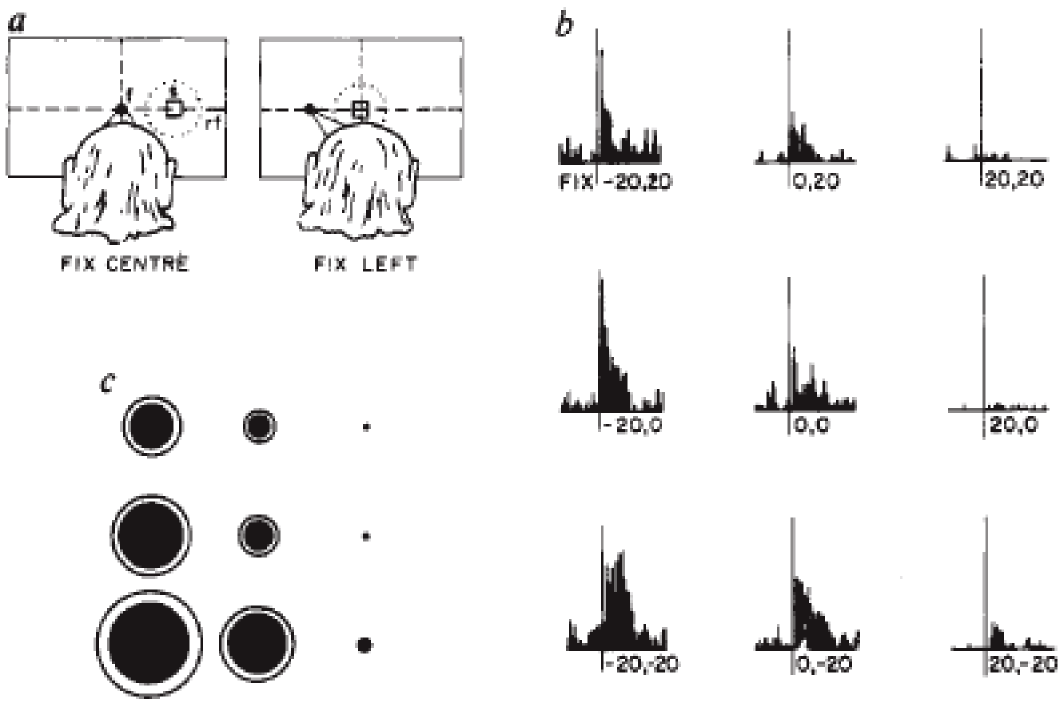
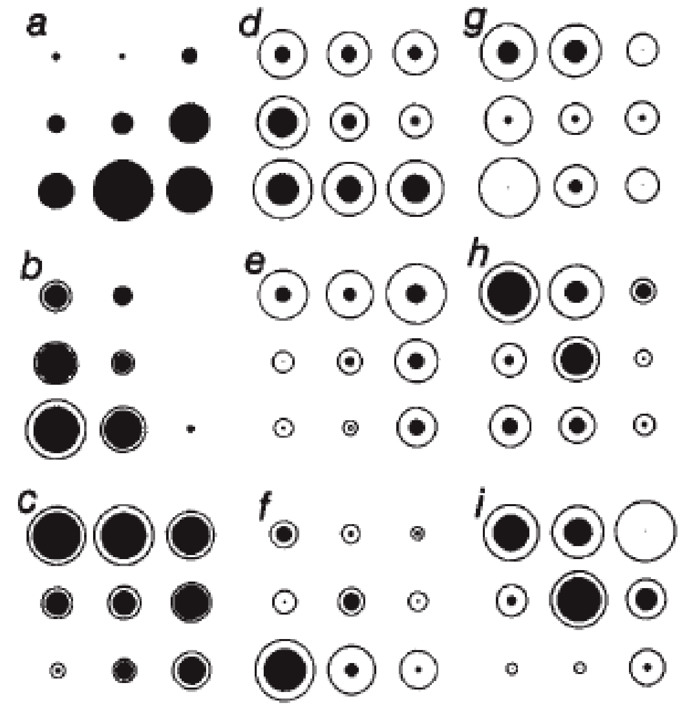



Figure 1: ALVINN Architecture

# Gain Fields

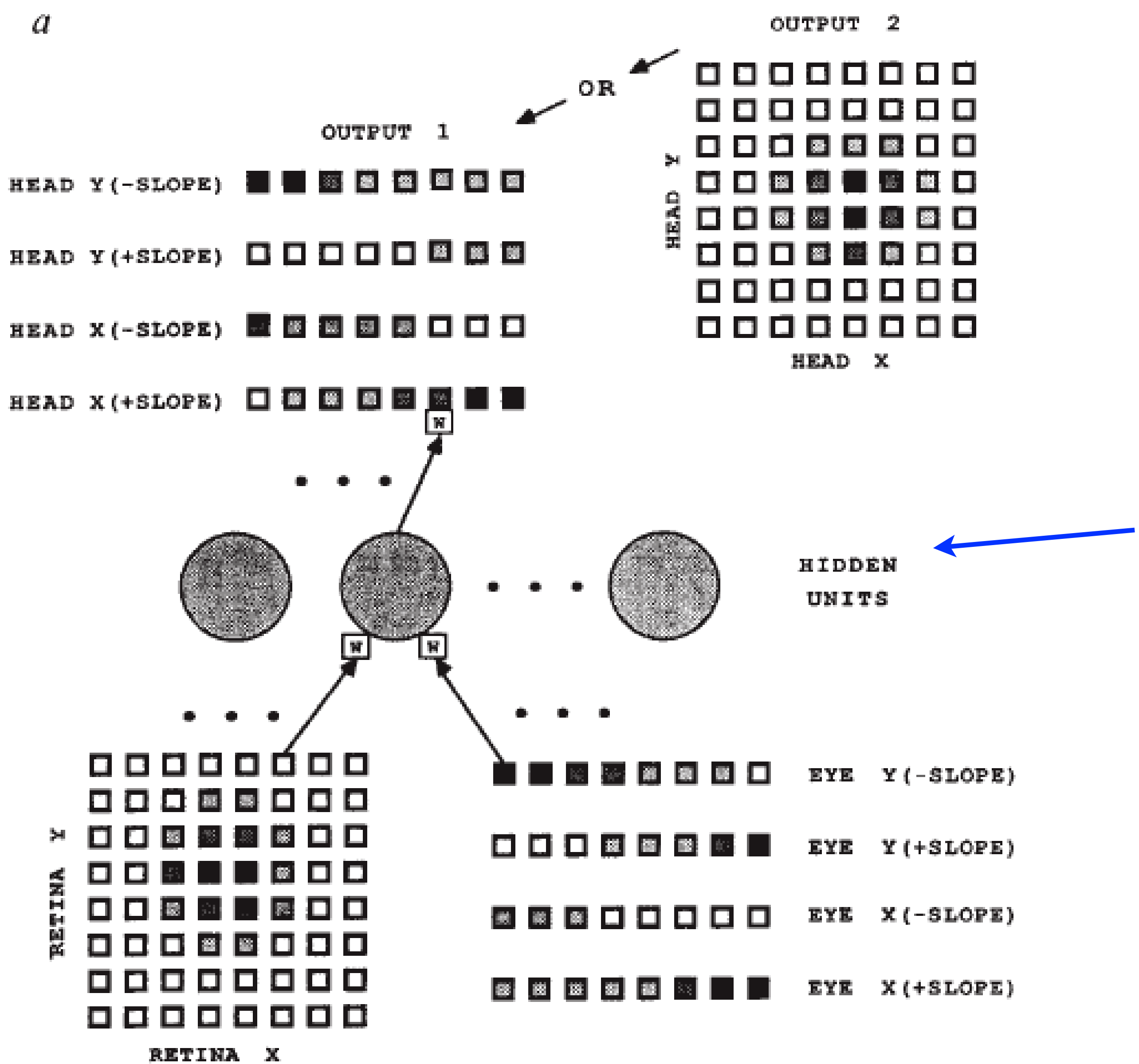
## (Zipser & Anderson, 1987)

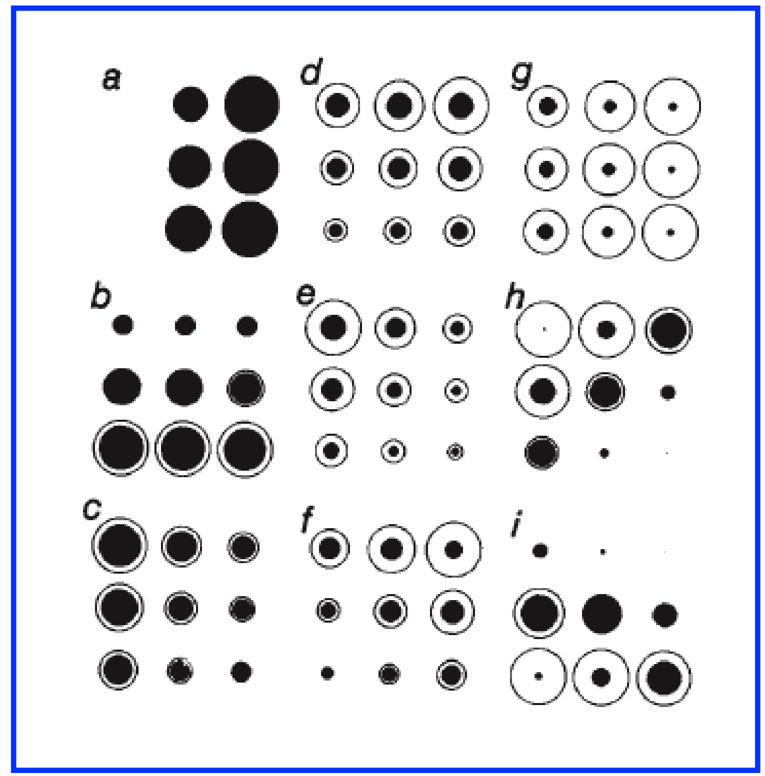


# Gain Fields

## (Zipser & Anderson, 1987)



# Deep networks appear to predict responses of V4 and IT neurons (Yamins & DiCarlo 2016)