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OPTIMAL PERCEPTUAL INFERENCE

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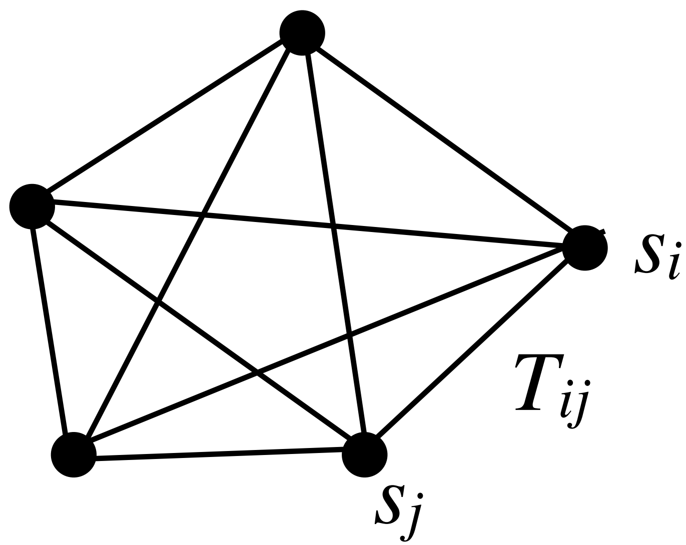
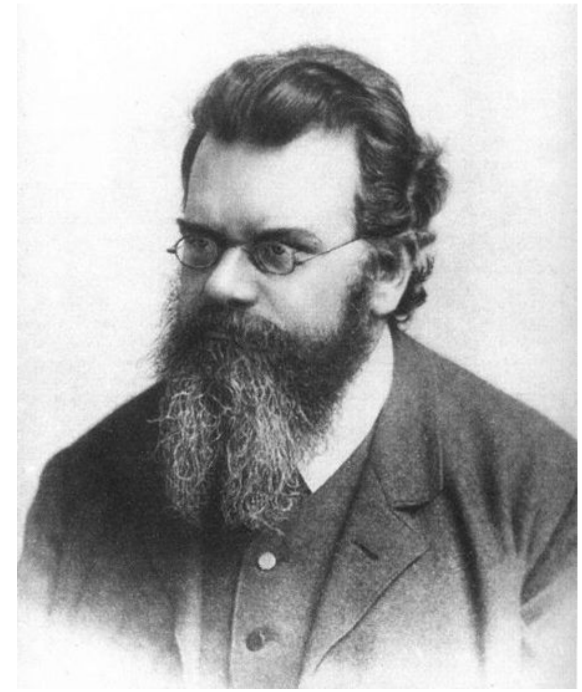
Introduction

One way of interpreting images is to formulate hypotheses about parts or aspects of the image and then decide which of these hypotheses are likely to be correct. The probability that each hypothesis is correct is determined partly by its fit to the image and partly by its fit to other hypotheses that are taken to be correct, so the *truth value of an* individual hypothesis cannot be decided in isolation. One method of searching for the most plausible combination of hypotheses is to use a relaxation process in which a probability is associated with each hypothesis, and the probabilities are then iteratively modified on the basis of the fit to the image and the known relationships between hypotheses. An attractive property of relaxation methods is that they can be implemented in parallel hardware where one computational unit is used for each possible hypothesis, and the interactions between hypotheses are implemented by direct hardware connections between the units.

The "Boltzmann machine"
(Hinton & Sejnowski, 1983)





Ludwig Boltzmann

$$E(\mathbf{s}) = -\frac{1}{2}\sum_{ij} T_{ij}\, s_i\, s_j$$

$$P(\mathbf{s}) = \frac{1}{Z} e^{-\beta\, E(\mathbf{s})}$$

Boltzmann-Gibbs distribution

$$
\begin{aligned}
P(\mathbf{x}) &= \frac{1}{Z} e^{\lambda\,\phi(\mathbf{x})} \\
Z &= \int e^{\lambda\,\phi(\mathbf{x})}\, d\mathbf{x}
\end{aligned}
$$

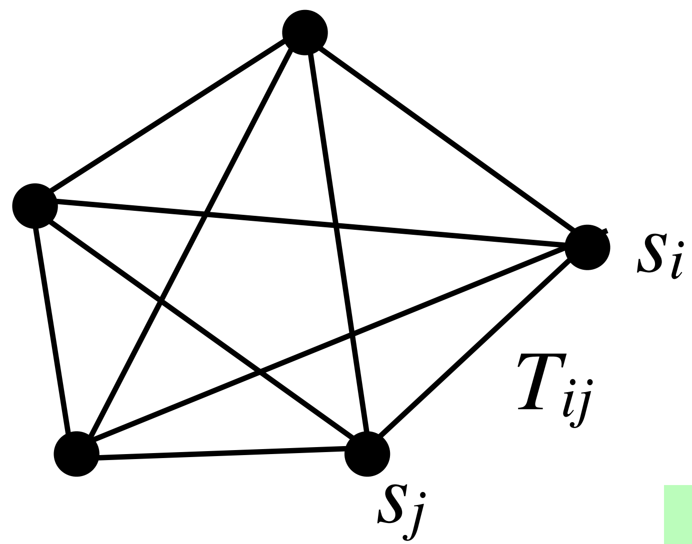
Learning rule:

$$
\begin{aligned}
\Delta\lambda &\propto \frac{\partial}{\partial\lambda} \langle \log P(\mathbf{x}) \rangle \\
&= \langle \phi(\mathbf{x}) \rangle - \langle \phi(\mathbf{x}) \rangle_{P(\mathbf{x})}
\end{aligned}
$$

$$\log P(\mathbf{x}) = \lambda\phi(\mathbf{x}) - \log Z$$

$$\begin{aligned}
\frac{\partial}{\partial\lambda}\langle\log P(\mathbf{x})\rangle &= \frac{\partial}{\partial\lambda}\langle\lambda\,\phi(\mathbf{x}) - \log Z\rangle \\
&= \langle\phi(\mathbf{x}) - \frac{\partial}{\partial\lambda}\log Z\rangle \\
&= \langle\phi(\mathbf{x}) - \frac{1}{Z}\frac{\partial Z}{\partial\lambda}\rangle \\
&= \langle\phi(\mathbf{x}) - \frac{1}{Z}\int\phi(\mathbf{x})\,e^{\lambda\,\phi(\mathbf{x})}d\mathbf{x}\rangle \\
&= \langle\phi(\mathbf{x}) - \int\phi(\mathbf{x})\,P(\mathbf{x})d\mathbf{x}\rangle \\
&= \langle\phi(\mathbf{x})\rangle - \langle\phi(\mathbf{x})\rangle_{P(\mathbf{x})}
\end{aligned}$$

The "Boltzmann machine"
(Hinton & Sejnowski, 1983)



λ_{ij} $\phi_{ij}(\mathbf{s})$

$$E(\mathbf{s}) = -\frac{1}{2}\sum_{ij} T_{ij}\, s_i\, s_j$$

$$P(\mathbf{s}) = \frac{1}{Z} e^{-\beta\, E(\mathbf{s})}$$

The Boltzmann machine learning rule

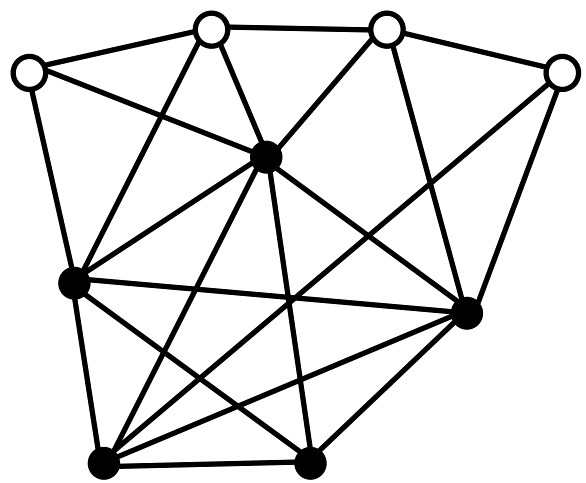
$$\Delta T_{ij} \quad \propto \quad \frac{\partial \langle \log P(\mathbf{s}) \rangle}{\partial T_{ij}}$$

$$= \quad \beta \left[\langle s_i \, s_j \rangle_{\text{clamped}} - \langle s_i \, s_j \rangle_{\text{free}} \right]$$

data

$P(\mathbf{s})$

Boltzmann machine with hidden units
(Hinton & Sejnowski)



‘hidden’ units, s^h

‘visible’ units, s^v

$$E(\mathbf{s}^v, \mathbf{s}^h) = -\sum_{i,j} T_{ij}^{vv}\, s_i^v s_j^v - \sum_{i,j} T_{ij}^{vh}\, s_i^v s_j^h - \sum_{i,j} T^{hh}\, s_i^h s_j^h$$

$$P(\mathbf{s}^v, \mathbf{s}^h) = \frac{1}{Z} e^{-E(\mathbf{s}^v, \mathbf{s}^h)}$$

$$P(\mathbf{s}^v) = \sum_{\mathbf{s}^h} P(\mathbf{s}^v, \mathbf{s}^h)$$

The Boltzmann machine learning rule

$$\begin{aligned}\Delta T_{ij} &\propto \frac{\partial \log P(\mathbf{s})}{\partial T_{ij}} \\ &= \beta\left[\langle s_i\, s_j\rangle_{\text{clamped}} - \langle s_i\, s_j\rangle_{\text{free}}\right]\end{aligned}$$

Clamped: $\left\{\begin{array}{ccc} \mathbf{s}^v & = & \mathbf{x} \\ \mathbf{s}^h & \sim & P(\mathbf{s}^h|\mathbf{s}^v) \end{array}\right.$

Free: $[\mathbf{s}^v, \mathbf{s}^h] \sim P(\mathbf{s}^v, \mathbf{s}^h) \equiv P(\mathbf{s})$

Gibbs sampling

To sample from $P(\mathbf{x})$:

$$x_1 \sim P(x_1 | x_2, ..., x_n)$$

$$x_2 \sim P(x_2 | x_1, x_3, ..., x_n)$$

$$x_3 \sim P(x_3 | x_1, x_2, x_4, ..., x_n)$$

$$\vdots$$

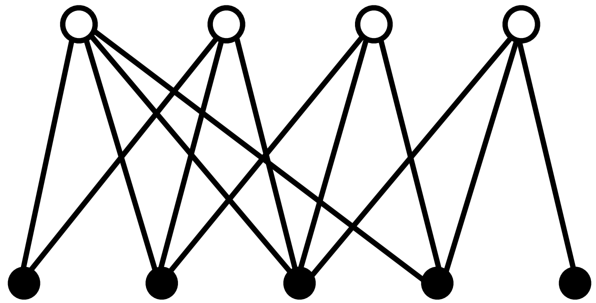
$$x_n \sim P(x_n | x_1, ..., x_{n-1})$$

Dynamics

$$P(s_i = 1|\{s_{\bar{i}}\}) = \frac{P(s_i = 1, \{s_{\bar{i}}\})}{P(s_i = 1, \{s_{\bar{i}}\}) + P(s_i = -1, \{s_{\bar{i}}\})}$$

Thus:
$$\begin{aligned} P(s_i = 1|\{s_{\bar{i}}\}) &= \sigma(2\beta\, h_i) \\ h_i &= \sum_{j \neq i} T_{ij}\, s_j \end{aligned}$$

Restricted Boltzmann machine (RBM)



'hidden' units, s^h

'visible' units, s^v

$$E(\mathbf{s}^v, \mathbf{s}^h) = -\sum_{i,j} T_{ij}^{vh}\, s_i^v\, s_j^h$$

$$P(\mathbf{s}^v, \mathbf{s}^h) = \frac{1}{Z} e^{-E(\mathbf{s}^v, \mathbf{s}^h)} \quad \Longrightarrow \quad \begin{array}{l} P(s_i^h | s_{\bar{i}}^h, \mathbf{s}^v) = \sigma(\sum_j T_{ij}^{hv}\, s_j^v) \\ P(s_i^v | s_{\bar{i}}^v, \mathbf{s}^h) = \sigma(\sum_j T_{ij}^{vh}\, s_j^h) \end{array}$$

$$P(\mathbf{s}^v) = \sum_{\mathbf{s}^h} P(\mathbf{s}^v, \mathbf{s}^h) = \frac{1}{Z} e^{\sum_i \log(1 + e^{T_i^{vh} \cdot \mathbf{s}^v})}$$

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PLOS COMPUTATIONAL BIOLOGY

Modeling Higher-Order Correlations within Cortical Microcolumns

Urs Köster[1]*, Jascha Sohl-Dickstein[2], Charles M. Gray[3], Bruno A. Olshausen[1]

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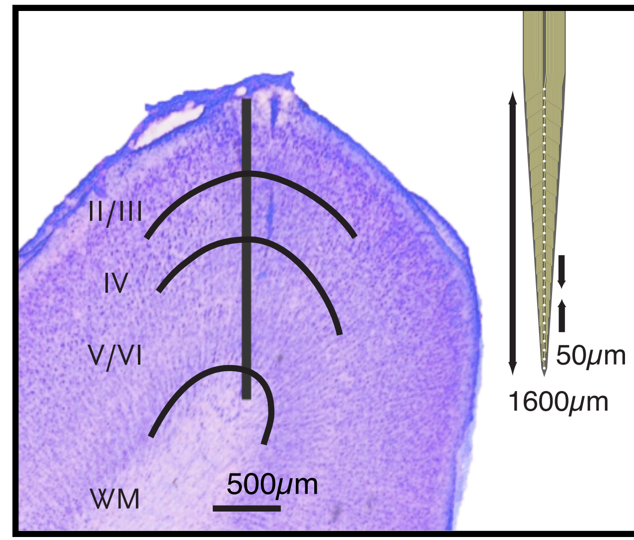
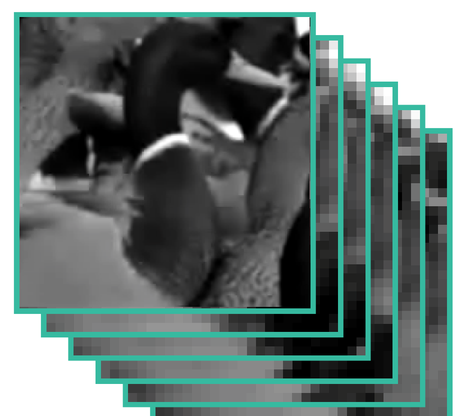
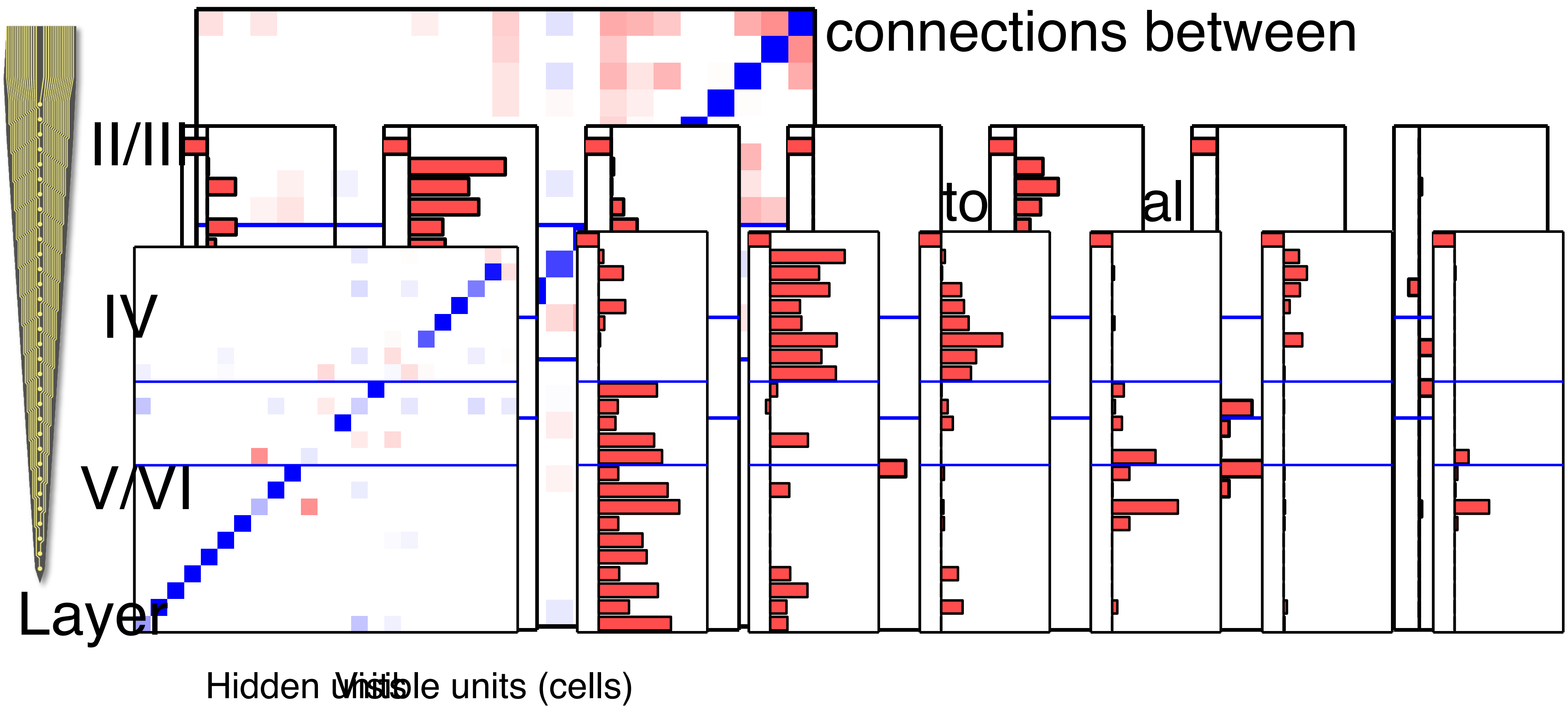
- Silicon polytrode, 32 channels span all laminae
- Anesthetized cat area V1
- Stimuli consist of natural scene movies at 150 fps



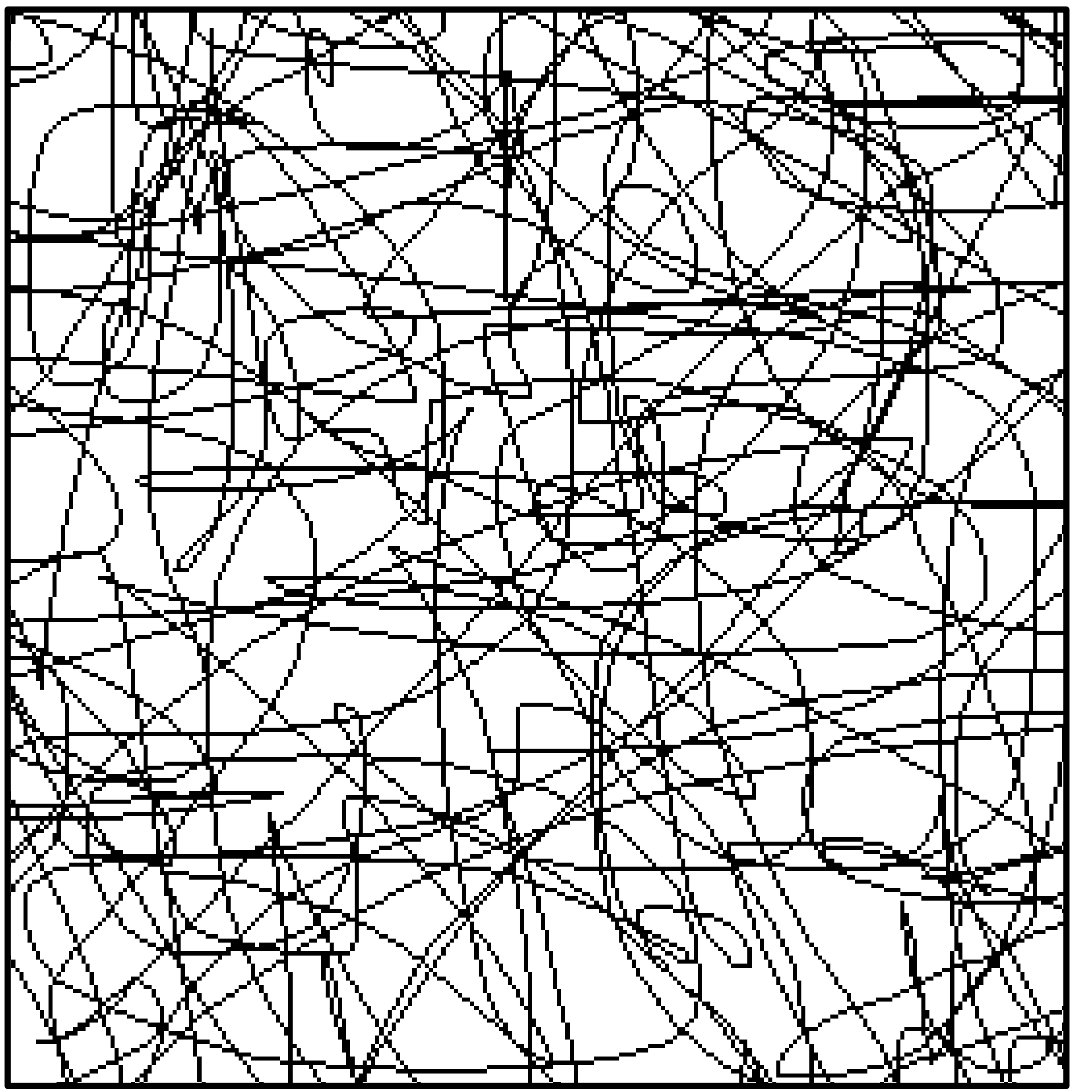


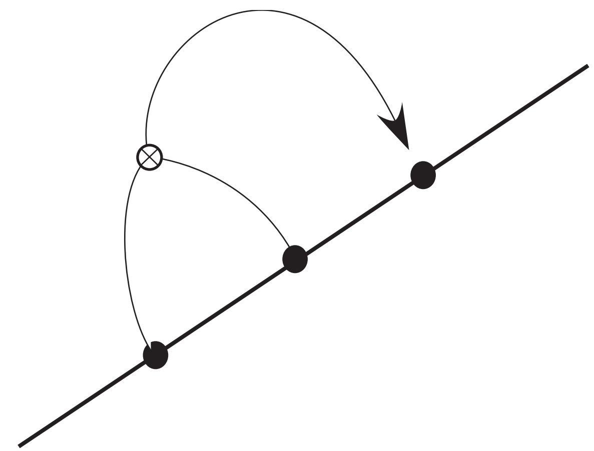
Results: Model structure

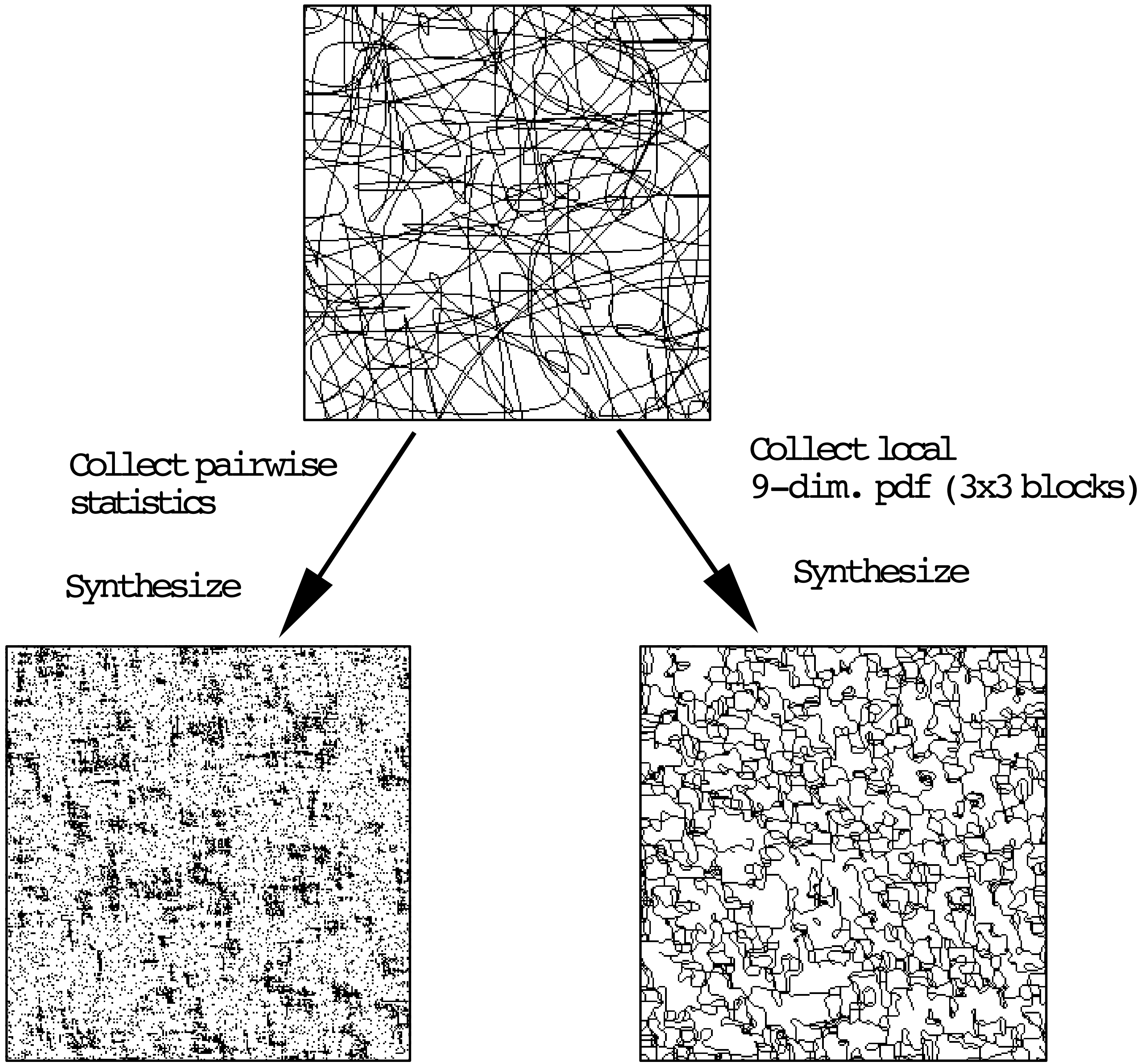
- Ising model: Pairwise coupling parameters
- RBM with vertical connections only



‘Lines world’



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