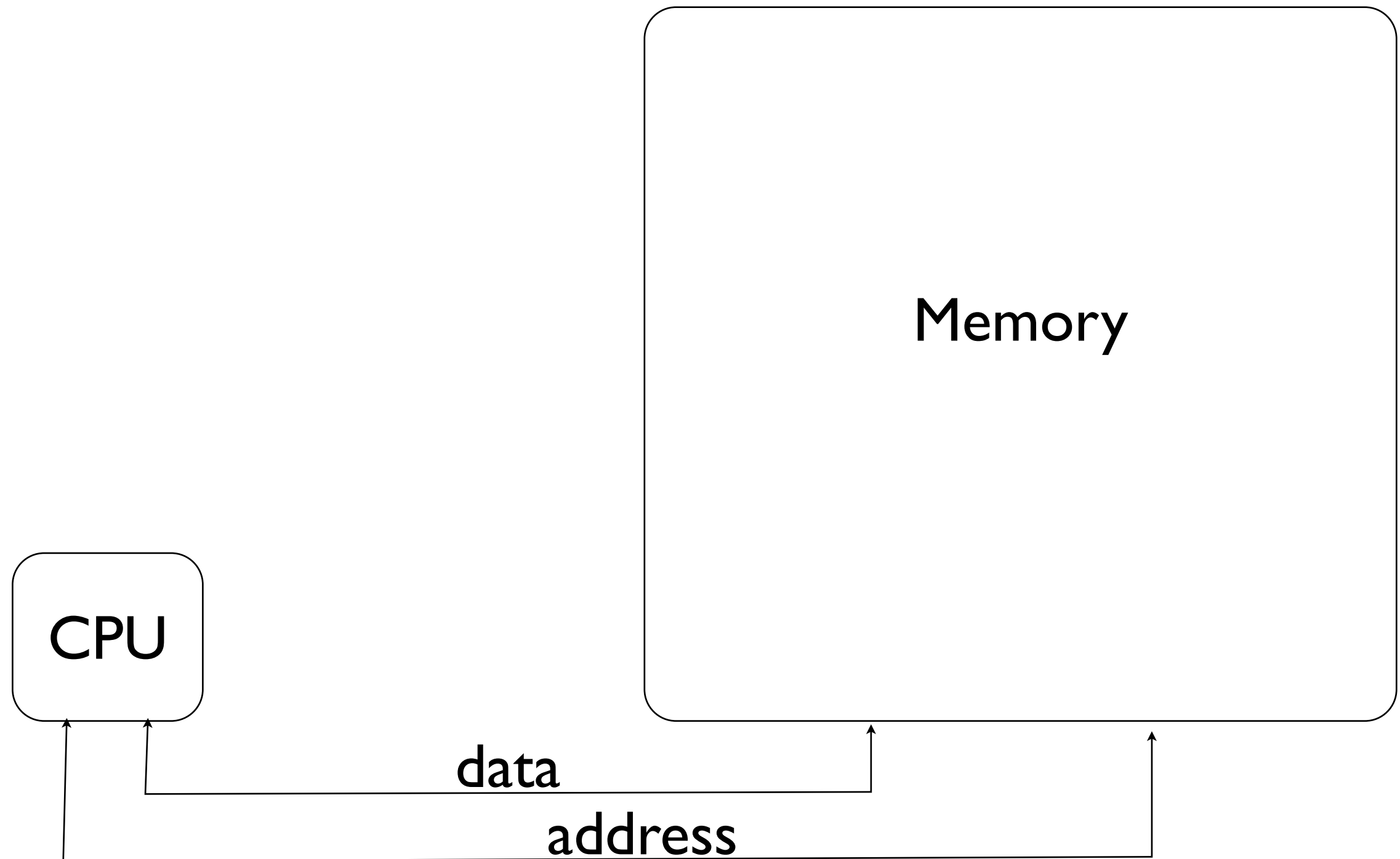
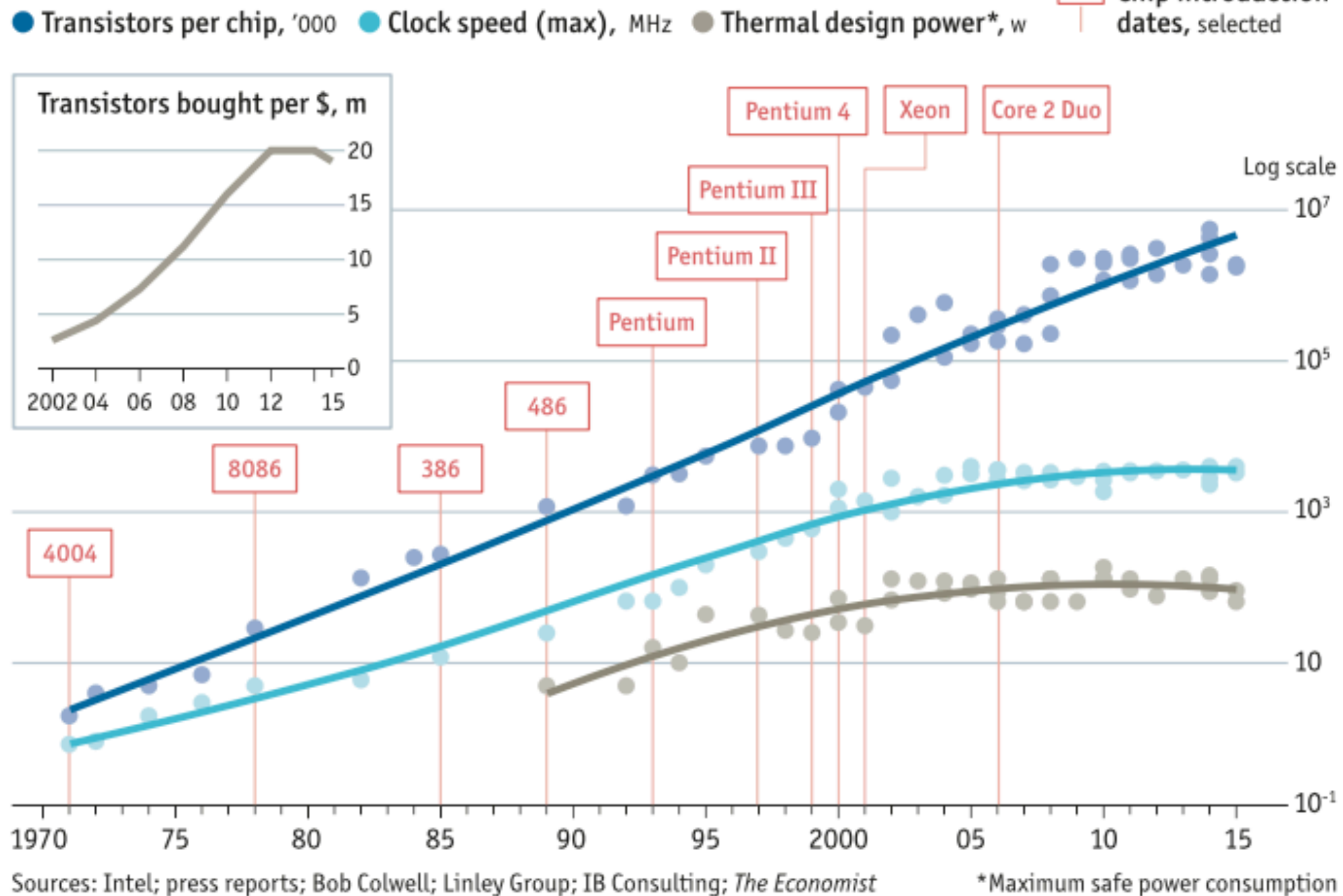


Physics of Computation

“Von Neumann” computing architecture

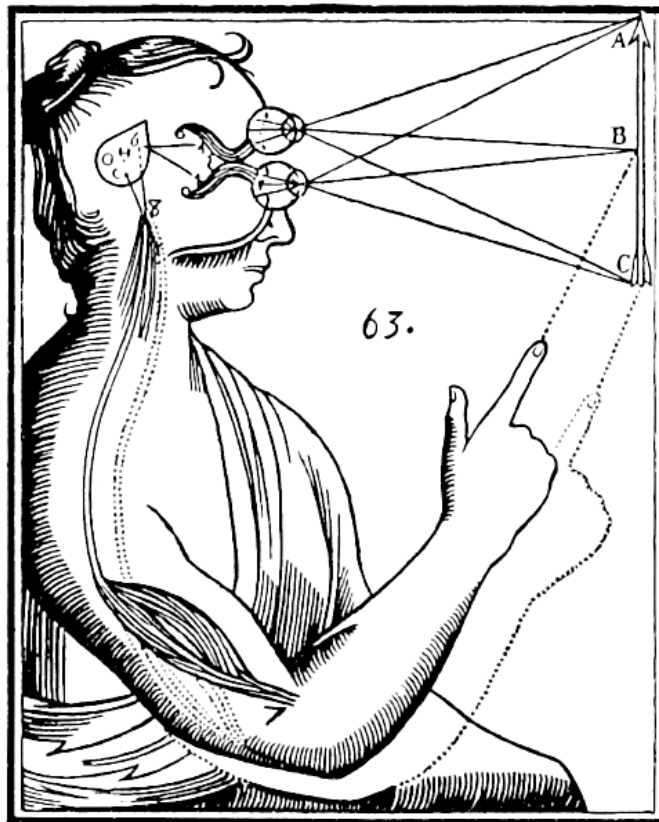


Stuttering

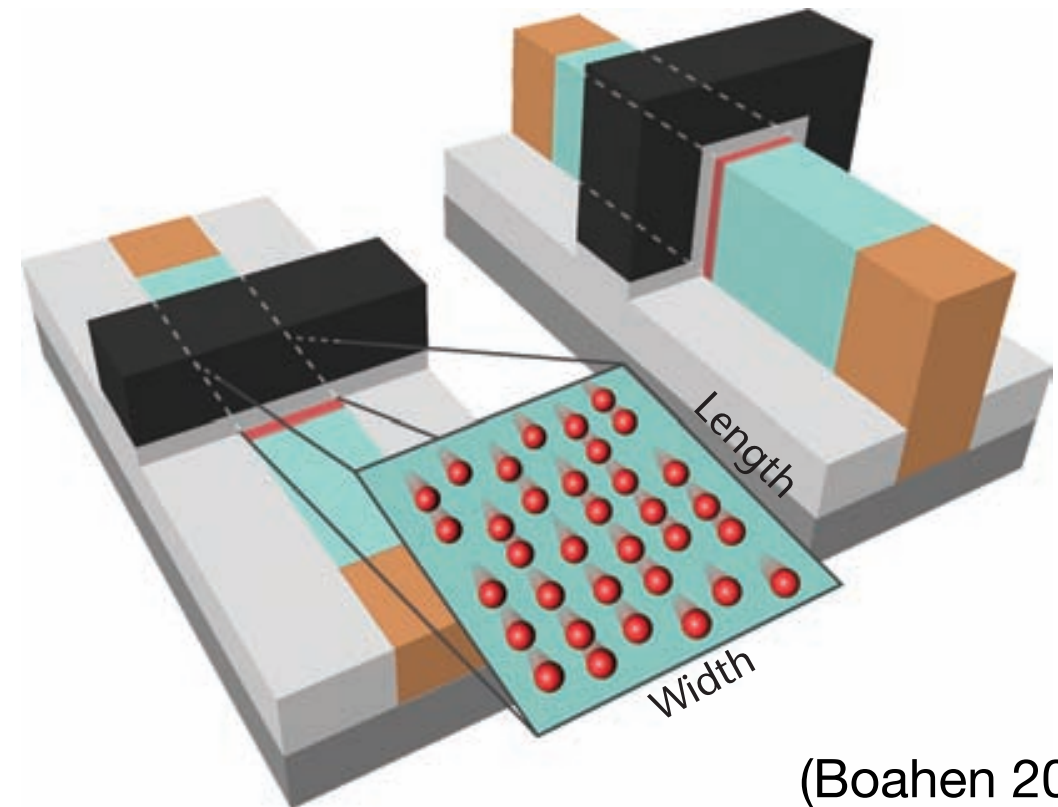


From “After Moore’s Law,” *The Economist*, March 2016

Brains vs. machines



Brain-like functions are more *probabilistic* in nature and use *different data representations*.



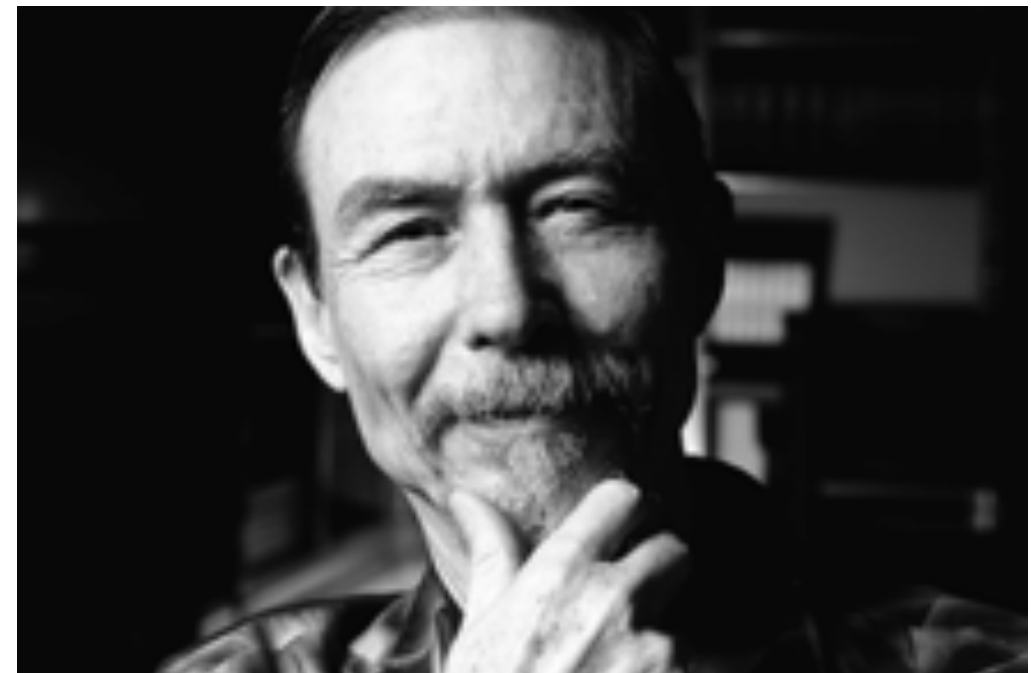
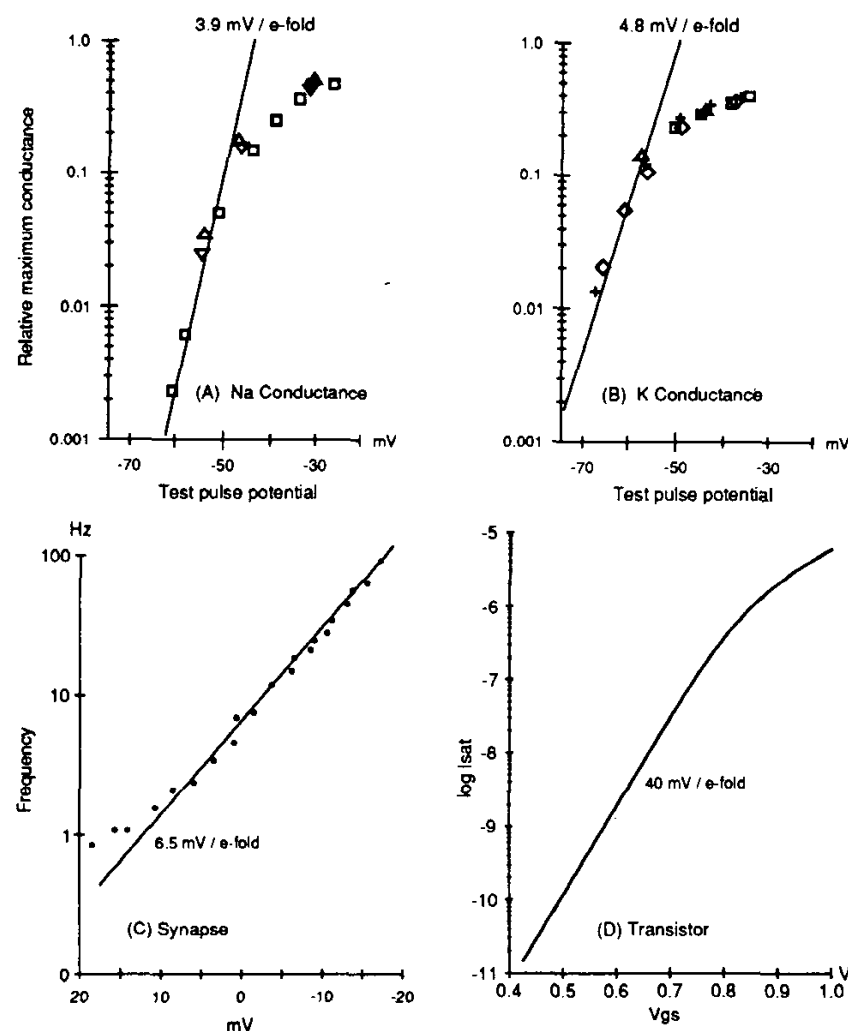
(Boahen 2017)

How to compute with nanoscale, low-power, *stochastic* circuit components?

Neuromorphic computing

*The fact that we can build devices that implement the same basic operations as those the nervous system uses leads to the inevitable conclusion that we should be able to build entire systems based on the **organizing principles** used by the nervous system.*

—Carver Mead (1990)



Lessons from the Early Days of Semiconductors - Carver Mead - 4/24/2019

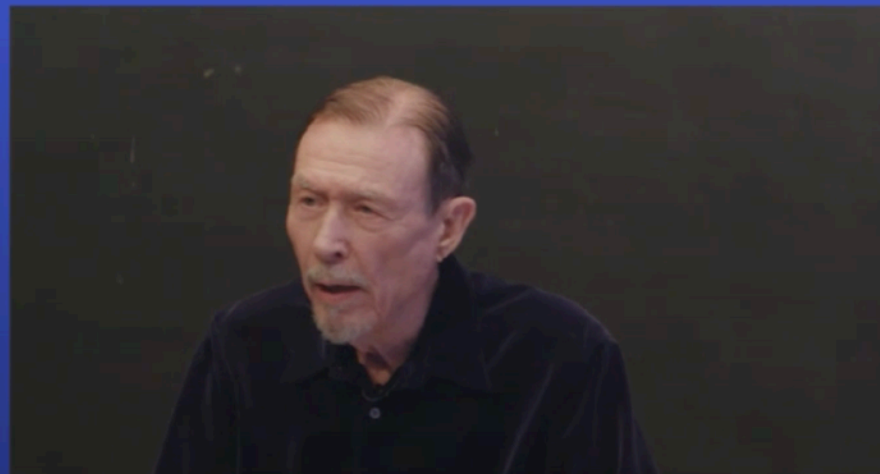
<https://www.youtube.com/watch?v=qhJaq3kl6Dc&feature=youtu.be>

RADIO COMMUNICATIONS

1745 Leyden Jar
1769 Franklin – Electricity
1799 Volta – Battery
1831 Faraday – Induction
1836 Ruhmkorff – Induction Coil
1853 Kelvin – Analysis of Discharge
1865 Maxwell – E&M Thy
1880 Edison – Thermionic Diode
1888 Hertz – Electromagnetic Waves
1895 Marconi – Radio Comms
1902 Poulson – CW Arc Transmitter
1906 Pickard – Silicon Detector
1907 de Forest – Thermionic Triode
1913 Langmuir – High Vacuum Triode
1916 de Forest – Com. Broadcast
1917 – 1919 Radio Ban WWI
1920 Com. Broadcast – KDKA
1922 Over 500 AM Com. Stations
1927 Nationwide Broadcast – NBC
1927 Farnsworth – First Television
1950s TV ubiquitous
1973 Cooper – Cell Phone
1981 1G Cell Networks

ELECTRONS IN VACUUM

1745 Leyden Jar
1769 Franklin – Electricity
1799 Volta – Battery
1831 Faraday – Induction
1836 Callen, Ruhmkorff – Induction Coil
1853 William Thomson – Spark Analysis
1858 Feddersen – Oscillating Spark
1880 Edison – Thermionic Emission
1890 Fleming – Thermionic Diode
1898 Elster & Geitel – Electron e/m
1899 J.J. Thomson – Electron e & m
1907 de Forest – Thermionic Triode
1913 Langmuir – High-vacuum Triode
1922 Lilienfeld – Field Emission
1960s Ken Shoulders – Micro-triode
1960s GE – TIMM



SOLID-STATE DEVICES

1799 Volta – Battery
1833 Faraday – Semiconductors
1877 Braun – Contact Rectification
1879 Hall – Hall Effect
1906 Pickard – Silicon Detector
1907 – 1911 Baedeker – Semiconductor Doping
1922 Grondahl – CuO Rectifier
1923 Lossev – Oscillating Crystal
1925 Lilienfeld – MESFET
1928 Lilienfeld – MOSFET
1928 – 1931 Bloch et. al. – Band Picture
1942 Schottky – Barrier Thy & Expt
1948 Bardeen & Brattain – Point-Contact Transistor
1948 Shockley – Junction Transistor
1952 Pankove – Alloy Germanium Transistor
1954 Pacific Semiconductors Inc.
1955 Derik & Frosch – Oxide Masking
1955 Shockley Labs
1956 Tannenbaum et. al. – Diffused Si Transistor
1957 Fairchild
1959 Kilby – All-Semiconductor Circuit
1959 Hoerni – Planar Process
1959 Noyce – Integrated Circuit
1960 Kahng & Atalla – MOSFET working
1963 Wanlass – CMOS
1965 Mead – MESFET working
1965 Dennard – DRAM
1965 Moore's Law

Caltech

Nernst potential (aka 'reversal potential')

$$V = -\frac{kT}{q} \log \frac{N_{\text{in}}}{N_{\text{ex}}} \quad \text{or} \quad N_{\text{in}} = N_{\text{ex}} e^{-\frac{q}{kT} V}$$

Current-voltage relation of voltage-gated channels

$$\frac{\theta}{1 - \theta} = e^{-E_0/kT} e^{qnV/kT} \quad \theta = \text{fraction of channels open}$$

Current-voltage relation of MOS transistor

$$I = I_0 e^{-\frac{q V_{gs}}{kT}} \left(1 - e^{\frac{q V_{ds}}{kT}}\right) \quad \begin{array}{l} V_{gs} = \text{gate-source voltage} \\ V_{ds} = \text{drain-source voltage} \end{array}$$

All of these things are related by the same fundamental physical law...

Its the Boltzmann distribution!

Example: atmospheric pressure vs. elevation

$$v_{\text{drift}} = \frac{wt_f}{2m} = v_{\text{diff}} = -\frac{1}{2N} \frac{dN}{dh} kT \frac{t_f}{m}$$

$$\frac{1}{N} \frac{dN}{dh} kT = -w$$

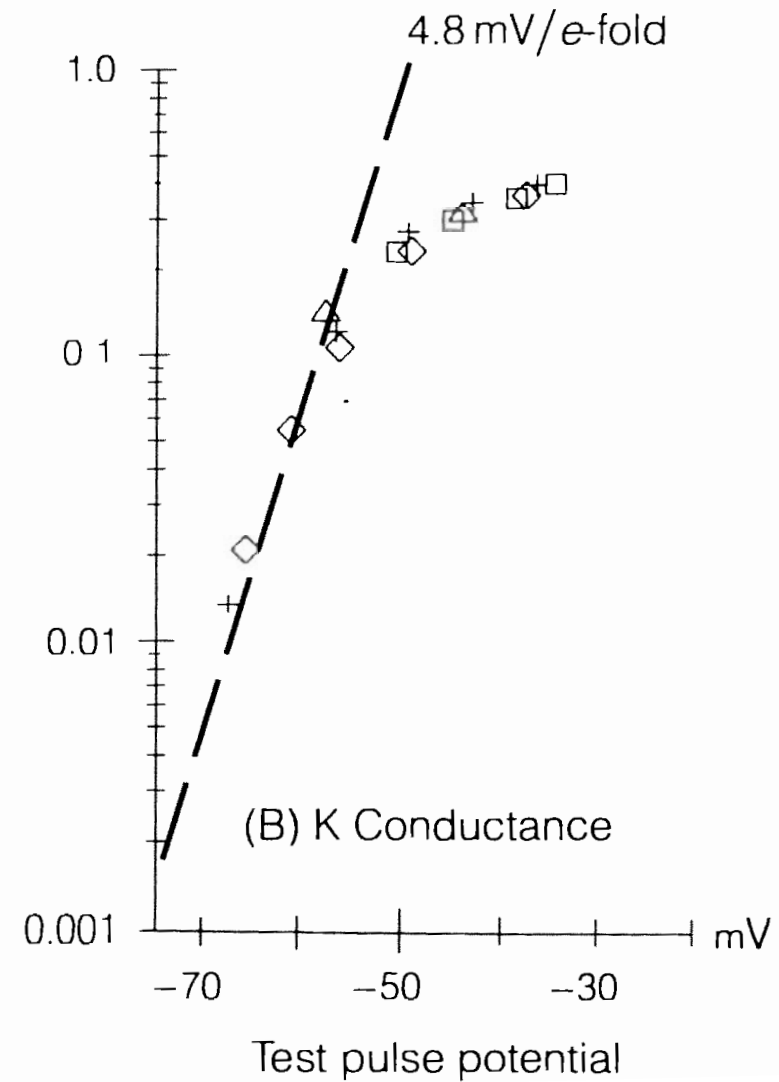
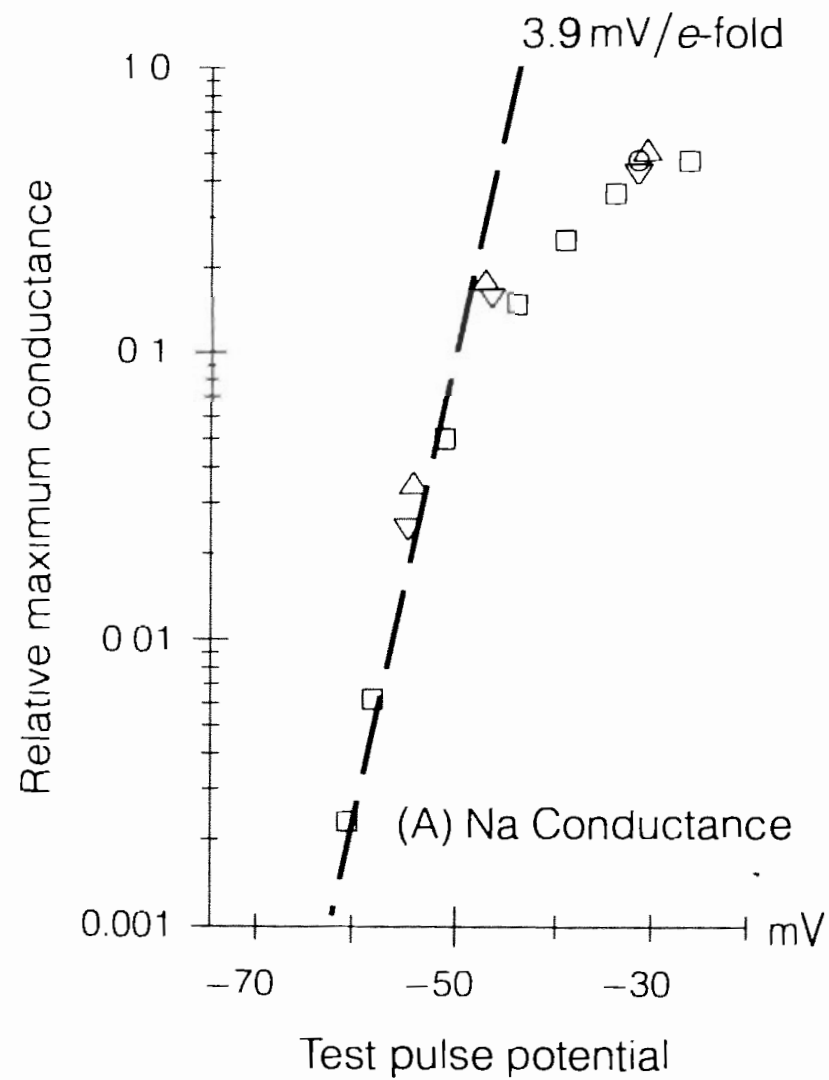
$$kT \ln \frac{N}{N_0} = -wh$$

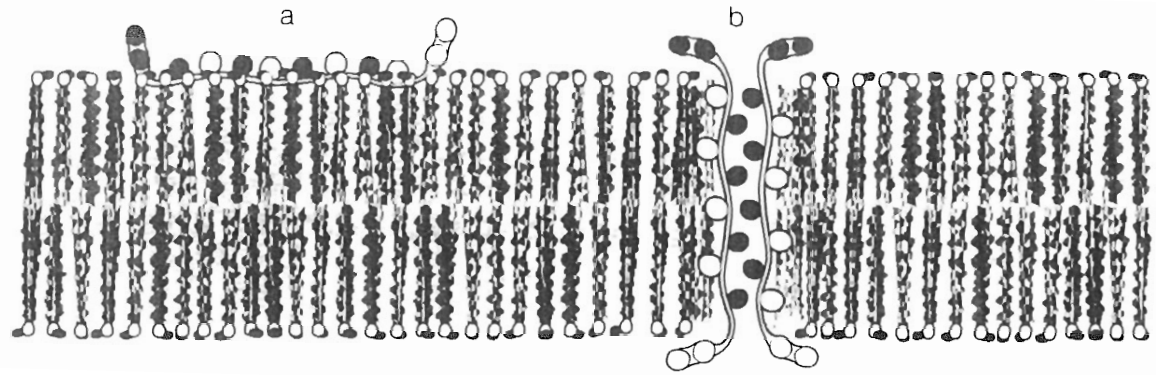
$$N = N_0 e^{-\frac{wh}{kT}}$$

or, for charge in an electric field:

$$N = N_0 e^{-\frac{q}{kT} V}$$

Voltage-gated channels





$$\frac{N_o}{N_c} = e^{-E_t/(kT)}$$

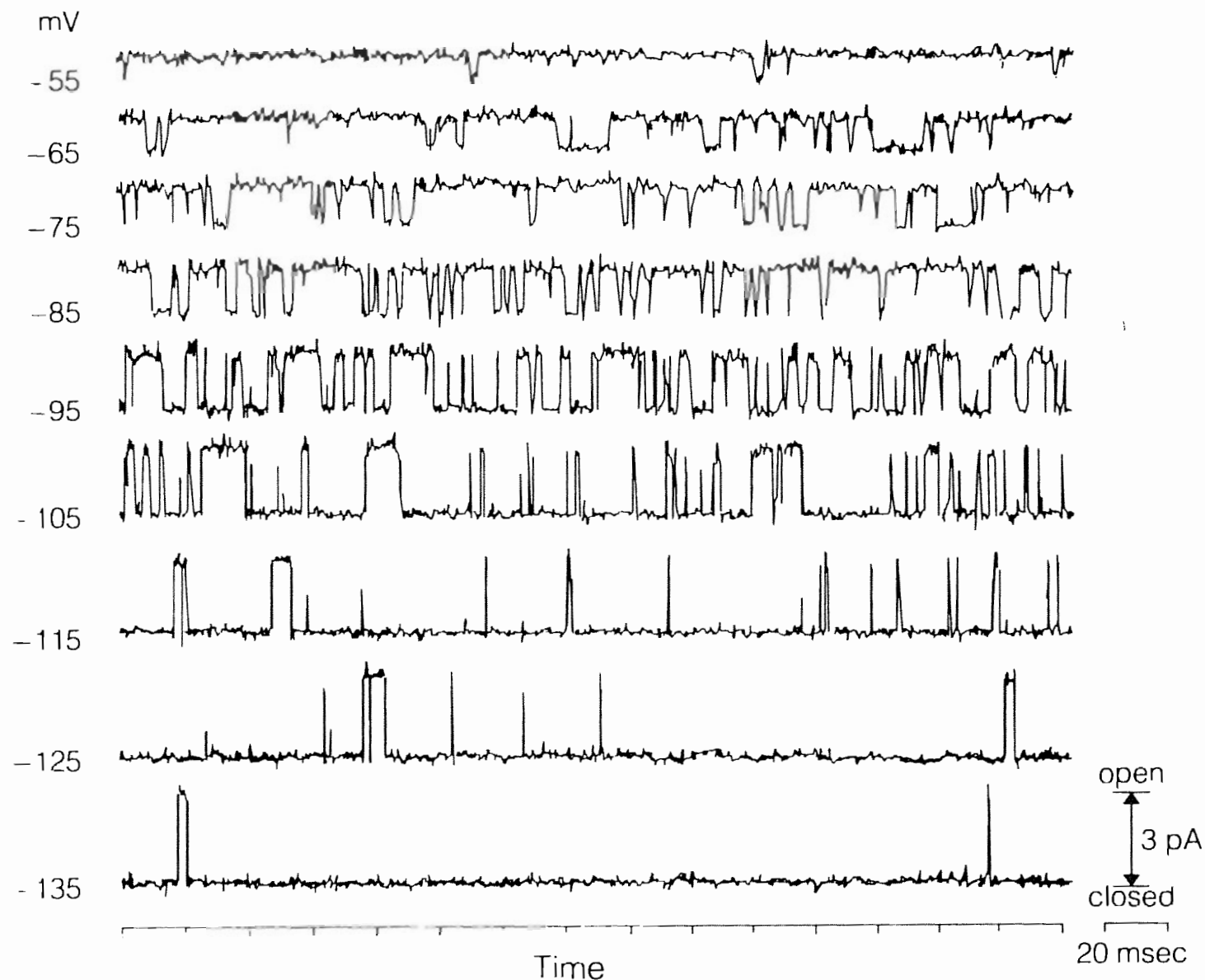
$$E_t = E_0 - Vnq$$

E_t = transition energy

E_0 = transition energy at $V=0$

$$\frac{\theta}{1 - \theta} = e^{-E_0/(kT)} e^{qnV/(kT)}$$

$$\theta = N_o/N$$



MOS transistor

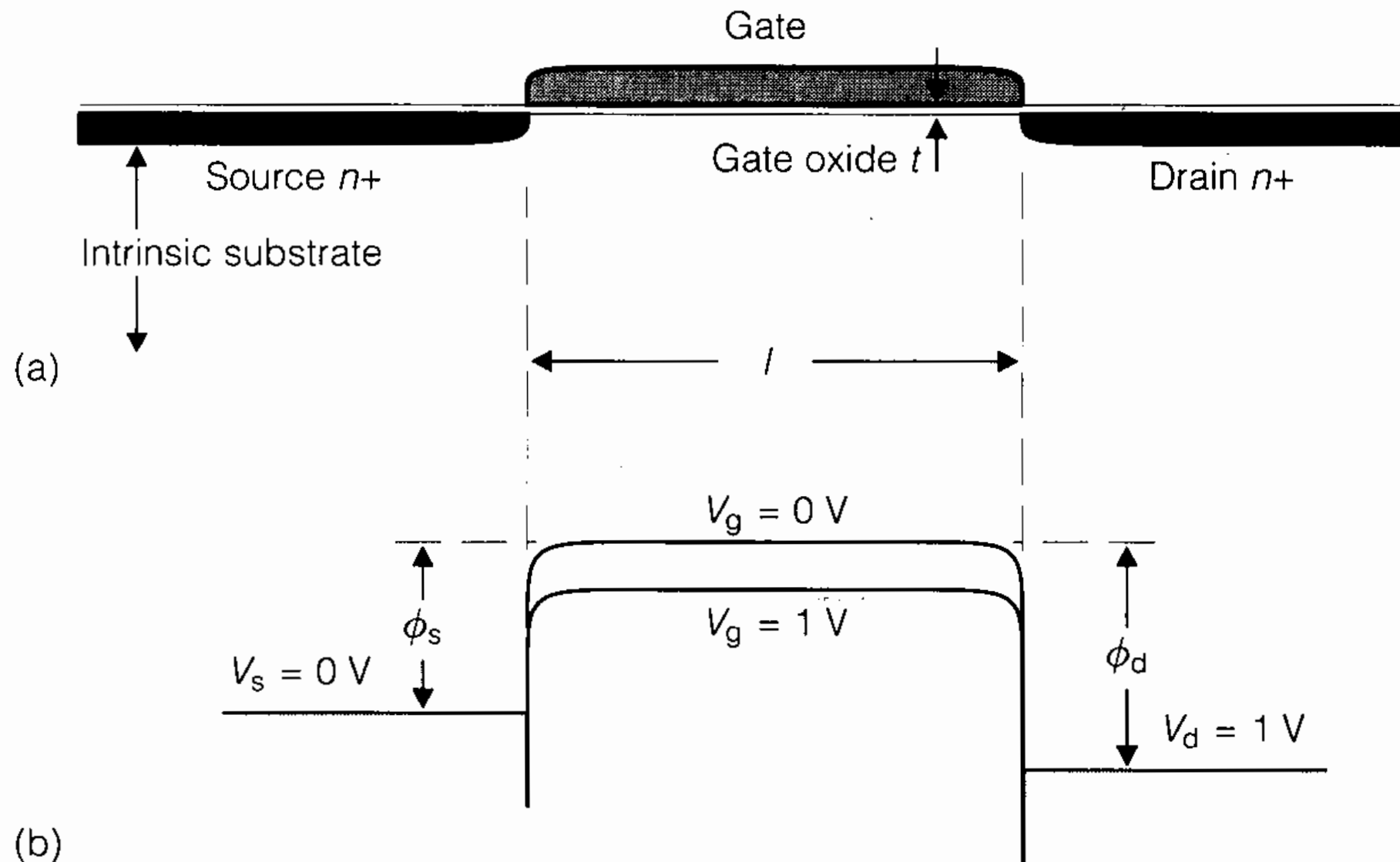
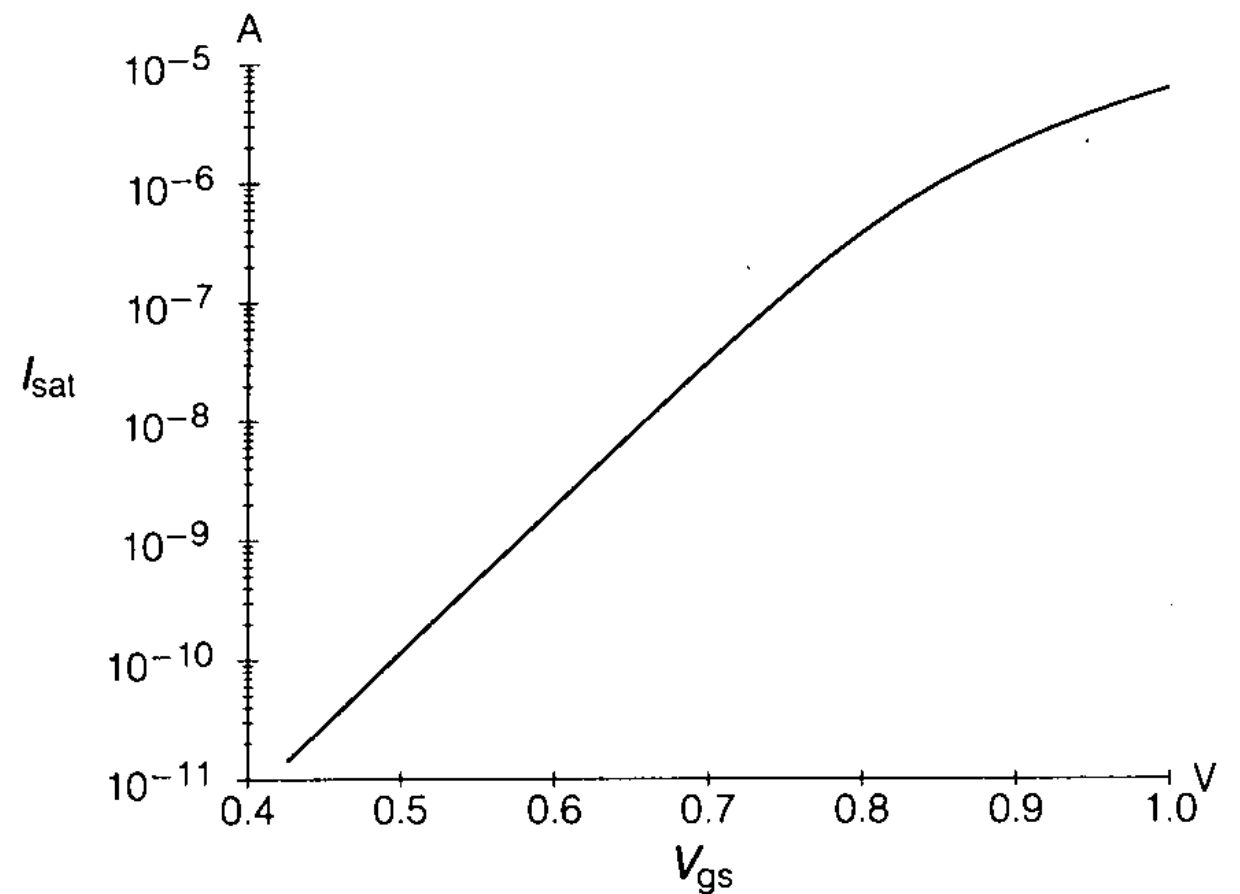
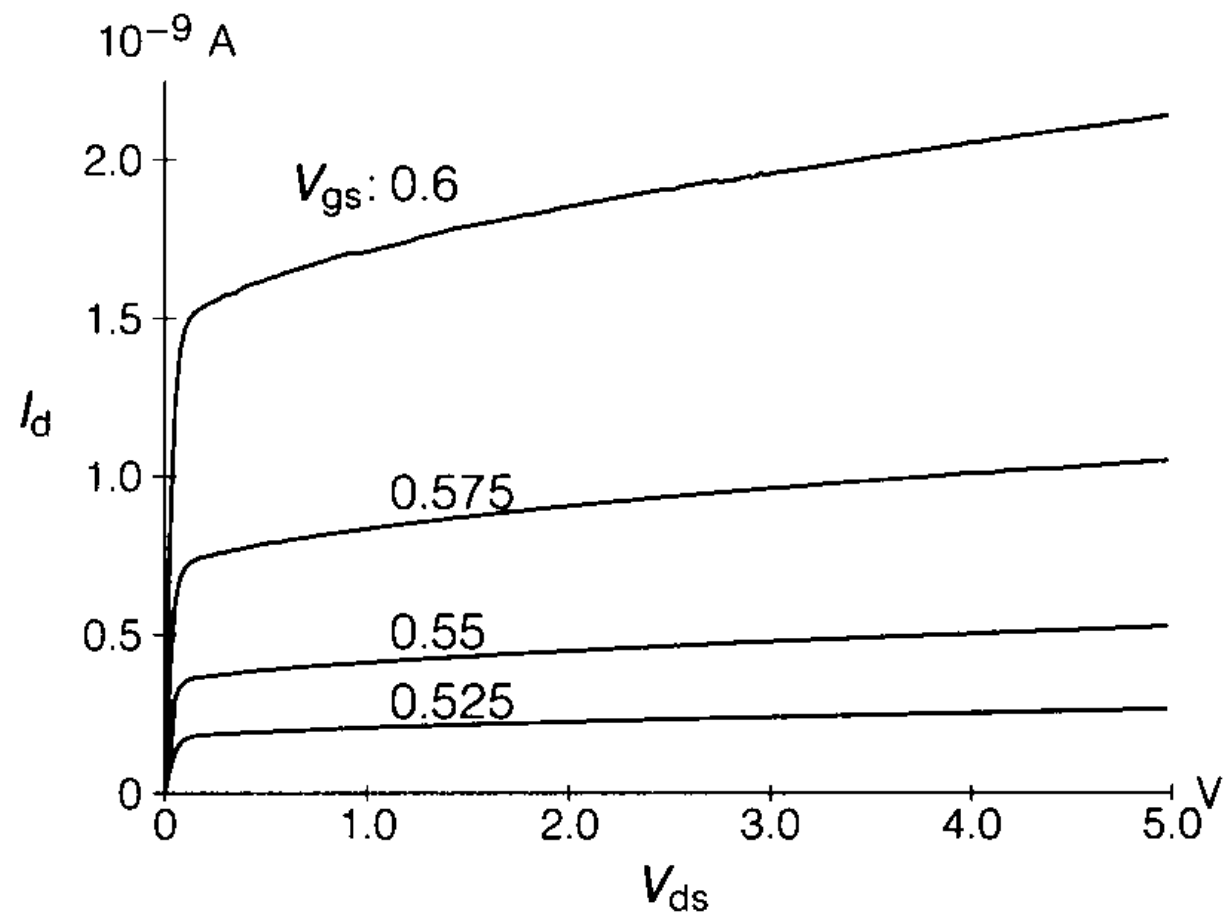


FIGURE 3.4 Cross-section (a) and energy diagram (b) of an n -channel transistor. In a typical 1988 process, the gate-oxide thickness is approximately 400 angstroms (0.04 micron), and the minimum channel length l is approximately 1.5 microns. When the circuit is in operation, the drain is biased positively; hence, the barrier for electrons is greater at the drain than at the source. Applying a positive voltage at the gate lowers the electron barrier at both source and drain, allowing electrons to diffuse from source to drain.

$$I = I_0 e^{-\frac{q V_{gs}}{kT}} \left(1 - e^{-\frac{q V_{ds}}{kT}}\right)$$

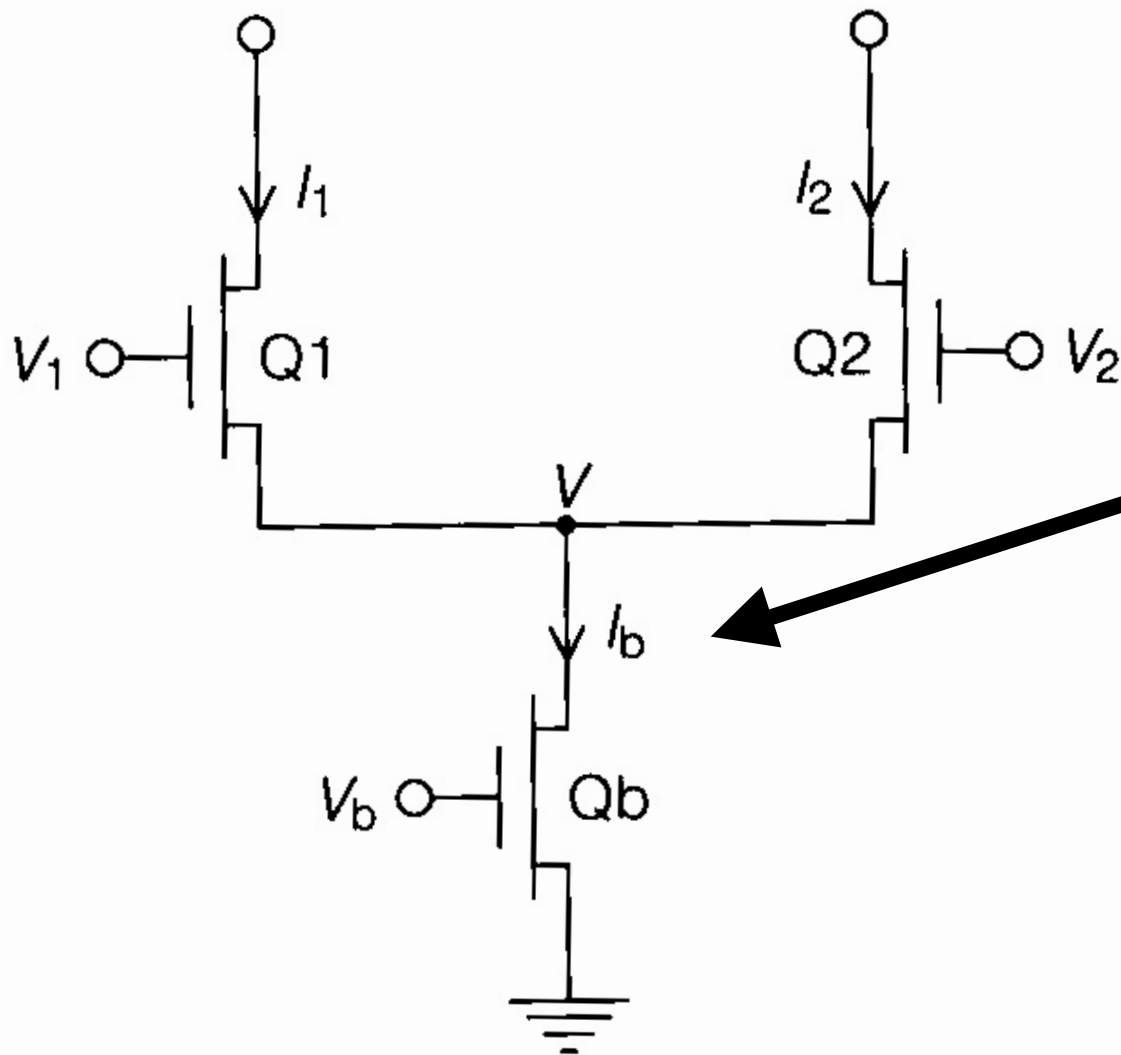
V_{gs} = gate-source voltage
 V_{ds} = drain-source voltage



The exponential current–voltage relation in the nerve is a result of the same physical laws responsible for the exponential transistor characteristic. There is an energy barrier between a state in which current can flow and one in which current cannot flow. The height of that barrier is dependent on a control voltage. The Boltzmann distribution determines the fraction of the total population that is in the conducting state. In the transistor, the electrons in the channel form the population in question, and these same electrons carry the current. In the nerve membrane, the channels form the population in question, and ions in the channels carry the current. In both cases, the number of individual charges in transit is exponential in the control voltage, and the transport of these charges results in a current that varies exponentially with the control voltage.

Transconductance amplifier

Differential pair



$$I_1 = I_0 e^{\kappa V_1 - V} \quad \text{and} \quad I_2 = I_0 e^{\kappa V_2 - V}$$

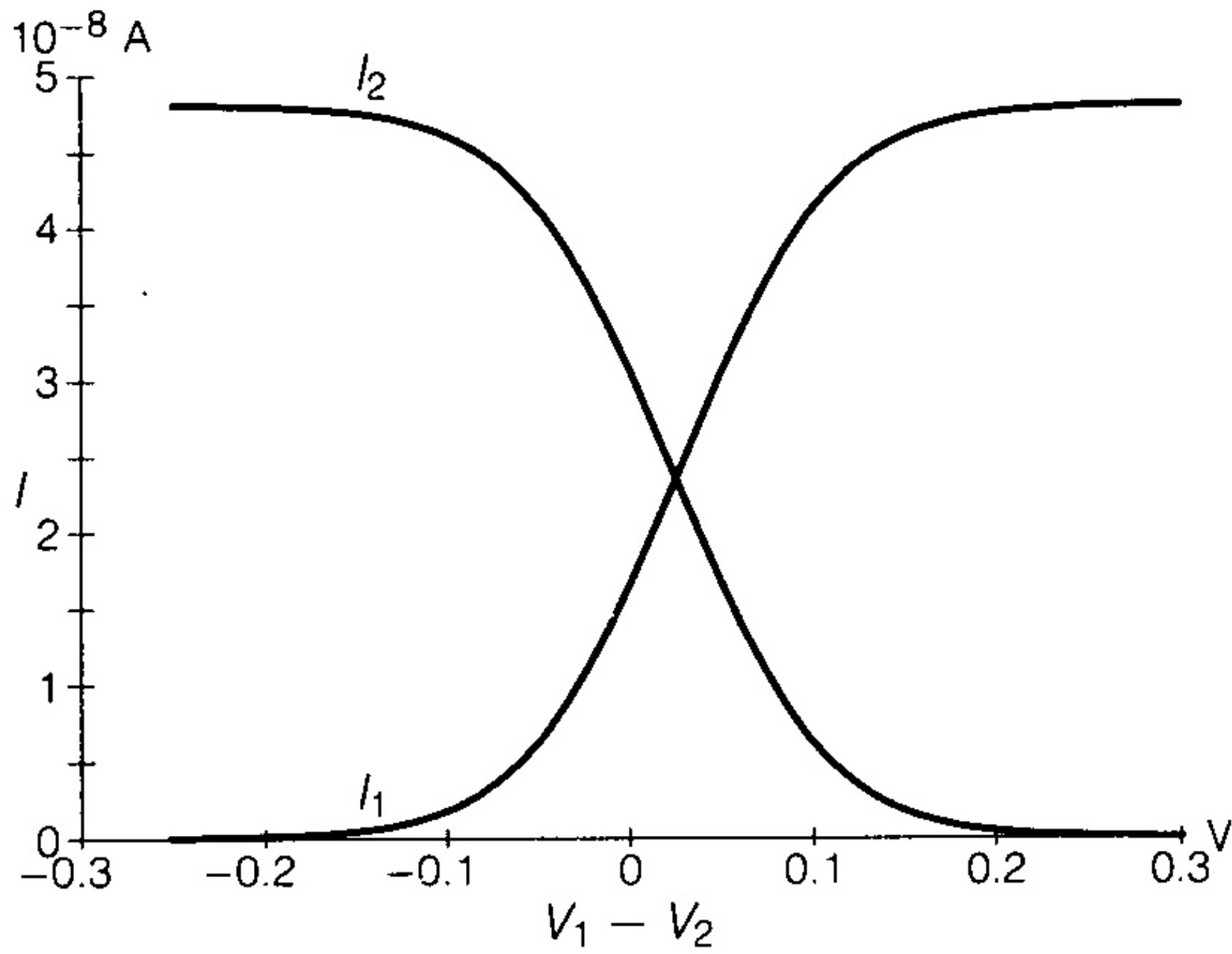
$$I_b = I_1 + I_2 = I_0 e^{-V} (e^{\kappa V_1} + e^{\kappa V_2})$$

$$\Rightarrow e^{-V} = \frac{I_b}{I_0} \frac{1}{e^{\kappa V_1} + e^{\kappa V_2}}$$

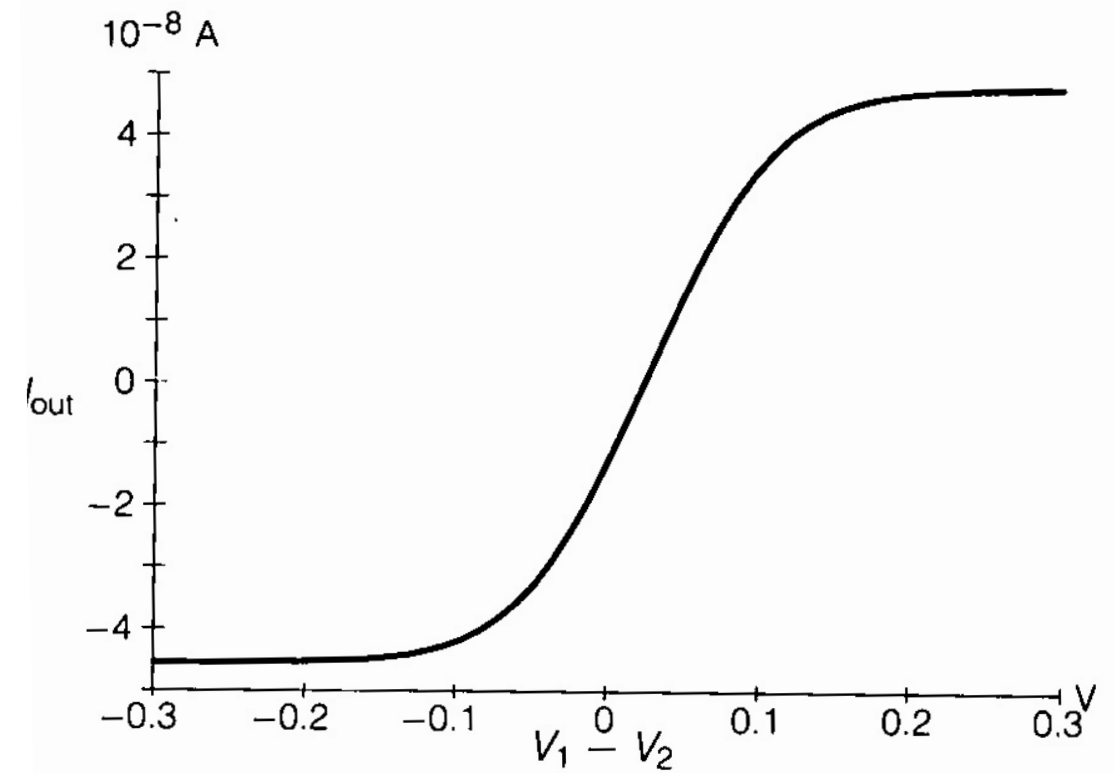
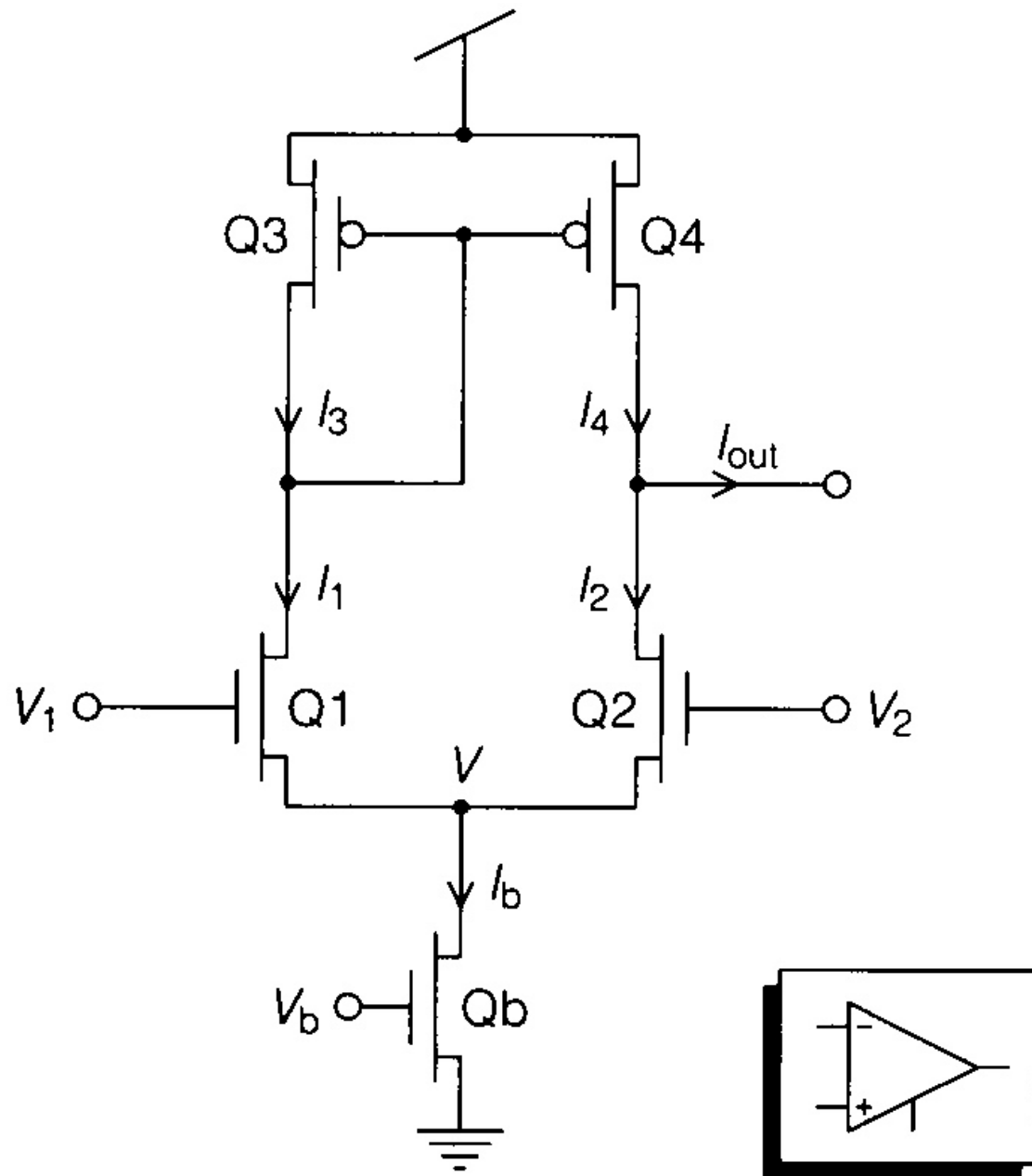
$$I_1 = I_b \frac{e^{\kappa V_1}}{e^{\kappa V_1} + e^{\kappa V_2}}$$

$$I_2 = I_b \frac{e^{\kappa V_2}}{e^{\kappa V_1} + e^{\kappa V_2}}$$

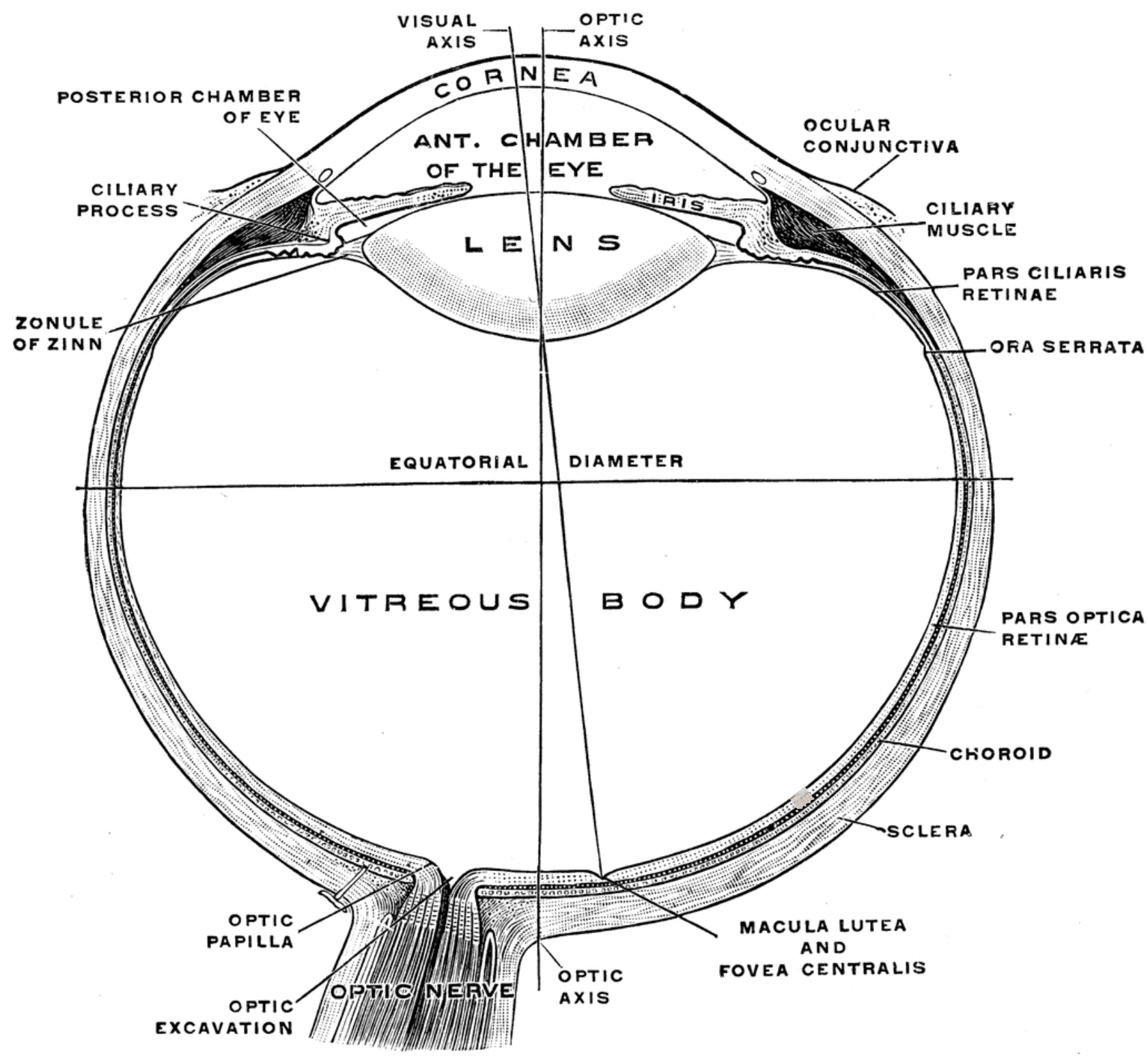
Differential pair

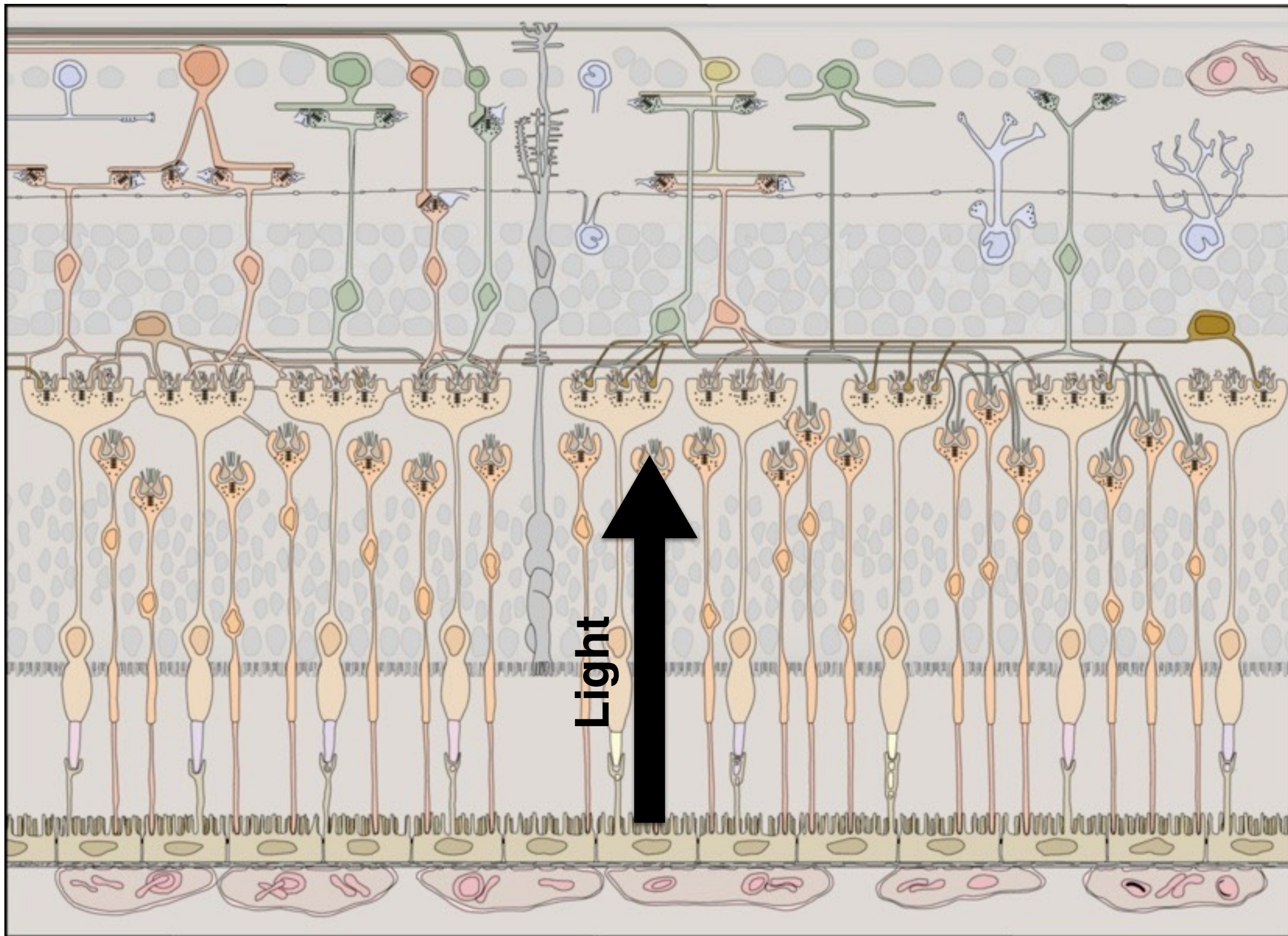


Transconductance amplifier

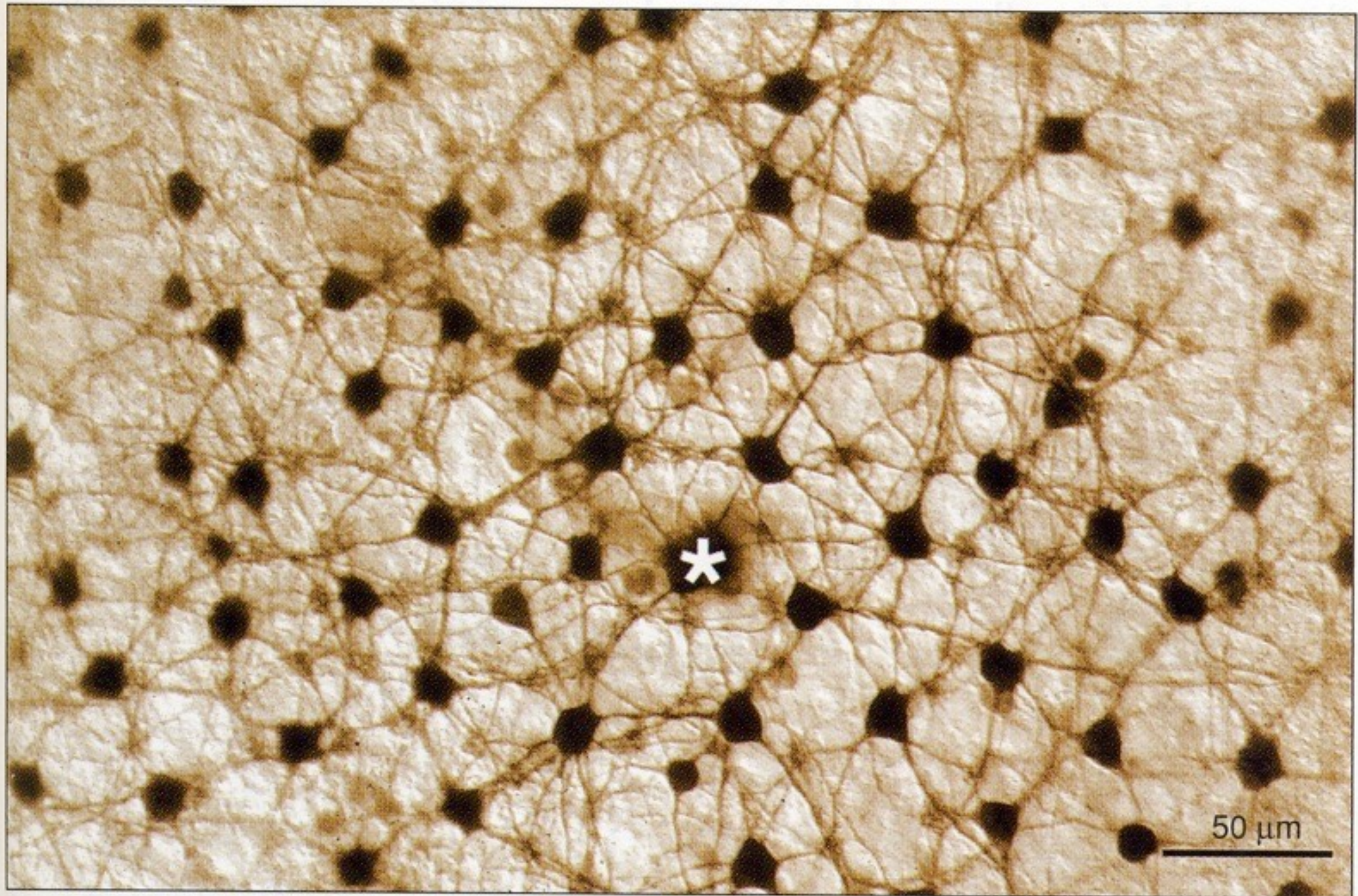


Silicon retina





HI horizontal cells connected via gap junctions

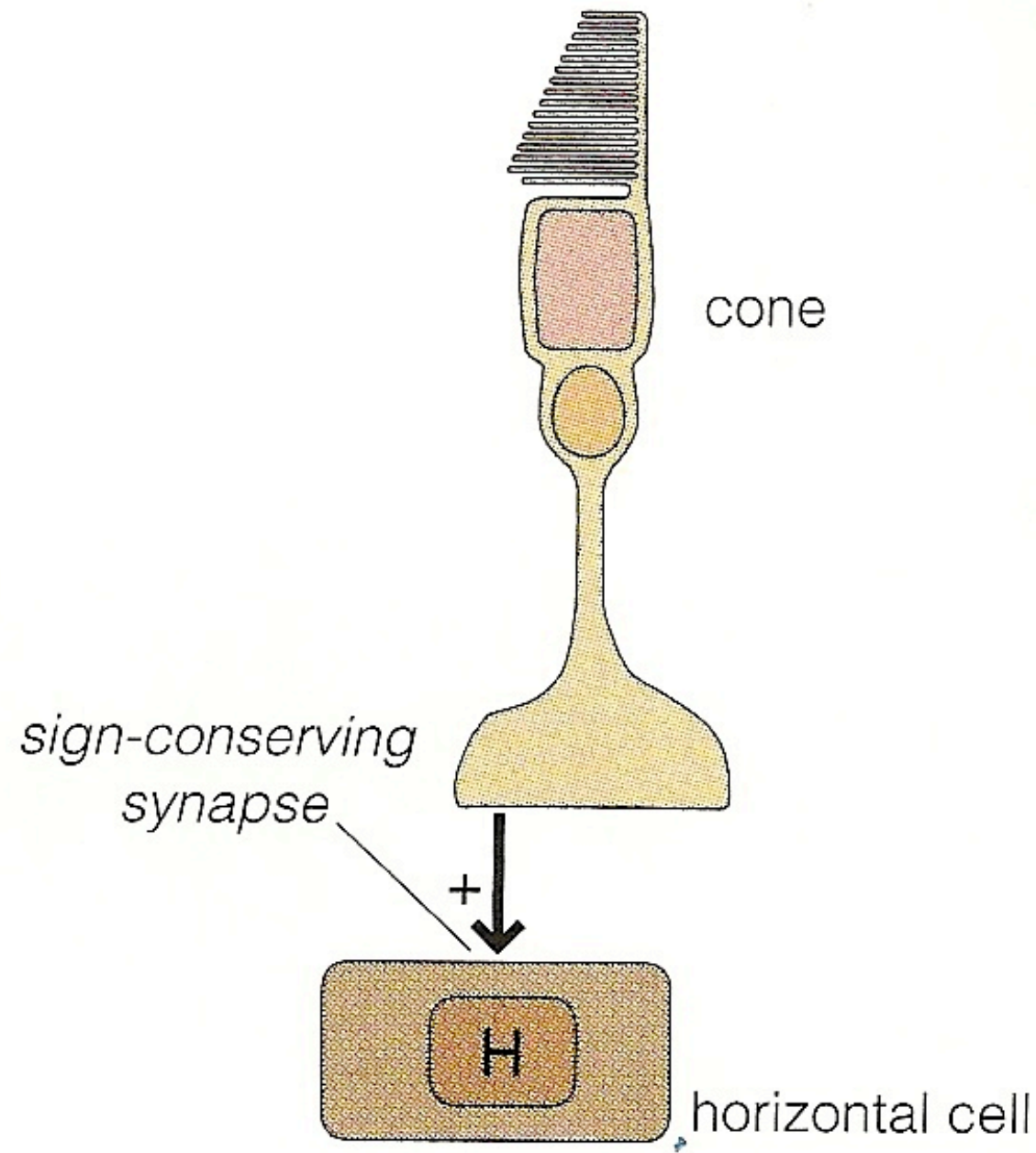


HI horizontal cells labeled following injection of one HI cell (*)

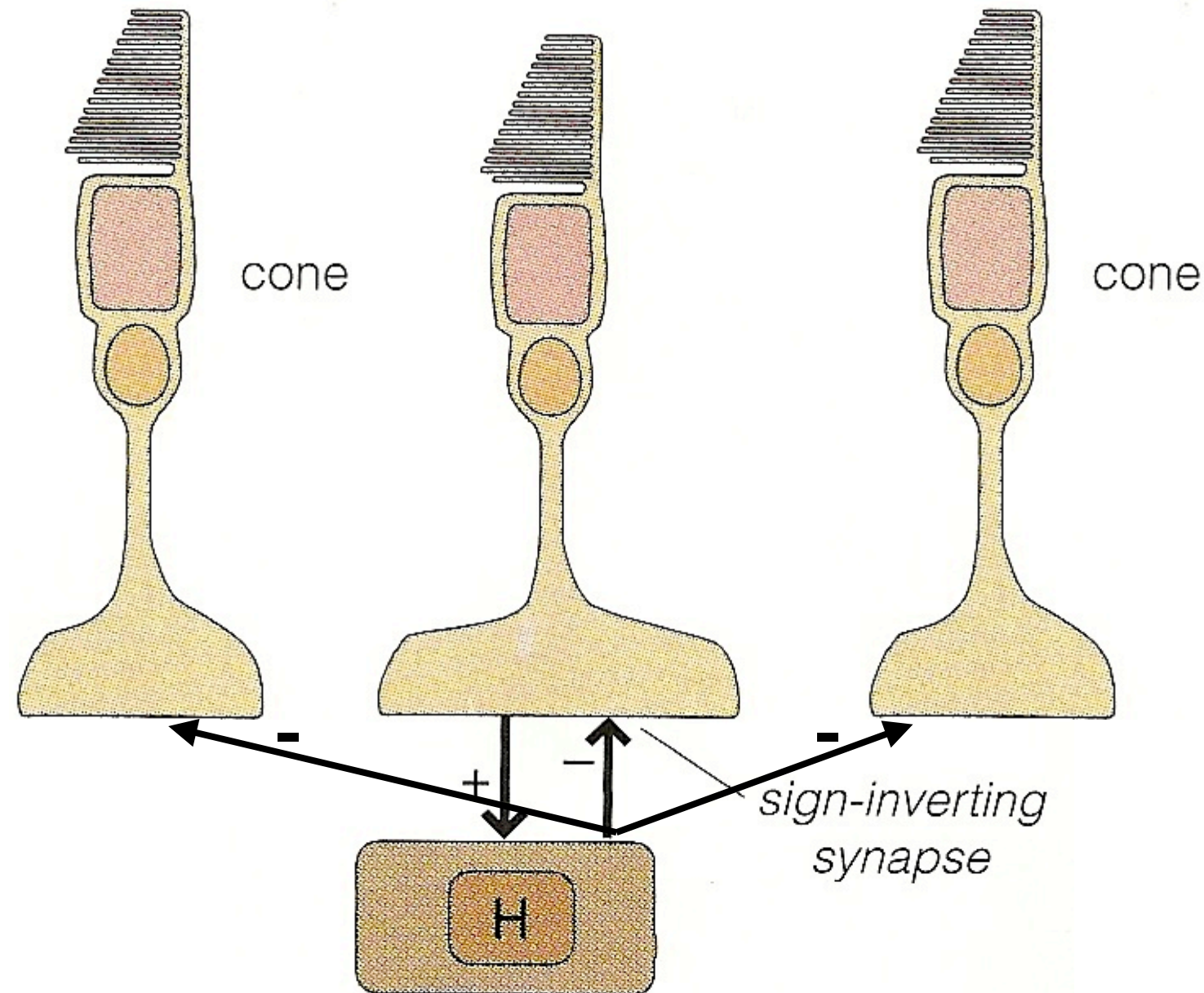
×300

after Dacey, Lee, and Stafford, 1996

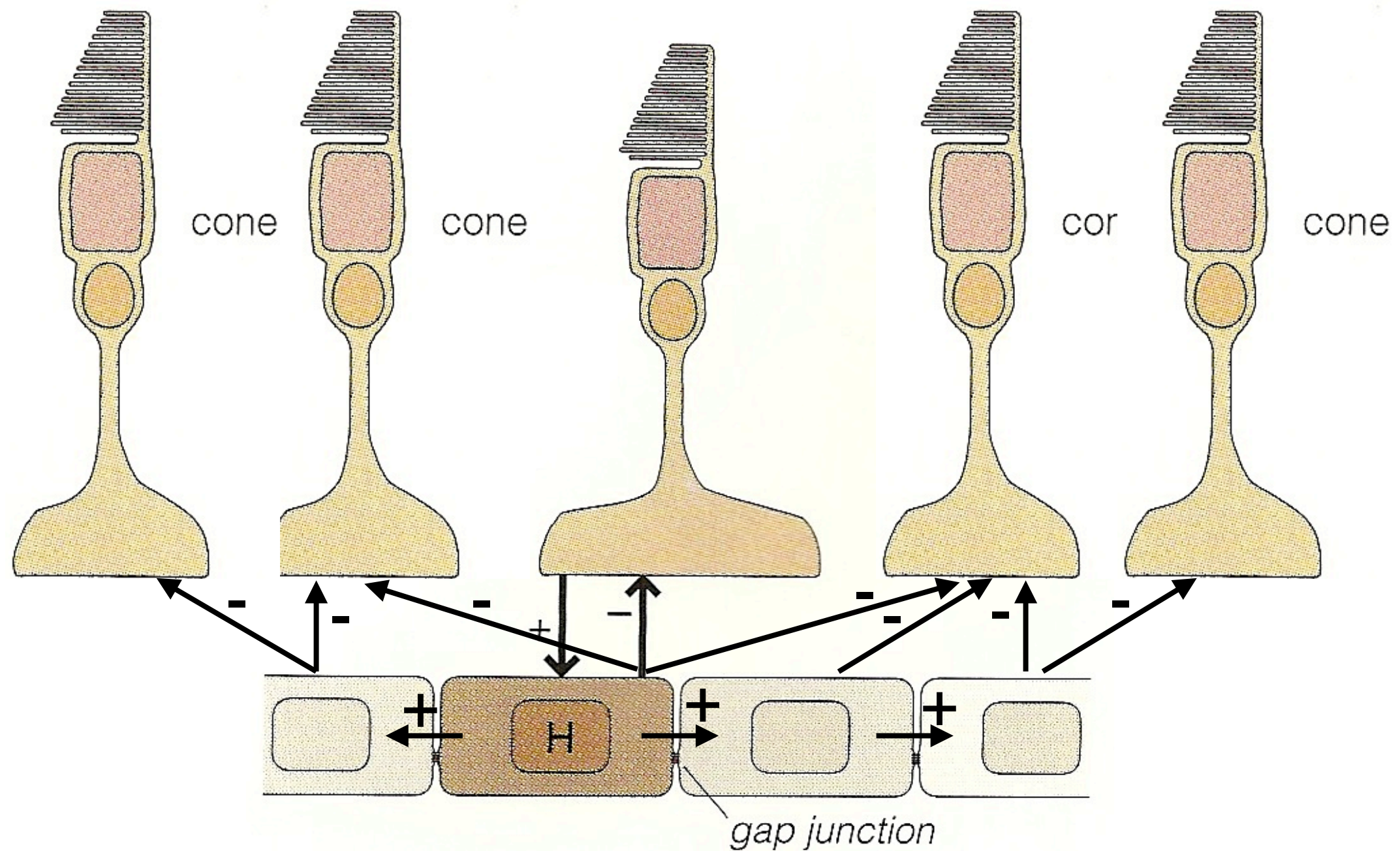
Hyperpolarization of photoreceptor results in hyperpolarization of horizontal cells



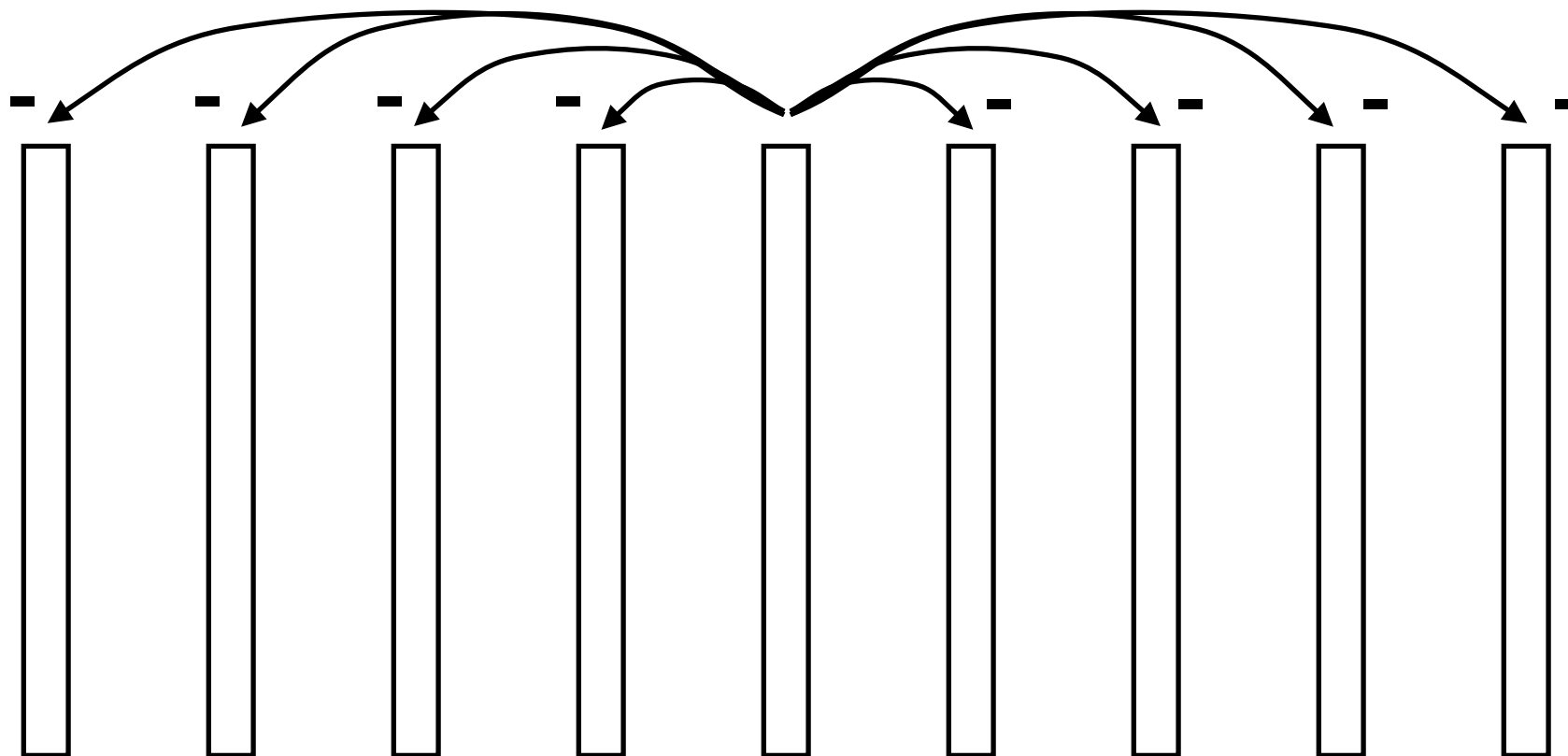
Hyperpolarization of horizontal cell results in depolarization of photoreceptors



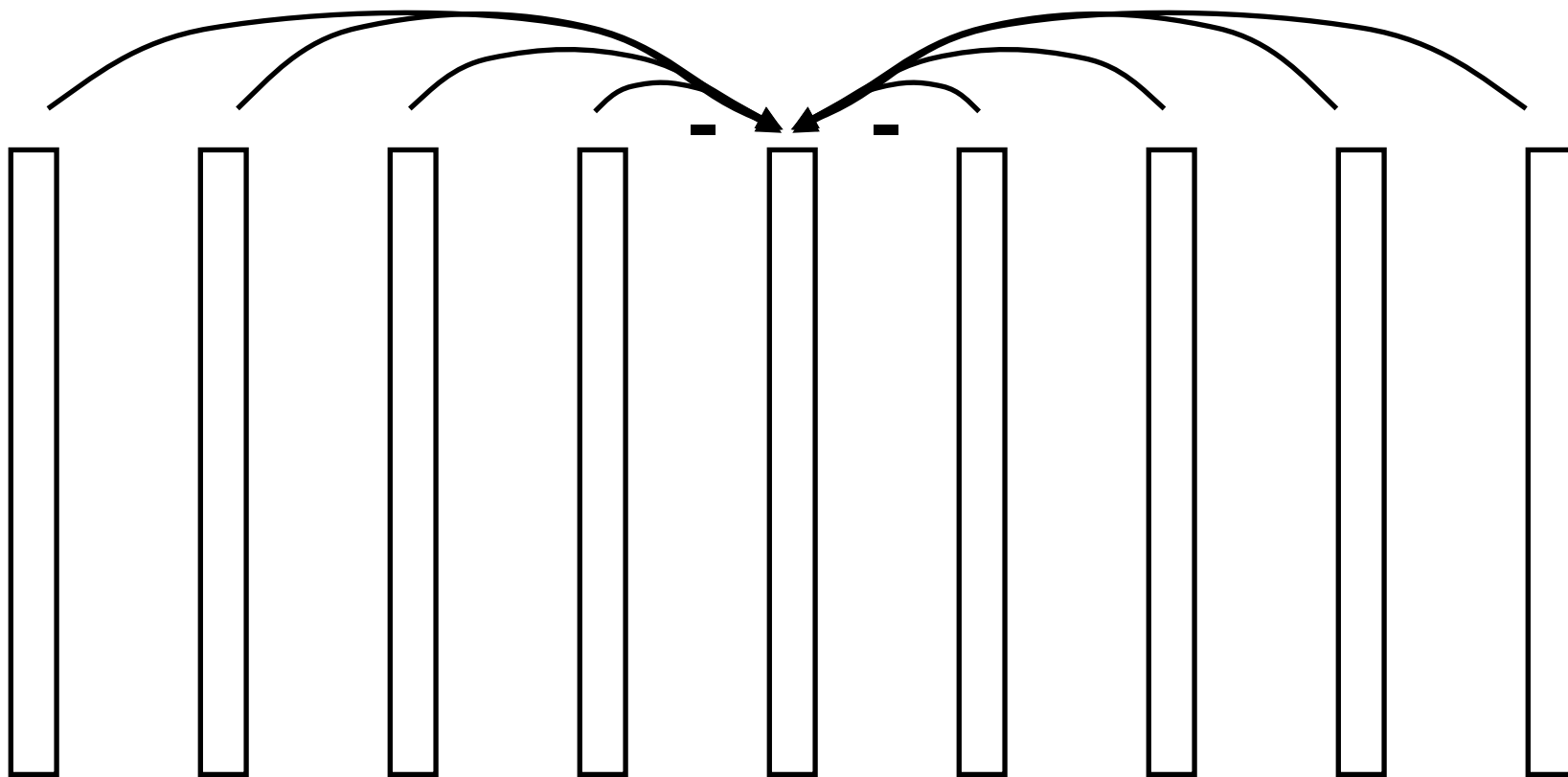
Hyperpolarization of horizontal cell spreads to other horizontal cells via gap junctions



Lateral inhibition

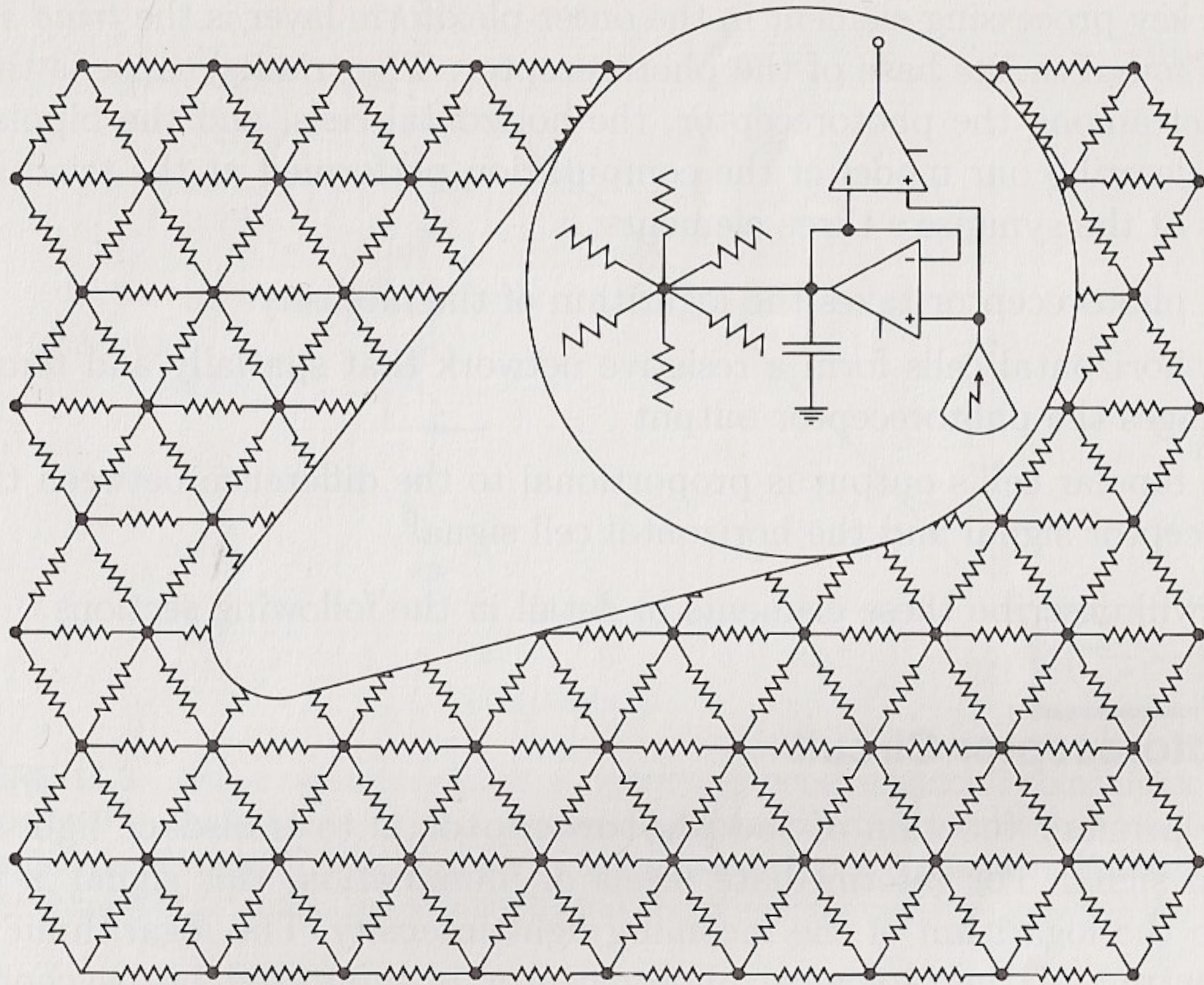


Lateral inhibition



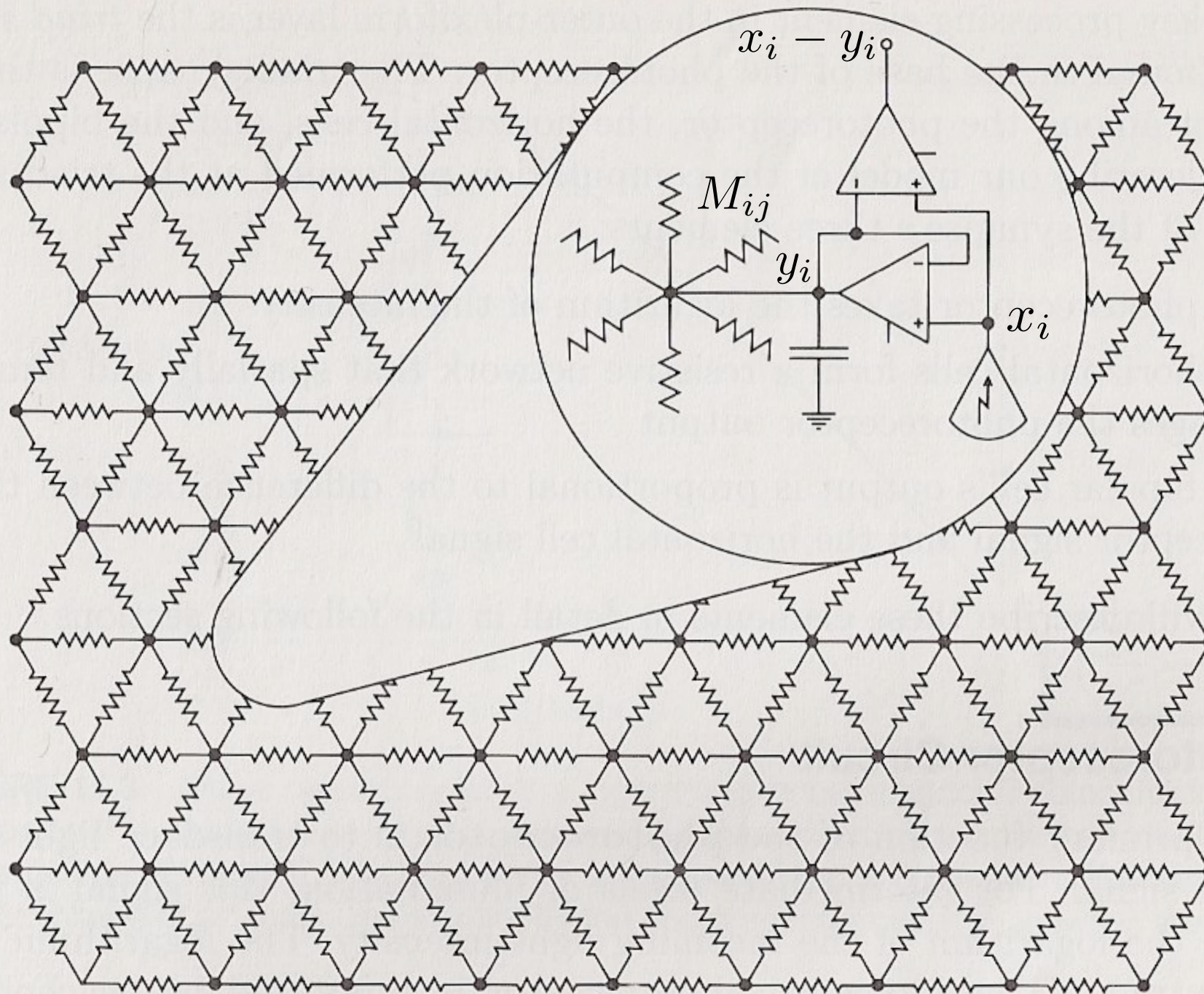
Analog VLSI retina

(Mead & Mahowald, 1989)



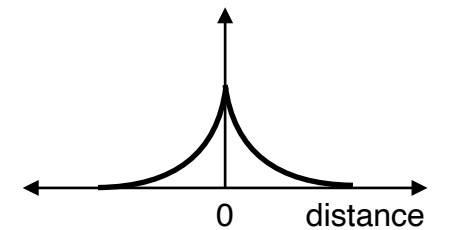
Analog VLSI retina

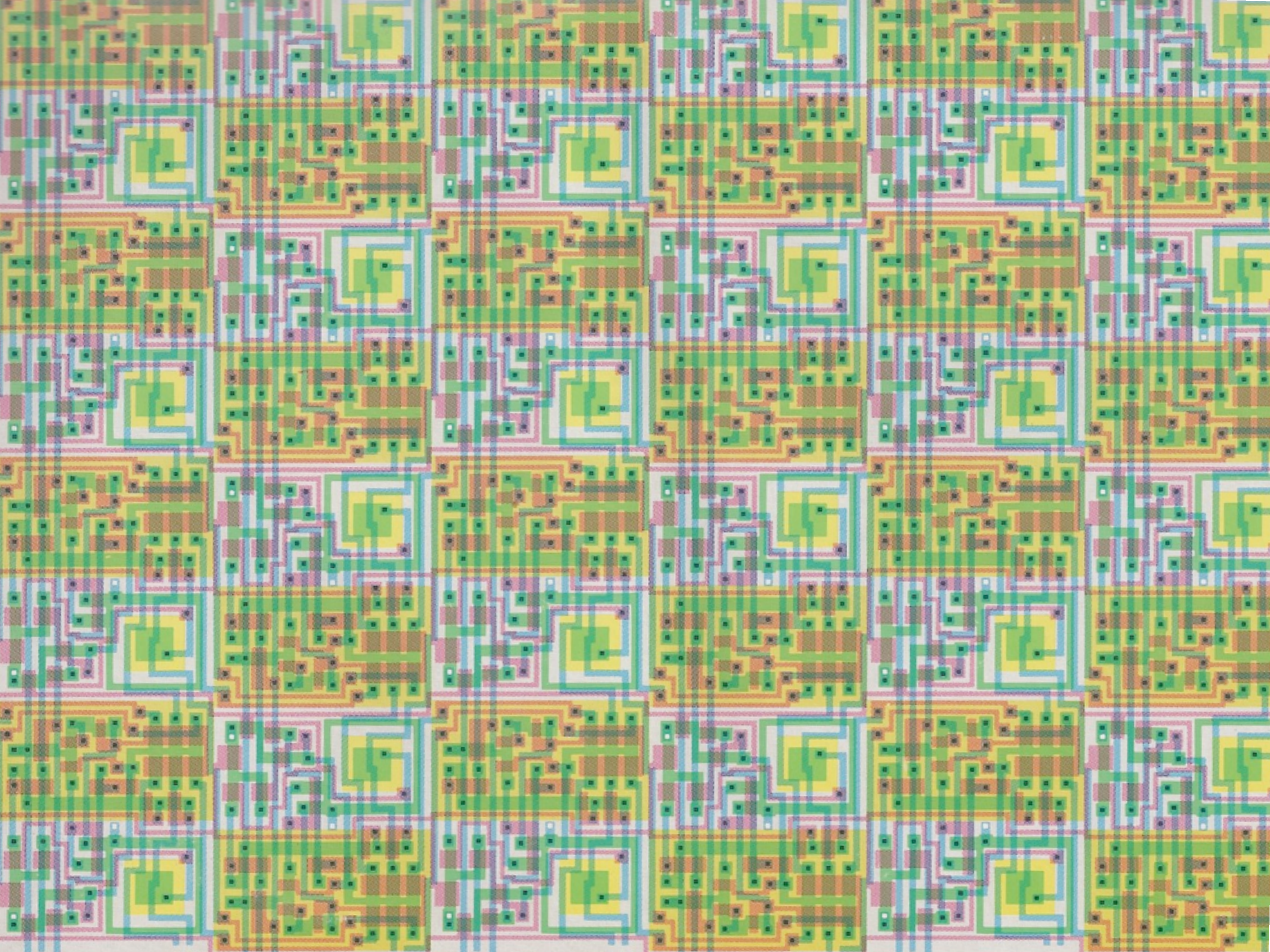
(Mead & Mahowald, 1988)



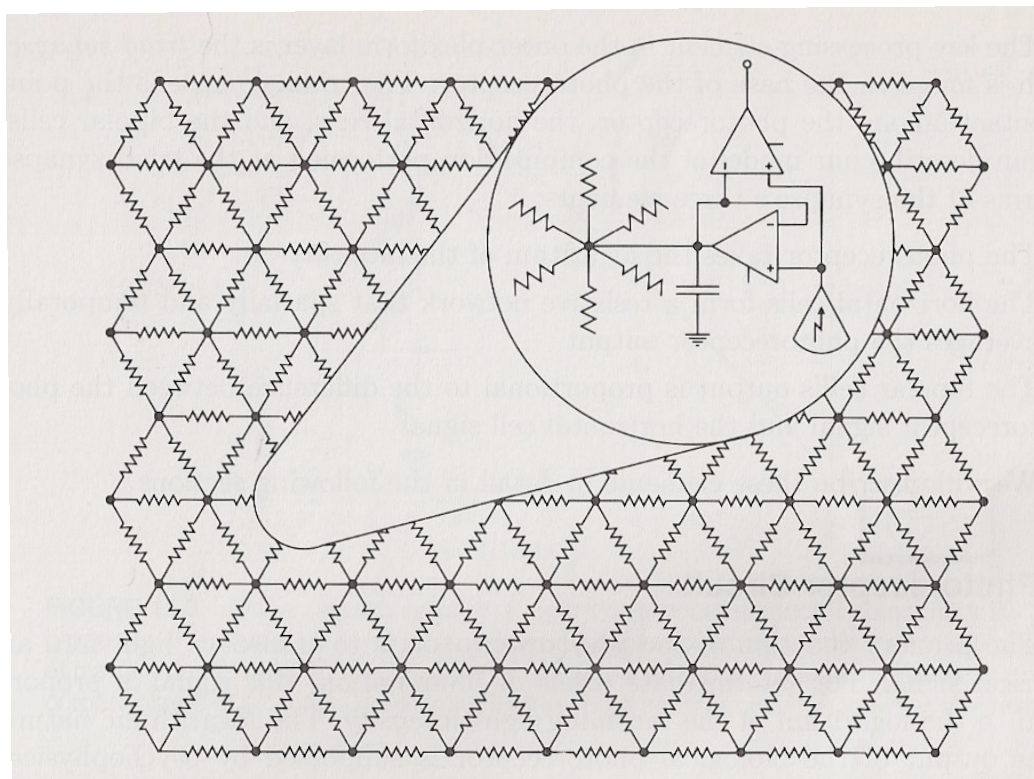
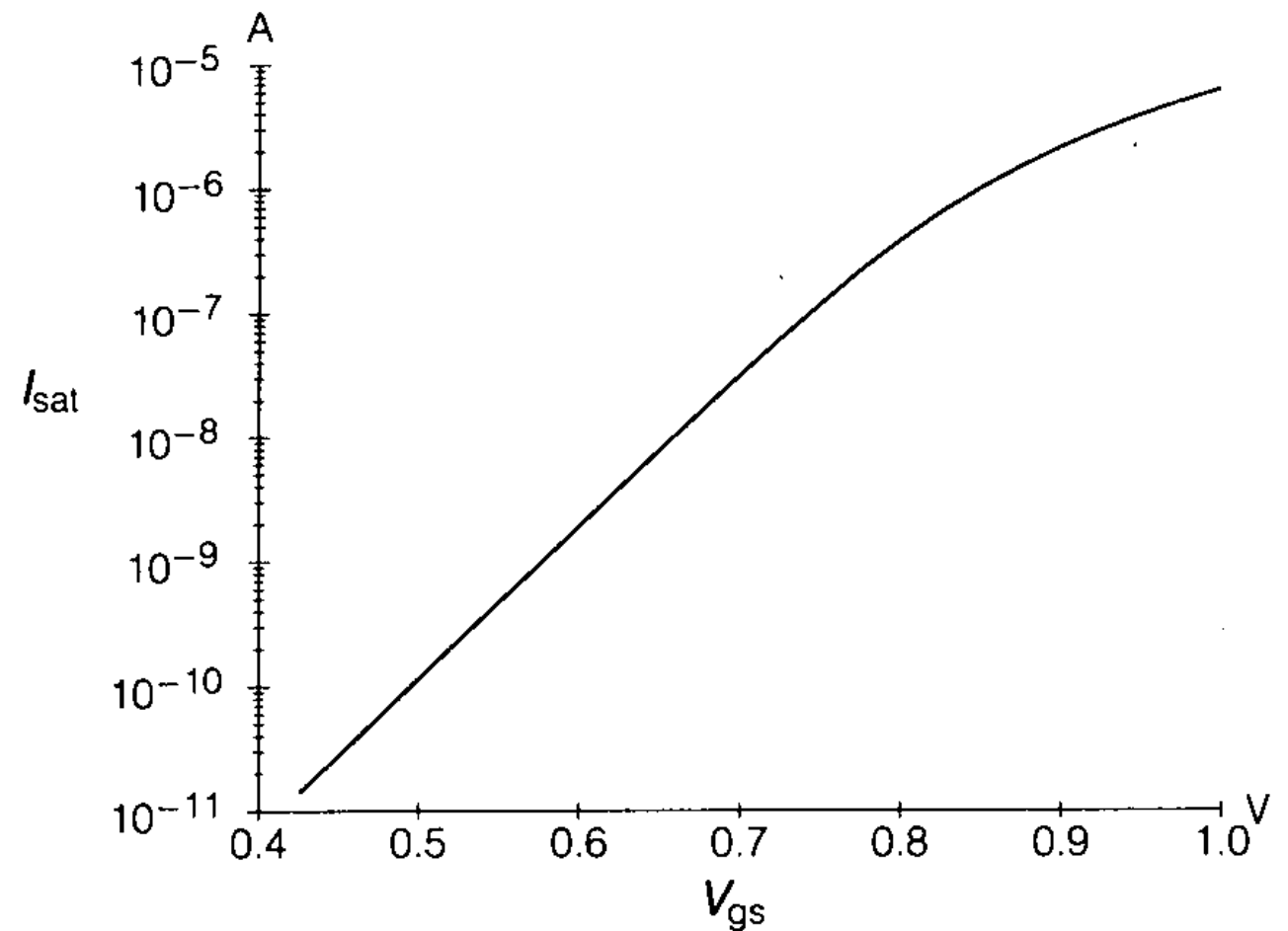
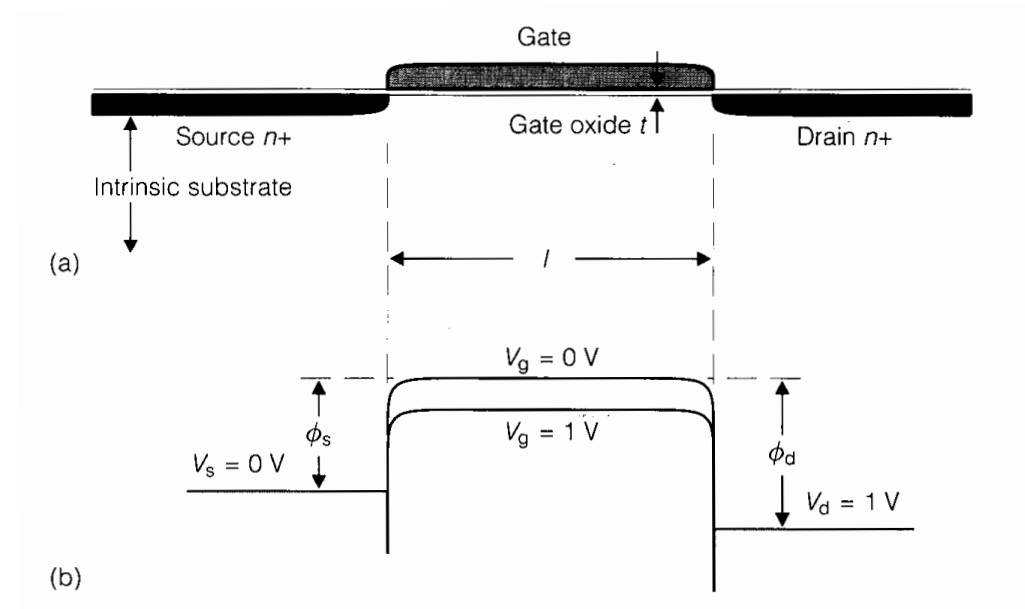
$$\tau \dot{y} + y = M y + x$$

$$y = \underbrace{(I - M)^{-1}} x$$

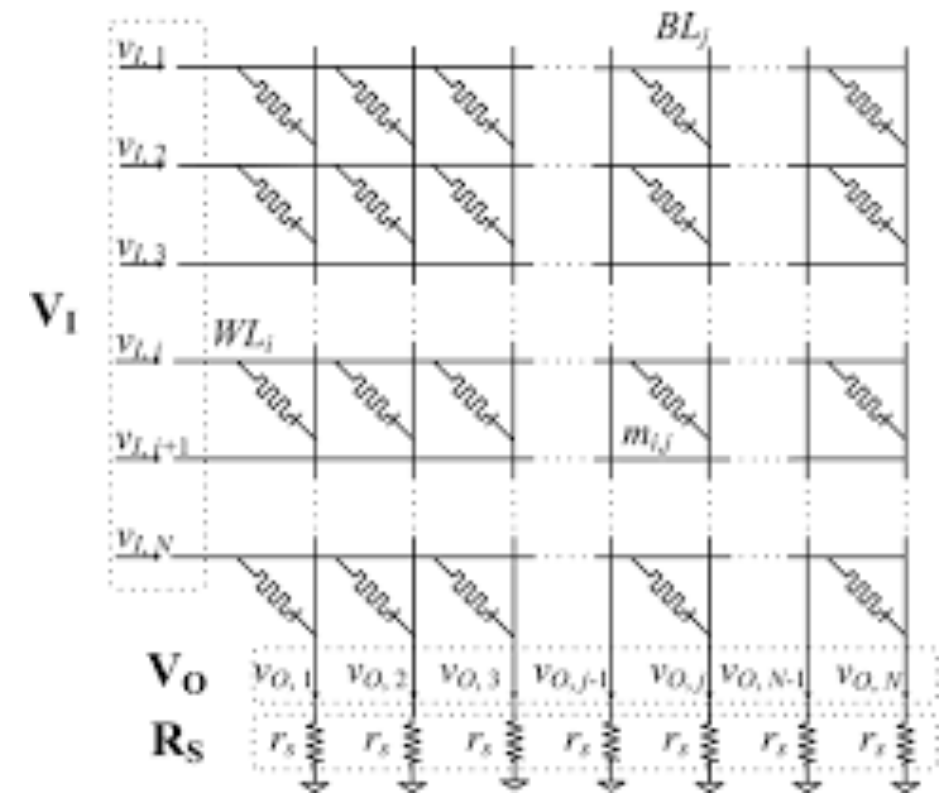
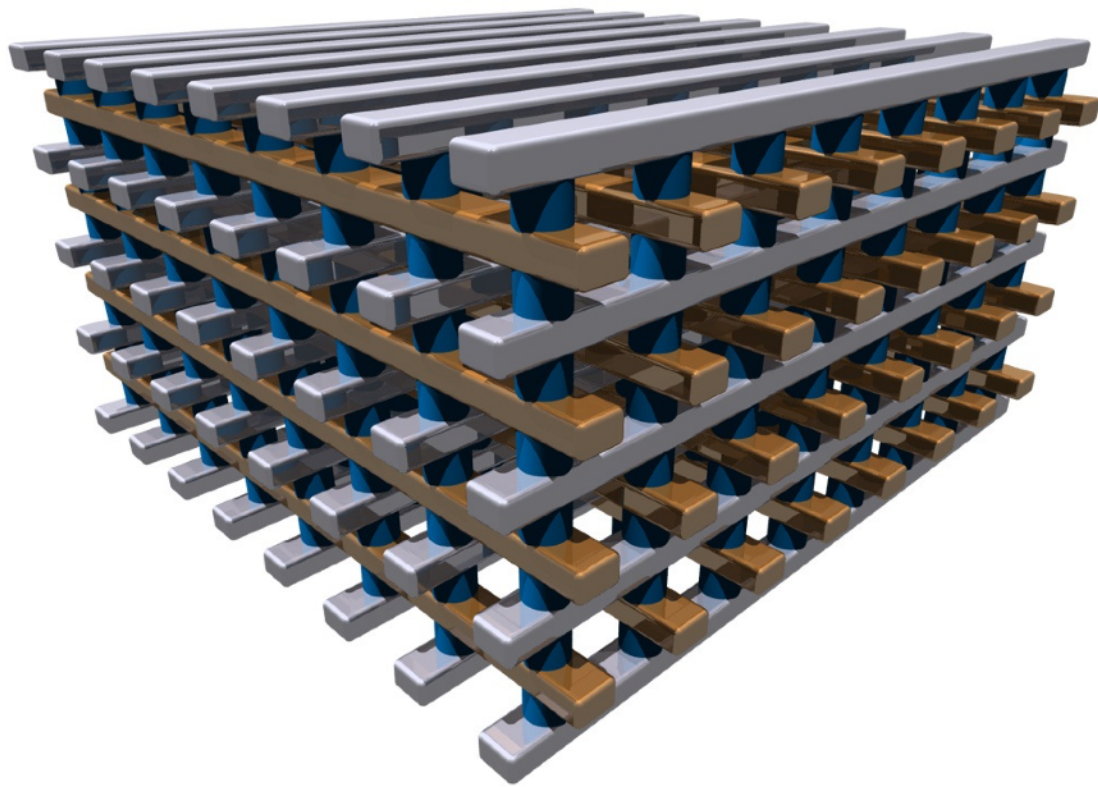




Analog VLSI (or neuromorphic computing) exploits intrinsic transistor physics and laws of electronics (Kirchhoff's law, Ohm's law) to do computation



3D RRAM crossbar array



Solving matrix equations in one step with cross-point resistive arrays

Zhong Sun^a, Giacomo Pedretti^a, Elia Ambrosi^a, Alessandro Bricalli^a, Wei Wang^a, and Daniele Ielmini^{a,1}

^aDipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano, 20133 Milan, Italy

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