

# Solving classification problems with HD computing/VSA

Neuroscience 299: Module 9  
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# About me

- BS Mechatronics Engineering: Tecnológico de Monterrey, Mexico, 2012.
- MS Systems and Control: Eindhoven University of Technology, The Netherlands, 2015.
- PhD Electrical Engineering: KU Leuven, Belgium, 2020. Dissertation: Hardware Aware Probabilistic Machine Learning Models.
- Postdoc at Berkeley Wireless Research Center working with Prof. Jan Rabaey since February 2021.

# Outline

- Motivation and goals
- Background
- Classification with HD computing/VSAs
- Application Examples
- Useful resources and questions

# Motivation and goals

# Why should we discuss classification problems?

- Classification is a widely used task in many application domains.
- Amenable to the practical implementation of many of the concepts studied so far.
- Variety of implementations and compelling results are the outcome of interdisciplinary interest in the topic of HD computing/VSAs.

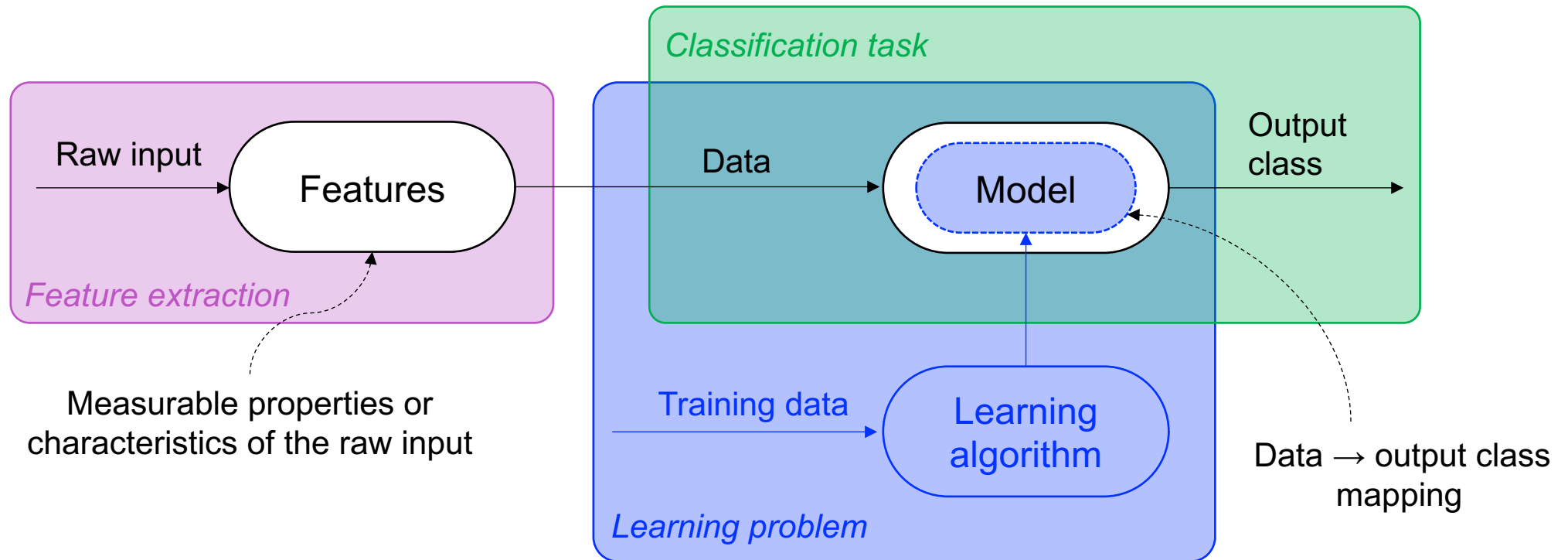
# Goals for this lecture

- An overview of how HD computing/VSAs can be used to solve a classification problem.
- Highlight the different approaches available in the literature for each step of the “classification pipeline”.
- Encourage you to think of how other properties or traits of HD computing/VSAs can be exploited in the classification realm and beyond.

# Background

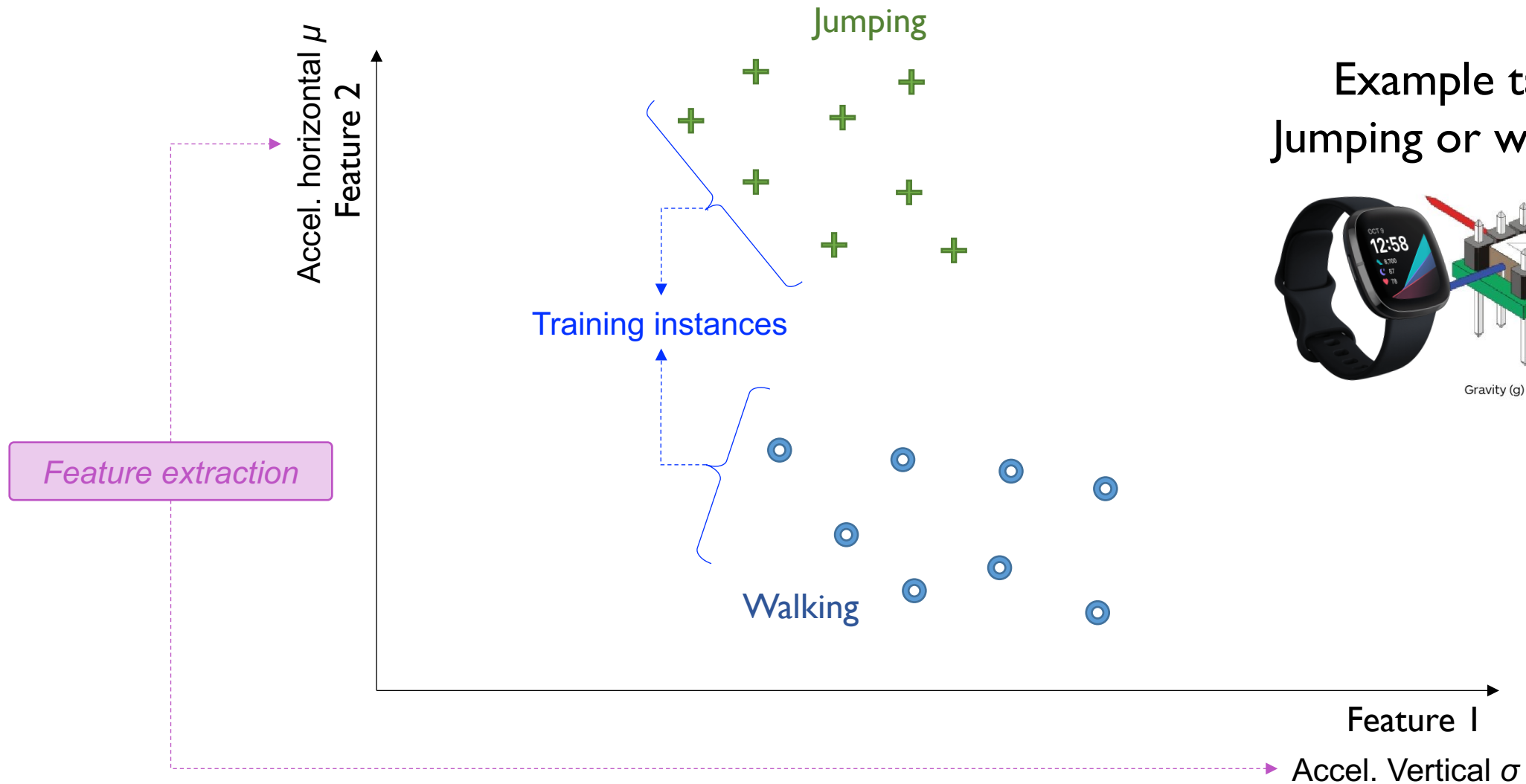
The “classification pipeline”

# The classification pipeline in Machine Learning

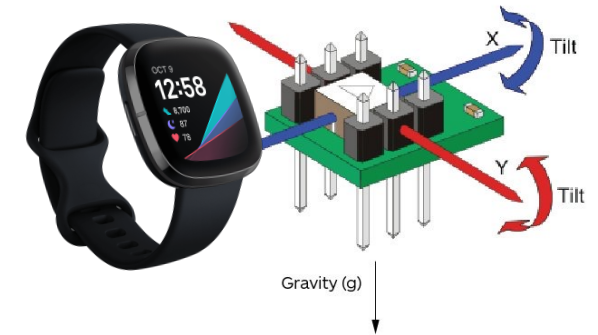




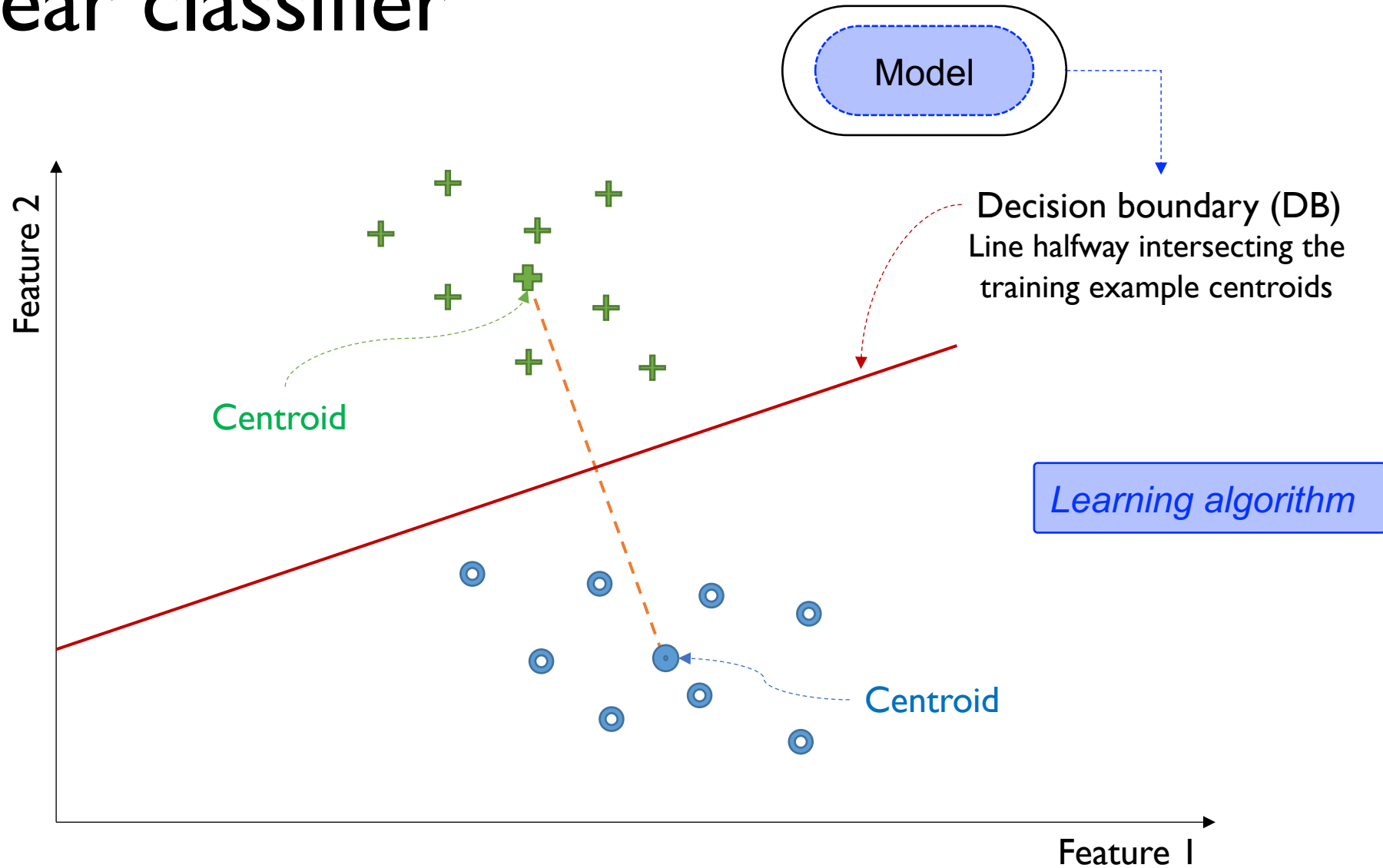
# Basic linear classifier



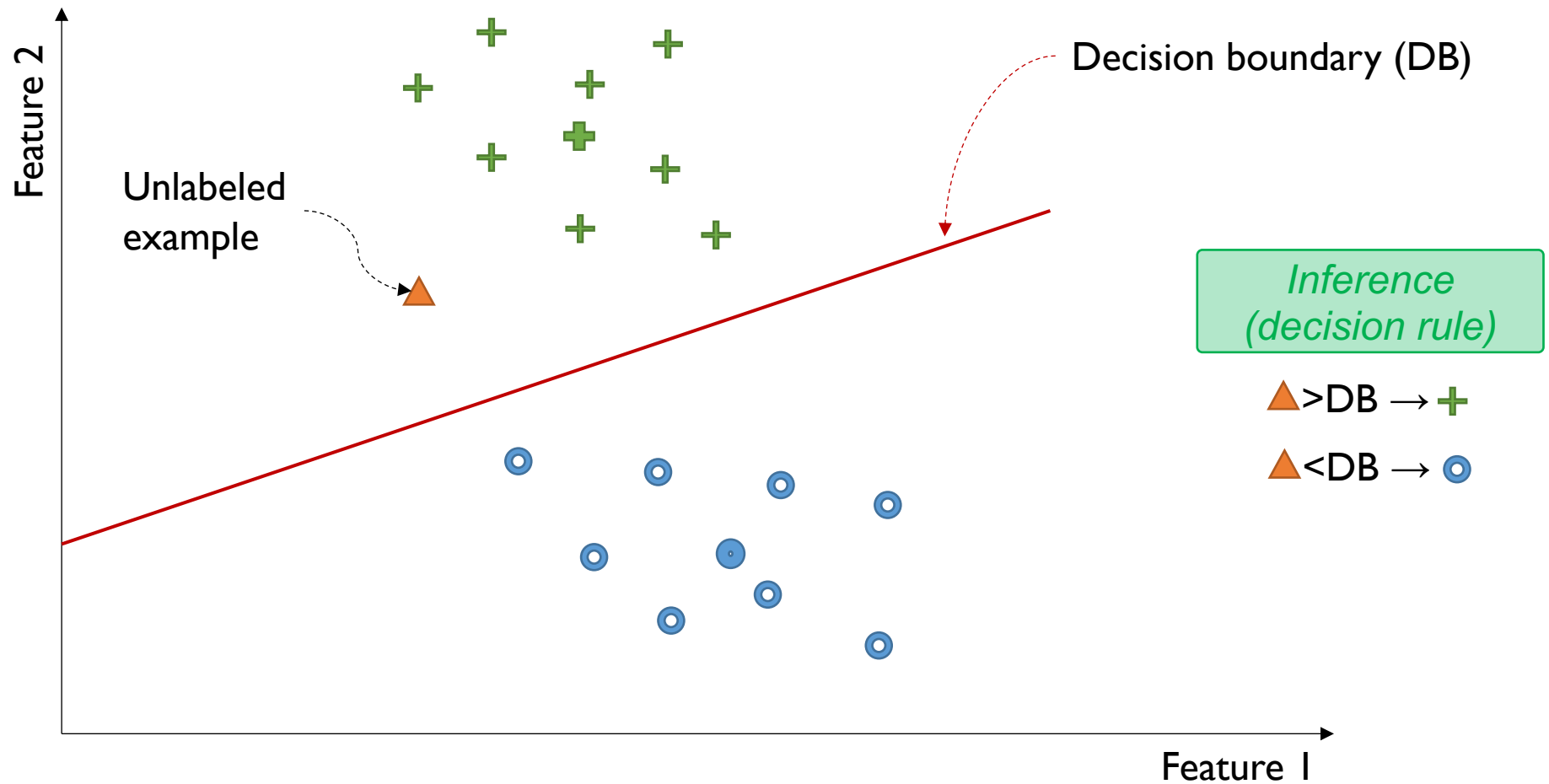
Example task:  
Jumping or walking?



# Basic linear classifier



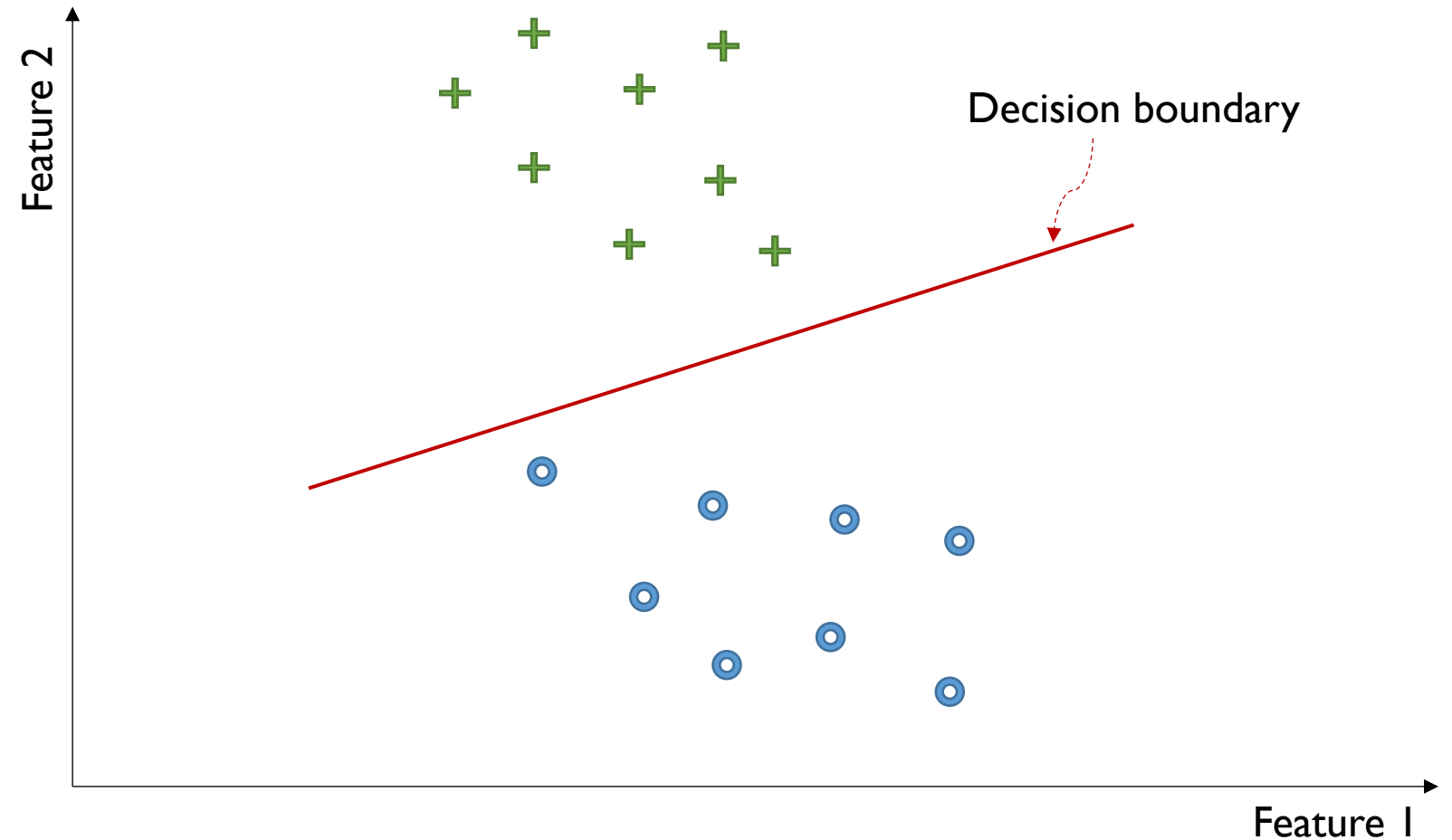
# Basic linear classifier



# Types of approaches/models\*

- **Geometric**

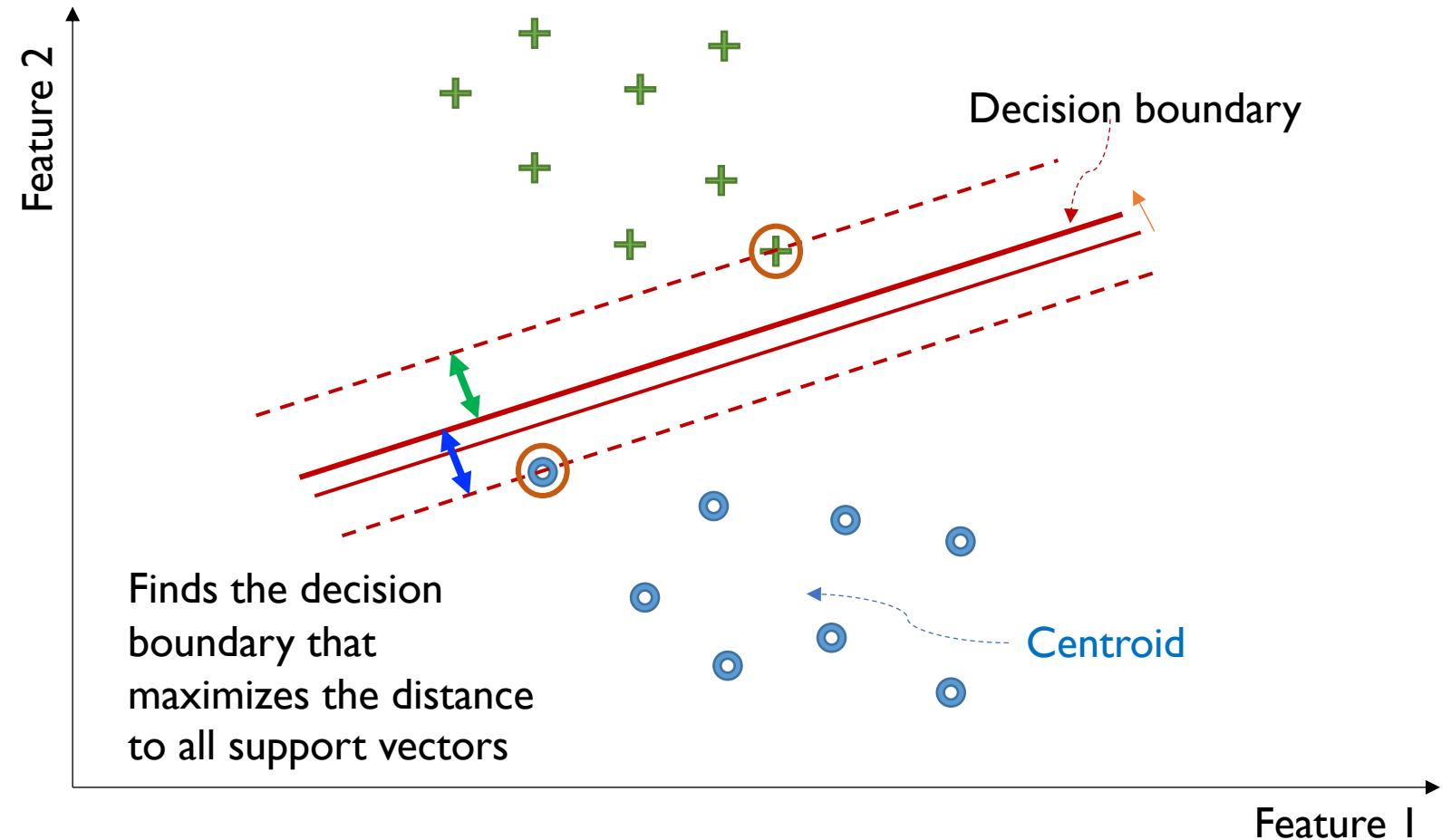
Linear classifier



# Types of approaches/models\*

- **Geometric**

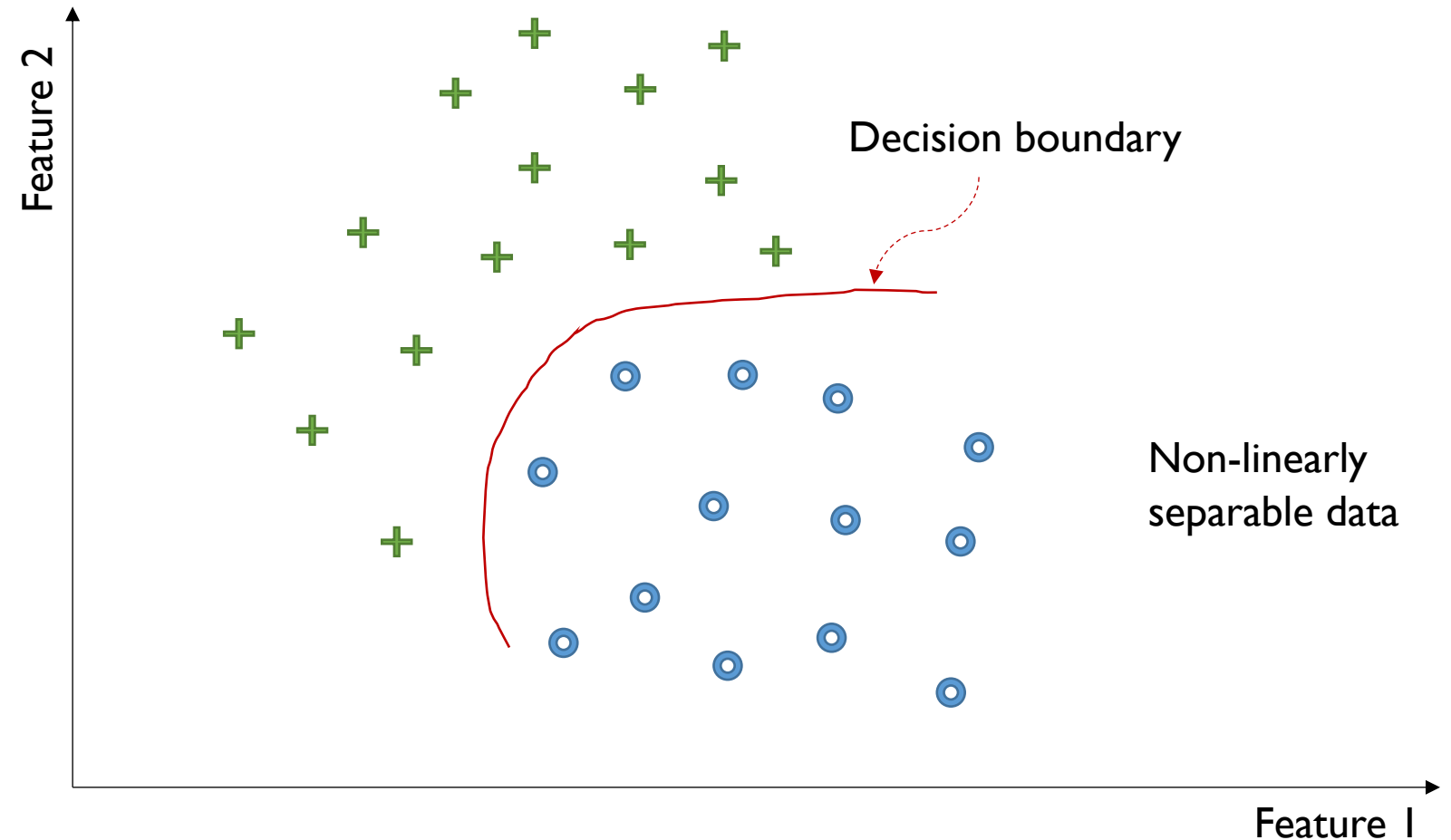
## Support Vector Machine



# Types of approaches/models\*

- **Geometric**

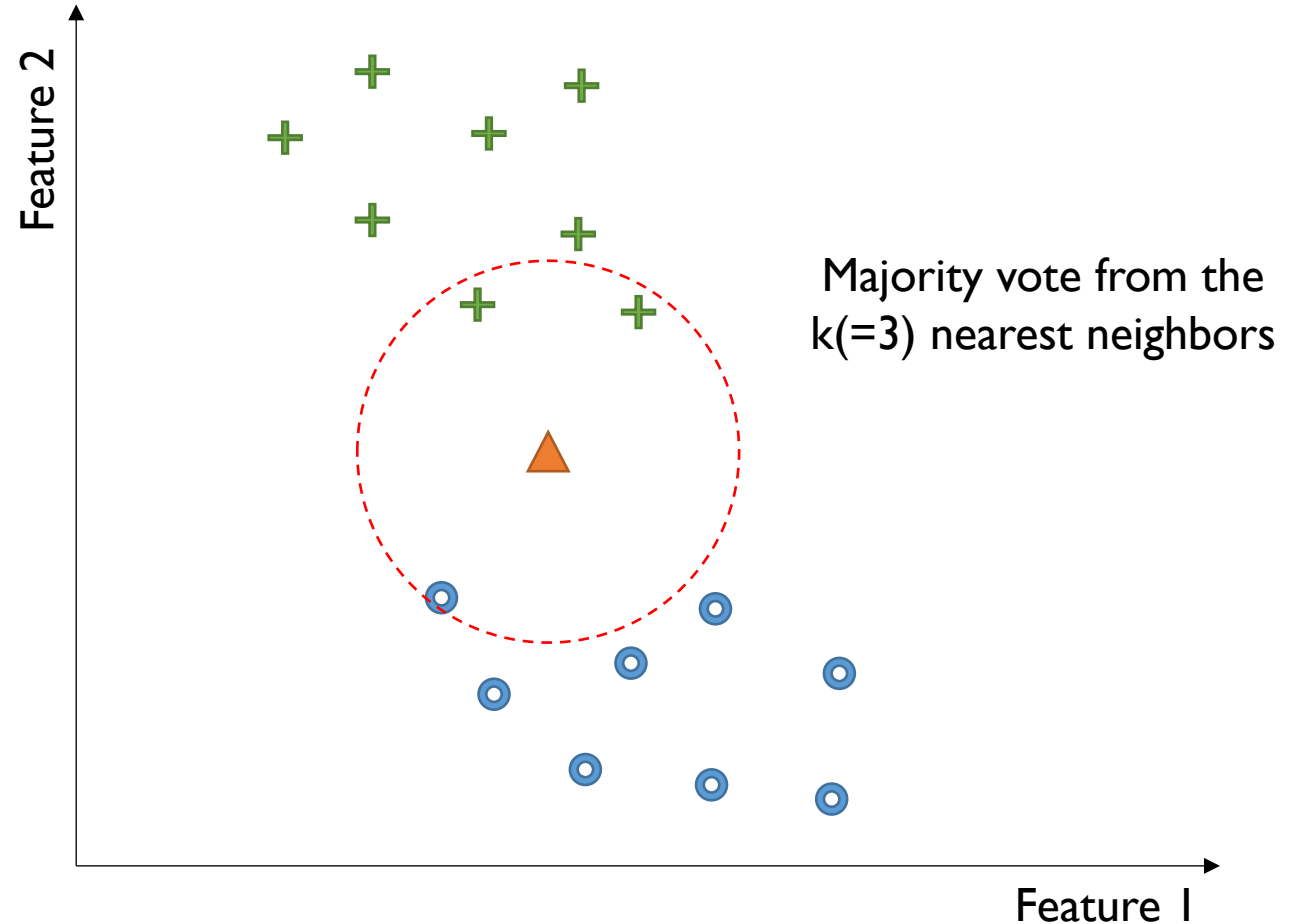
(Multilayer) perceptron,  
Deep neural networks



# Types of approaches/models\*

- Geometric
- **Distance based (geometric)**

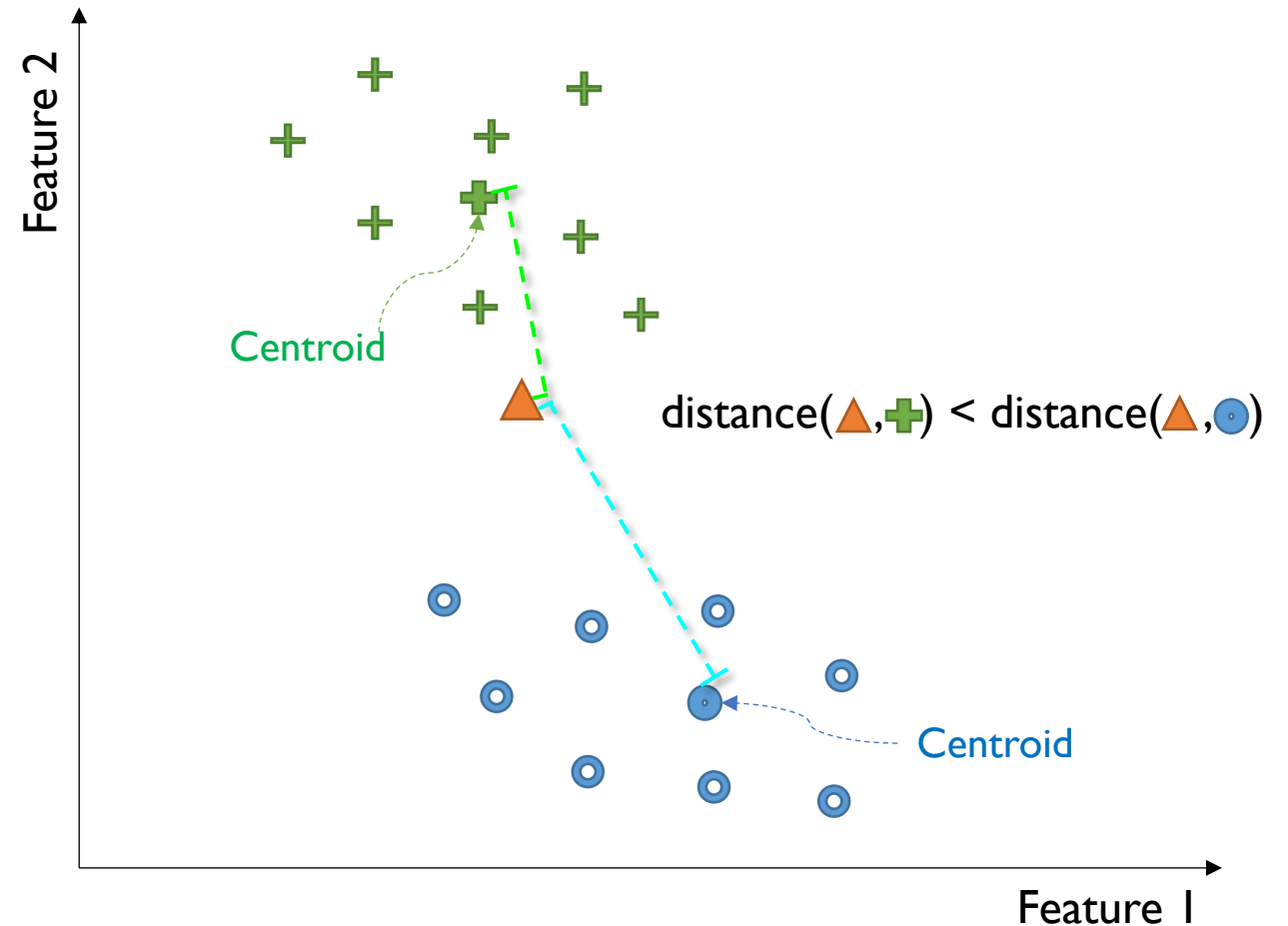
Nearest neighbor classifiers  
(e.g. K nearest neighbors)



# Types of approaches/models\*

- Geometric
- **Distance based (geometric)**

Nearest neighbor classifiers  
(e.g. nearest centroid)

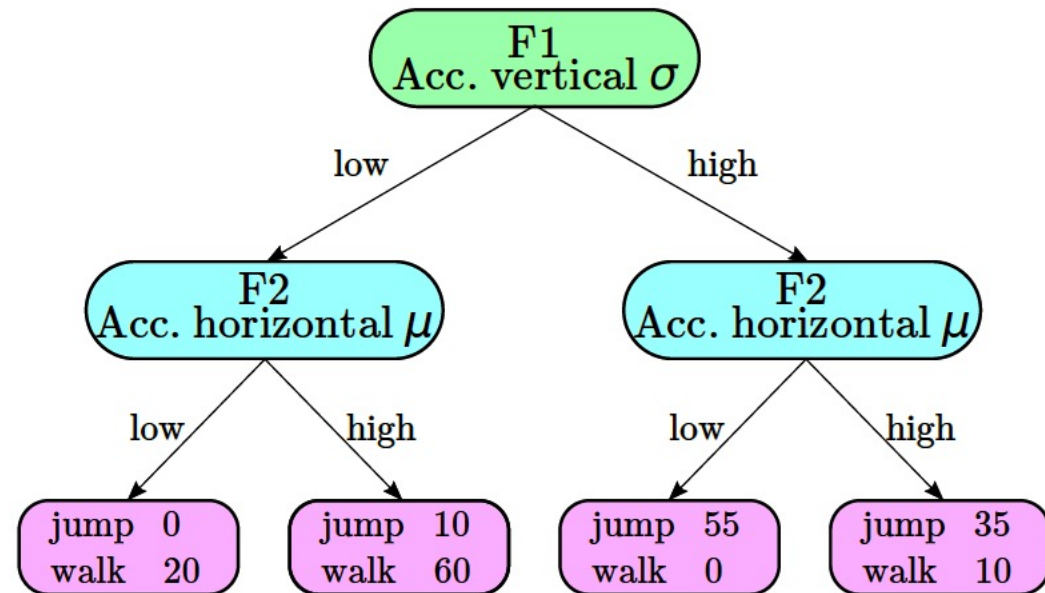




# Types of approaches/models\*

- Geometric
- Distance based (geometric)
- **Logical**

## Decision tree

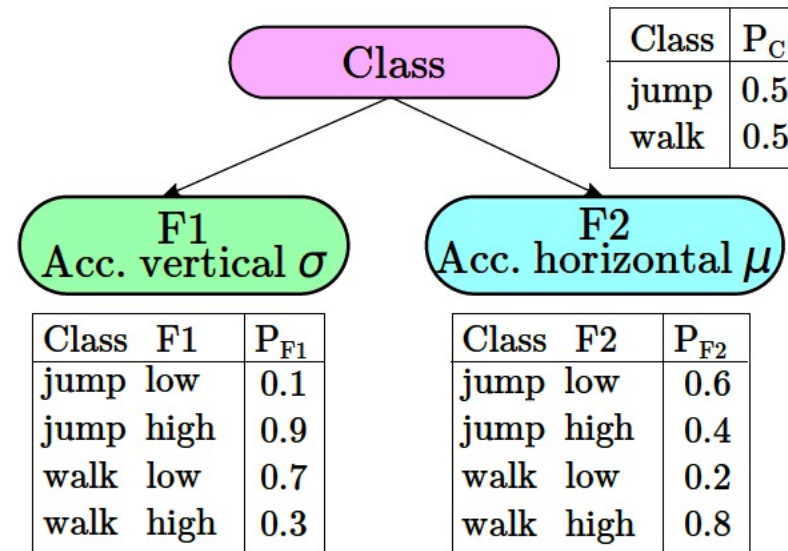


Taken from Galindez Olascoaga, L. I., Meert, W., & Verhelst, M (2021).  
Hardware-Aware Probabilistic Machine Learning Models. Springer, Cham.

# Types of approaches/models\*

- Geometric
- Distance based (geometric)
- Logical
- **Probabilistic**, etc.

## Bayesian network classifiers



$$\Pr(C=\text{jump}|F1=\text{high},F2=\text{low})$$

vs

$$\Pr(C=\text{walk}|F1=\text{high},F2=\text{low})$$

Taken from Galindez Olascoaga, L. I., Meert, W., & Verhelst, M (2021).  
Hardware-Aware Probabilistic Machine Learning Models. Springer, Cham.

# Types of approaches/models\*

- Geometric
  - Distance based (geometric)
  - Logical
  - Probabilistic, etc.
- 
- What kind of classification can HD computing/VSAs enable?

# Classification with HD computing/VSAs

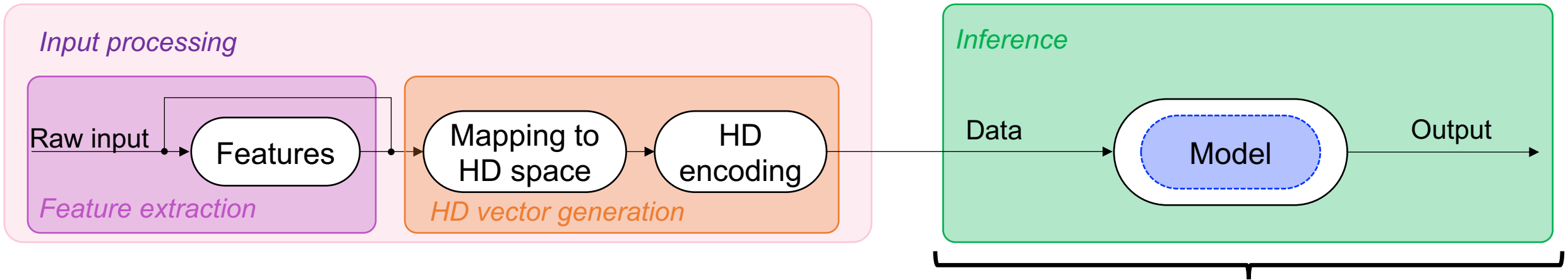
## Overview

# Main components of HD computing/VSAs

- High-dimensional vectors (HD vectors, hypervectors, HV) are approximately orthogonal to each other.
- Hyperdimensional arithmetic comprises three simple operators:
  - Bundling or superposition  $+$
  - Binding: typically, a multiplicative operation  $\otimes$
  - Permutation  $\rho$
- Similarity metrics: dot product, Hamming distance, etc. `distance( . )`
- Associative memory and item memory (codebook).

# Role of HD computing/VSAs in classification pipeline

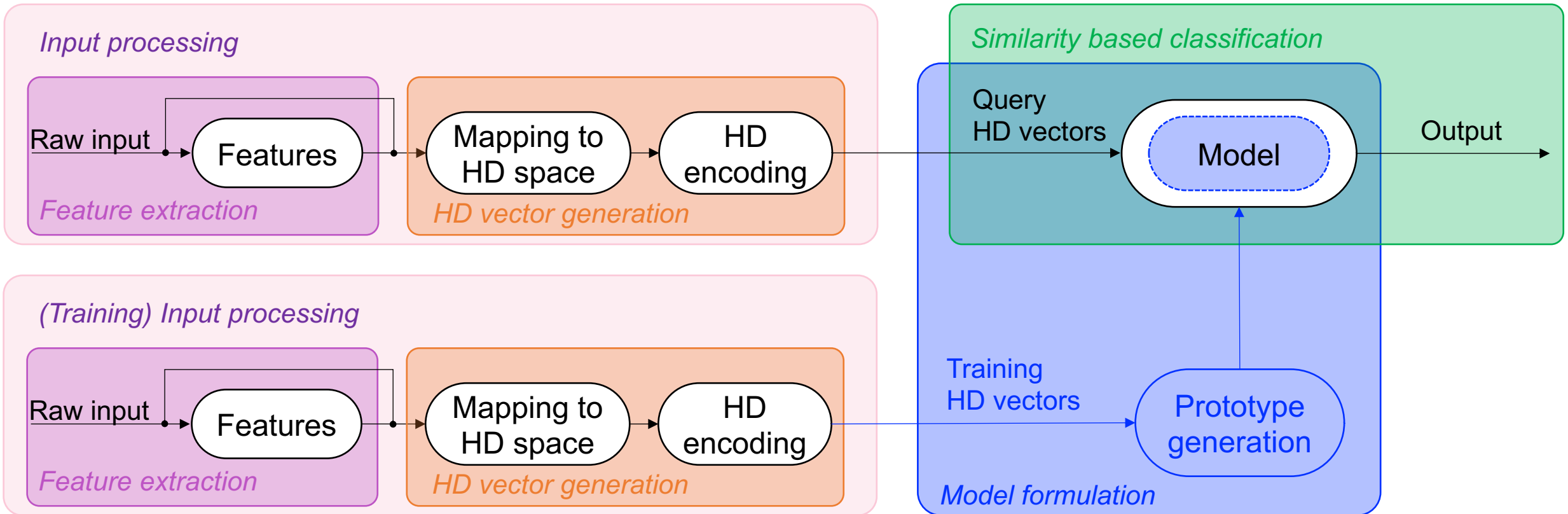
- Input data representation and encoding.



Can be done with HD computing/VSAs or within a hybrid scheme with other models. E.g. Module 10 on Relations to Neural Networks by Denis.

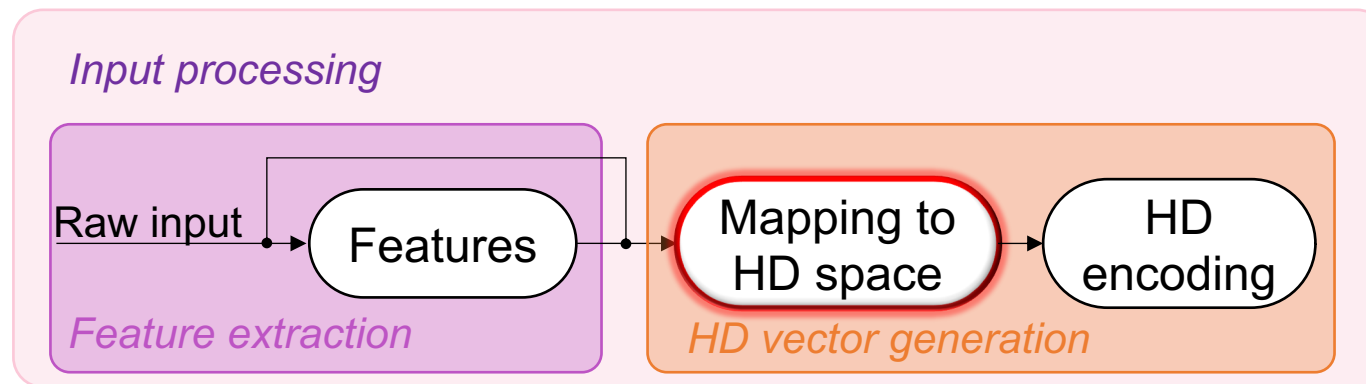
# Role of HD computing/VSAs in classification pipeline

- Model formulation with HD computing/VSAs.



# Classification with HD computing/VSAs

## I. Mapping inputs to HD space





# Mapping inputs to HD space

Determined by the type of input:

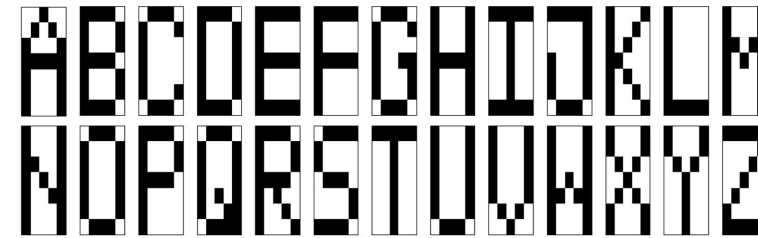
- **Symbolic or categorical:** Can be described by a finite alphabet of independent values or symbols. For example, letters in the alphabet (examples from Modules 1 by Pentti and Module 3 by Ryan).
- **Real-valued:** Continuous input data or discrete variables with correlated values. For example, accelerometer signals (see also Module 8 by Chris).

# Mapping symbolic inputs to HD space

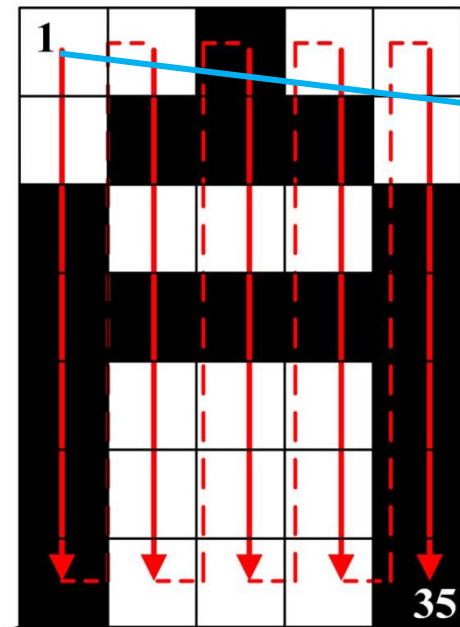
- Orthogonal mapping: a unique HD vector, randomly chosen, is assigned to each symbol.
- These unique HD vectors are saved in an *item memory* (I.M.) and remain fixed for the rest of the system execution.

# Mapping symbolic inputs to HD space

Example 1: character recognition



Each pixel is treated as a feature. Features are assumed to be uncorrelated.



Each feature is assigned a randomly generated binary HD vector, these are saved in *I.M.*

**Pix1**

1	0	0	1	0	0	0	1	1	1
---	---	---	---	---	---	---	---	---	---

**Black**

1	0	0	1	0	0	0	1	1	1
---	---	---	---	---	---	---	---	---	---

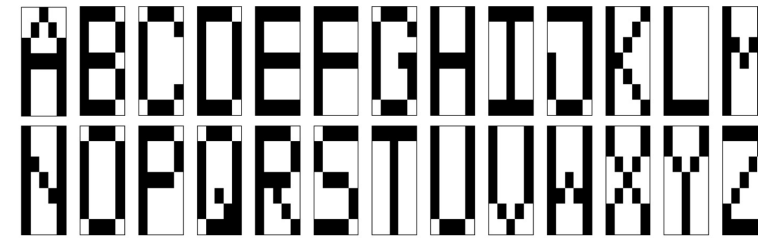
**White**

1	1	0	0	1	0	0	0	1	1
---	---	---	---	---	---	---	---	---	---

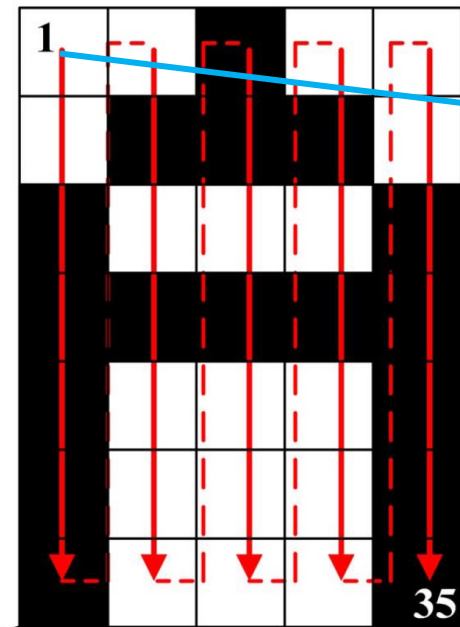
Values can be represented by shifting the pixel ID HD vector: 0 bits for black and 1 bit for white.

# Mapping symbolic inputs to HD space

Example 1: character recognition  
(key-value pair approach)



Each pixel is  
treated as a  
feature.  
Features are  
assumed to be  
uncorrelated.



Each feature is assigned a randomly generated  
binary HD vector, these are saved in *PixID I.M.*

**Pix1**

1	0	0	1	0	0	0	1	1	1
---	---	---	---	---	---	---	---	---	---

Each value is assigned a randomly generated  
binary HD vector, these are saved in the *Color I.M.*

**Black**

1	1	0	0	0	1	1	0	1	0
---	---	---	---	---	---	---	---	---	---

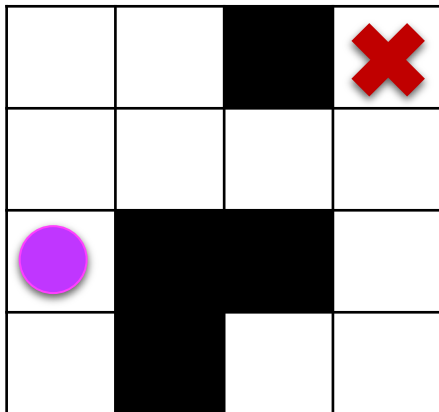
**White**

0	1	1	0	0	1	0	1	0	1
---	---	---	---	---	---	---	---	---	---

Encoding as {Pixel ID: value} pair: **Pix1**  $\otimes$  **White**

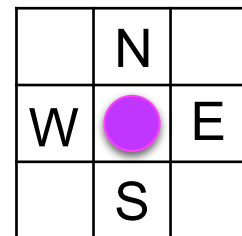
# Mapping symbolic inputs to HD space

## Example 2: 2D robot navigation



Agent navigating in 2D grid must reach goal while avoiding obstacles.

Sensors: presence or absence of obstacles in each cardinal direction



Sensor ID I.M.

**N**

1	0	0	1	0	0	0	1	1	1
---	---	---	---	---	---	---	---	---	---

 ... **W**

1	0	1	0	0	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---

Sensor value I.M.

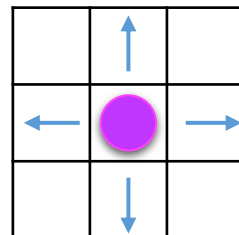
**true**

0	1	1	1	0	1	0	1	0	0
---	---	---	---	---	---	---	---	---	---

**false**

1	1	0	0	0	1	0	1	0	1
---	---	---	---	---	---	---	---	---	---

Actuation: the robot can move up, down, right or left



Actuation I.M.

**UP**

1	1	1	0	0	0	0	1	1	0
---	---	---	---	---	---	---	---	---	---

 ... **LEFT**

0	1	1	0	1	0	1	0	1	0
---	---	---	---	---	---	---	---	---	---

Related works:

Levy, Simon D., Suraj Bajracharya, and Ross W. Gayler. "Learning Behavior Hierarchies via High-Dimensional Sensor Projection." *AAAI Workshop: Learning Rich Representations from Low-Level Sensors*. 2013.

P. Neubert, S. Schubert, and P. Protzel, "Learning vector symbolic architectures for reactive robot behaviours," *IROS Workshop: Machine Learning Methods for High-Level Cognitive Capabilities in Robotics*, 2016.

# Mapping real-valued inputs to HD space

Real-valued inputs call for locality preserving encodings (LPEs). Introduced in module 8 by Chris.

Approaches I will discuss today:

- Discrete mapping: discretize the real-valued input and assign HD vectors such that they preserve relevant correlations.
- Random projection: directly project real-valued inputs to HD space by means of scalar multiplication. Often used as an intermediate representation.

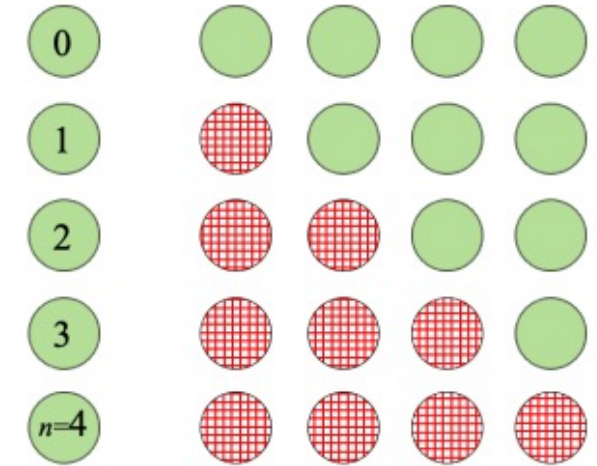
Other powerful approaches, still largely unexplored for practical applications:

- Vector Function Architectures (VFAs), kernel LPEs (in the context of HD computing/VSAs).

# Similarity preserving discrete mapping

*Linear similarity*\* preserving mapping:

1. Discretize real-valued input into  $m+1$  levels.
2. Randomly assign HD vector to level 0 ( $HV_0$ ).
3. Gradually flip a predefined number of bits starting from  $HV_0$  such that  $HV_0$  and  $HV_{m+1}$  are uncorrelated.



\*Linear similarity also preserved by e.g. thermometer code.  
(Taken from Chris's slides, module 8)

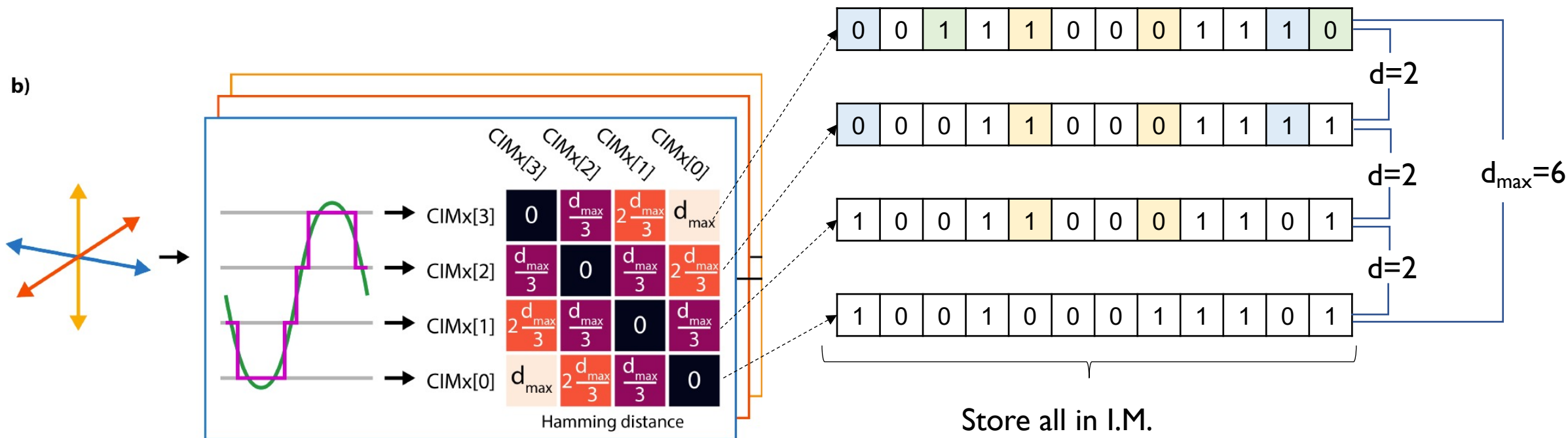
## Considerations:

- Sample without replacement for the bit flipping.
- Need to store the HD vectors for all levels.

# Similarity preserving discrete mapping

*Linear similarity* preserving mapping:

Example: accelerometer data





# Similarity preserving discrete mapping

*Approximately linear similarity preserving mapping\**:

1. Discretize real-valued input into  $m+1$  levels.
2. Randomly assign HD vectors to the first and last levels  $HV_0$  and  $HV_{m+1}$
3. Construct intermediate levels by concatenating sections of  $HV_0$  and  $HV_{m+1}$ .

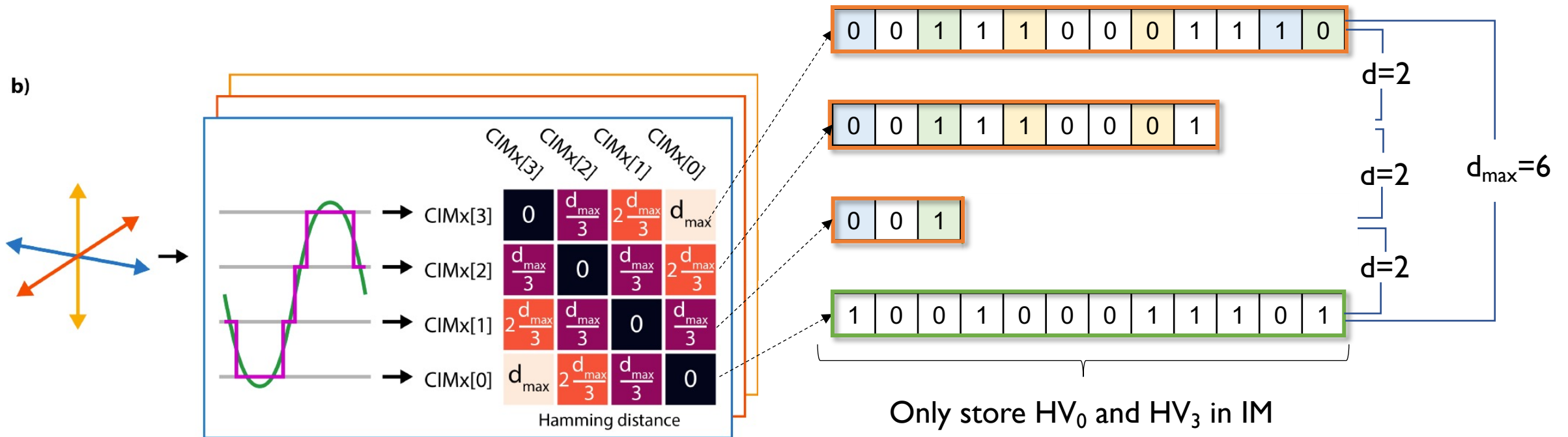
Considerations:

- Might only need to store first and last level.
- *Linear similarity* preservation not guaranteed.

# Similarity preserving discrete mapping

*Approximately linear similarity preserving mapping:*

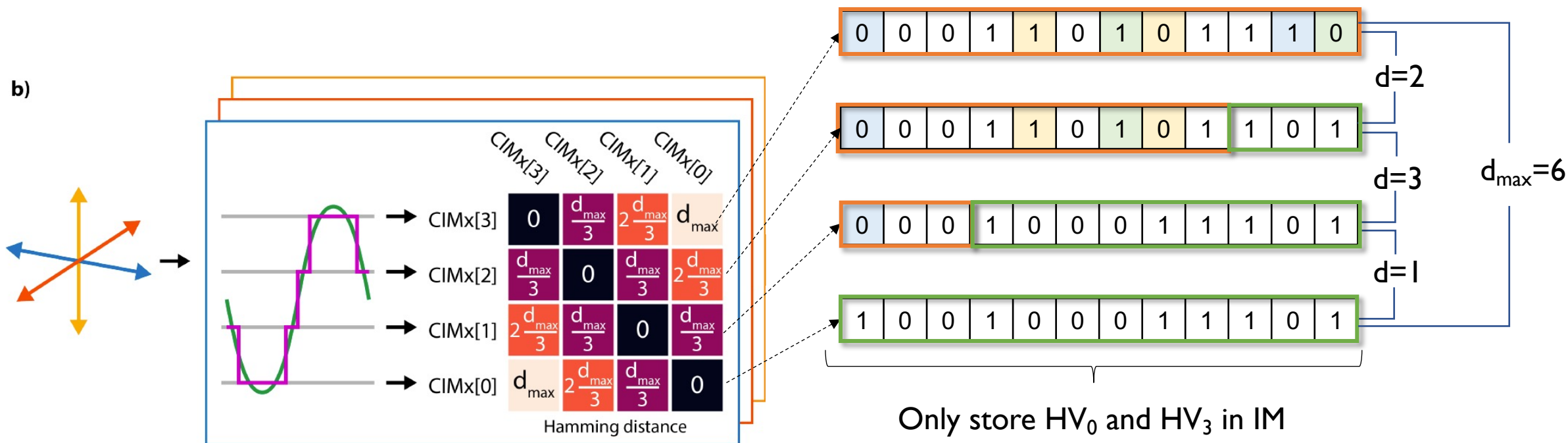
Example: accelerometer data



# Similarity preserving discrete mapping

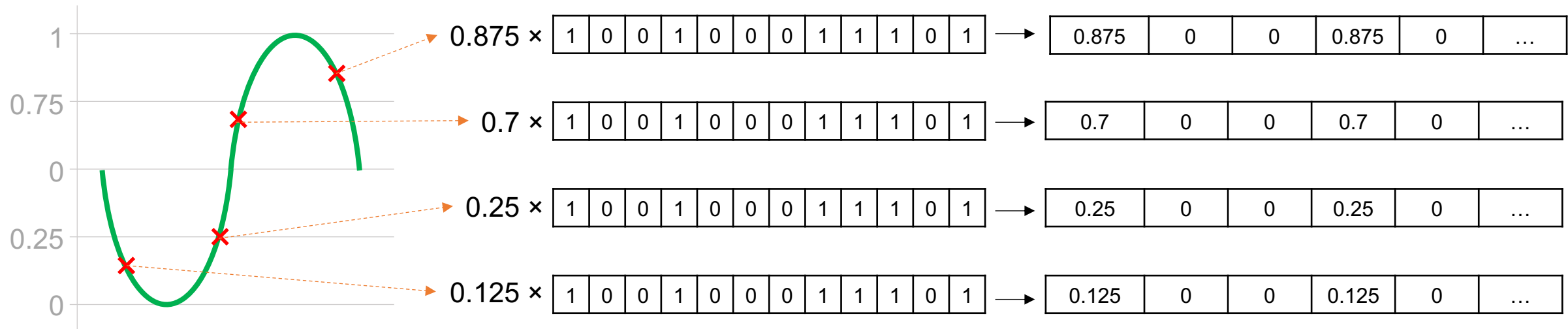
*Approximately linear similarity preserving mapping:*

Example: accelerometer data



# Random projection

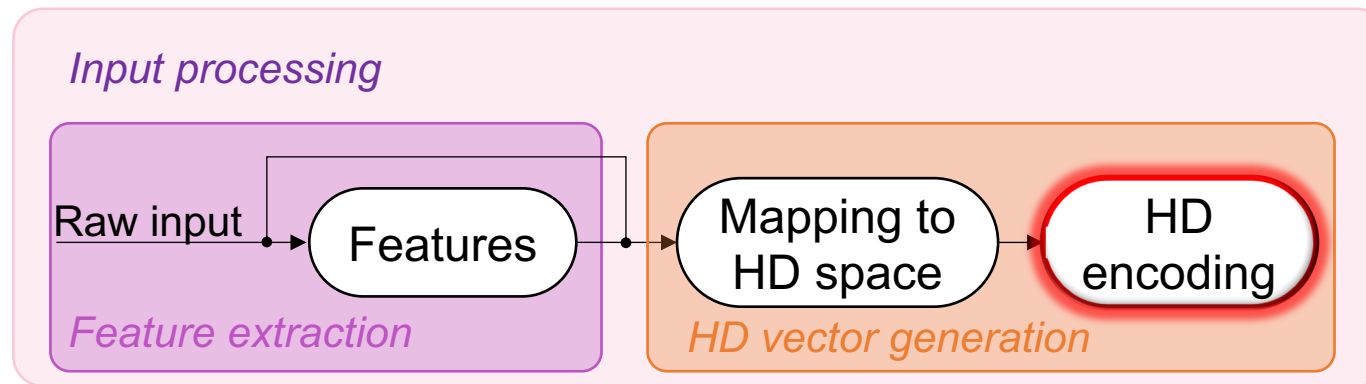
- Scalar multiplication of real value and HD vector.
- Useful when most appropriate quantization is unknown.
- Can be used as an intermediate representation, and normalized back to  $\{-1, 1\}$  or  $\{0, 1\}$  at a later step\*.



\* See also Rachkovskij, D.A., I. S. Misuno, and S.V. Slipchenko. "Randomized projective methods for the construction of binary sparse vector representations." *Cybernetics and Systems Analysis* 48.1 (2012): 146-156.

# Classification with HD computing/VSAs

## 2. Encoding

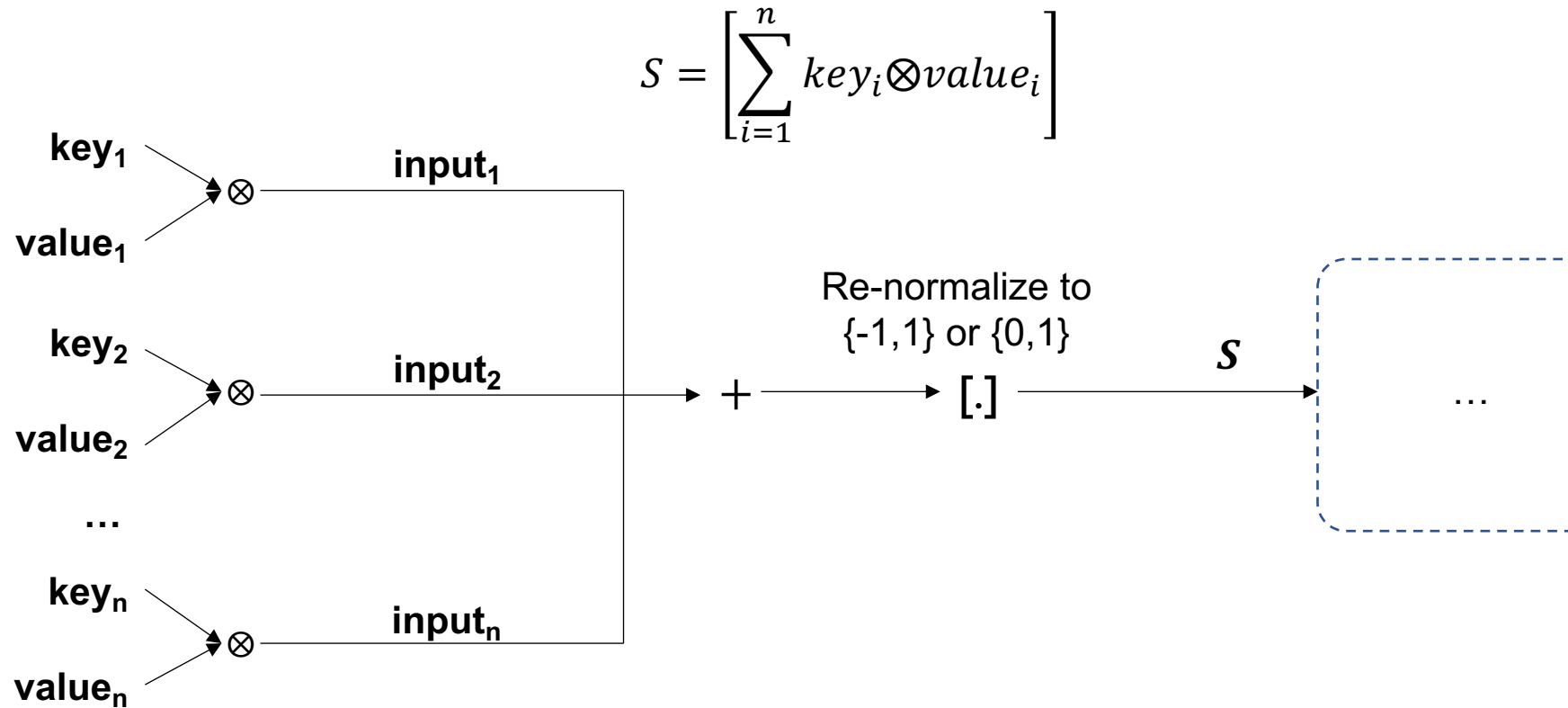


# Encoding

- Exploit the properties of HD computing/VSAs, for example:
  - High capacity of HD vectors.
  - Binding: {key:value} / (variable:value) pairs.
  - Construction of data structures: sets, sequences, histograms, etc.  
Refer to Module 4: Representation and Manipulation of data structures by Denis.

# Spatial encoding

- Combine inputs available at a given time-aligned sample into a single HD vector.
- Set of key-value pairs (refer to Module 4 on data structures by Denis).



# Spatial encoding

- Example: 2D robot navigation

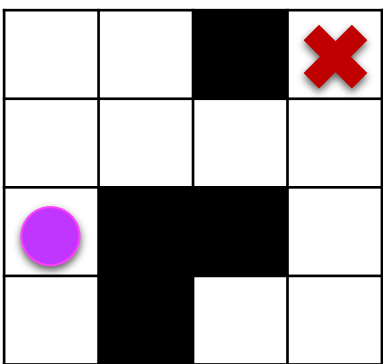
Sensor ID I.M.

N	1	0	0	1	0	0	0	1	1	1
S	0	1	1	0	0	1	0	1	0	0
E	0	1	0	0	1	1	1	1	0	1
W	1	1	0	1	0	0	1	1	0	0

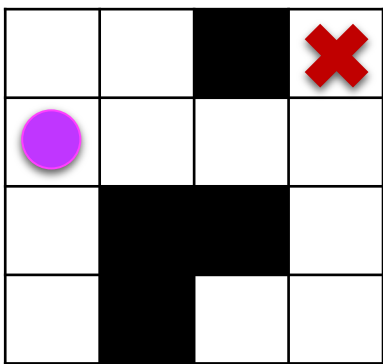
Sensor value I.M.

True	0	1	0	0	1	1	1	1	0	1
False	1	1	0	1	0	0	1	1	0	0

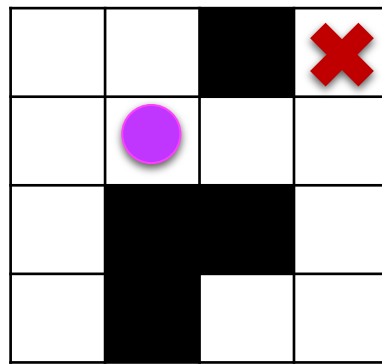
t=1



t=2



t=3



$$S_{t=1}=[N\otimes F + S\otimes F+E\otimes T +W\otimes T]$$

$$S_{t=2}=[N\otimes F + S\otimes F+E\otimes F +W\otimes T]$$

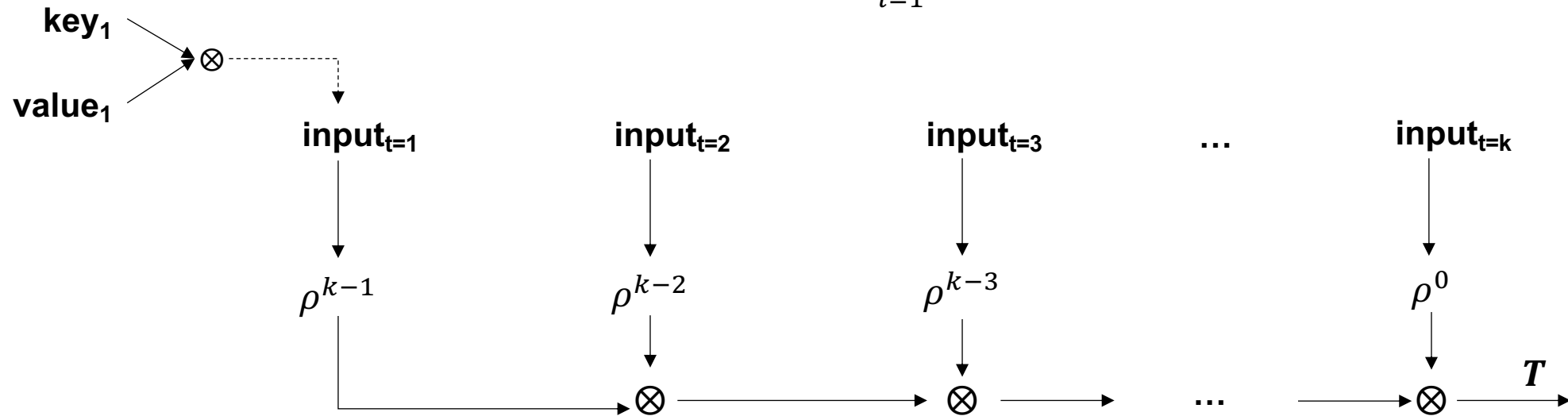
$$S_{t=3}=[N\otimes F + S\otimes T+E\otimes F +W\otimes F]$$



# Temporal encoding

- Represent time dependencies of the inputs through sequences or n-grams over a specific window of length  $k$ .
- Sequence (of key-value pairs). Refer to Module 4.

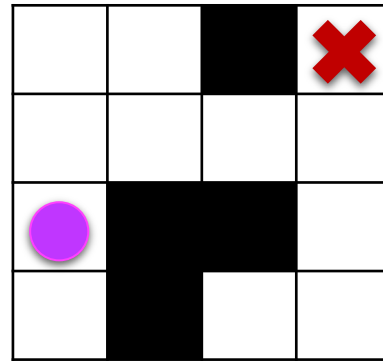
$$T = \prod_{i=1}^k \rho^{k-i}(\text{input}_{t=i})$$



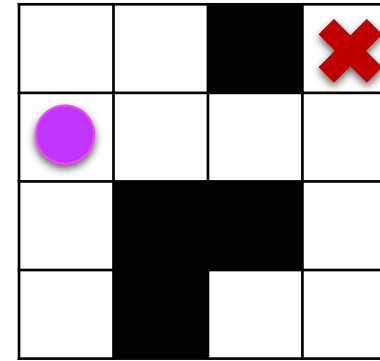
# Temporal encoding

- Example: 2D robot navigation

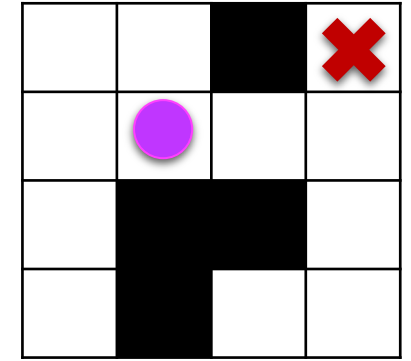
t=1



t=2



t=3



Sensor ID I.M.

N	1	0	0	1	0	0	0	1	1	1
S	0	1	1	0	0	1	0	1	0	0
E	0	1	0	0	1	1	1	1	0	1
W	1	1	0	1	0	0	1	1	0	0

Sensor value I.M.

True	0	1	0	0	1	1	1	1	0	1
False	1	1	0	1	0	0	1	1	0	0

$$S_{t=1} = [N \otimes F + S \otimes F + E \otimes T + W \otimes T]$$

$$S_{t=2} = [N \otimes F + S \otimes F + E \otimes F + W \otimes T]$$

$$S_{t=3} = [N \otimes F + S \otimes T + E \otimes F + W \otimes F]$$

$$T = \rho^2(S_{t=1}) \otimes \rho(S_{t=2}) \otimes S_{t=3}$$

# Histogram encoding

- Some applications may benefit from frequency distribution representations.

$$H = \sum_{i=1}^n input_i$$

E.g. Language identification (Lecture I by Pentti)

Trigrams from letter seed vectors

$$\mathbf{THE} = r(r(\mathbf{T})) * r(\mathbf{H}) * \mathbf{E}$$

Accumulate to form language profile  
(histogram of trigrams)

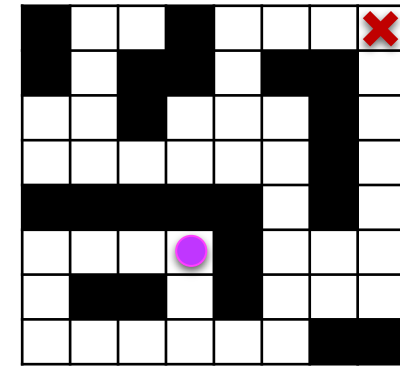
$$\mathbf{Eng1} += \mathbf{THE} + \mathbf{HE\#} + \mathbf{E\#Q} + \mathbf{\#QU} + \mathbf{QUI} + \mathbf{UIC} + \dots$$

Compare to other language profiles

Query most likely letter after e.g.TH

# Histogram encoding

- Example: 2D robot navigation, anomaly detection



Actuation I.M.

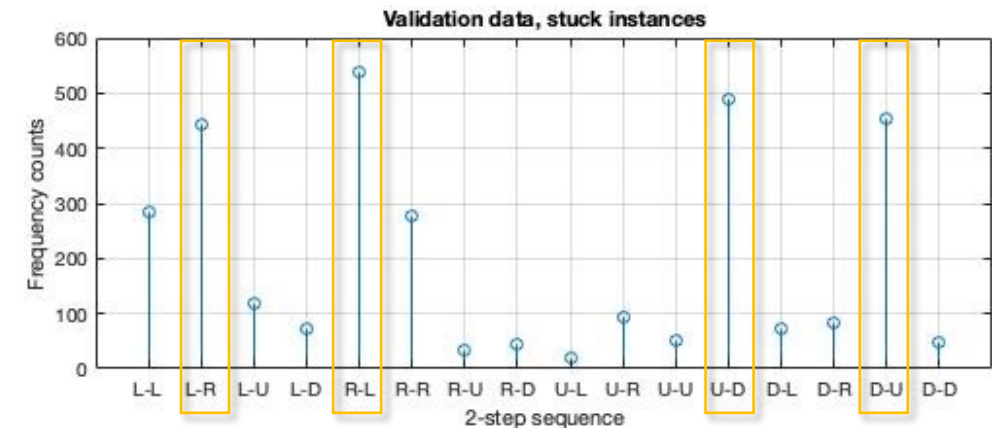
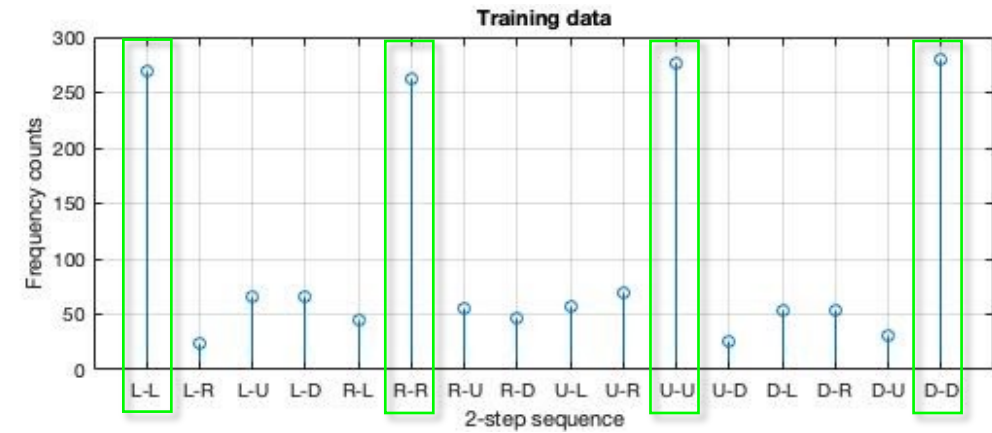
Up	1	0	0	1	0	0	0	1	1	1
Down	0	1	1	0	0	1	0	1	0	0
Right	0	1	0	0	1	1	1	1	0	1
Left	1	1	0	1	0	0	1	1	0	0

Histogram over x time steps

$$H = \sum_{i=1}^x S_{t_i}$$

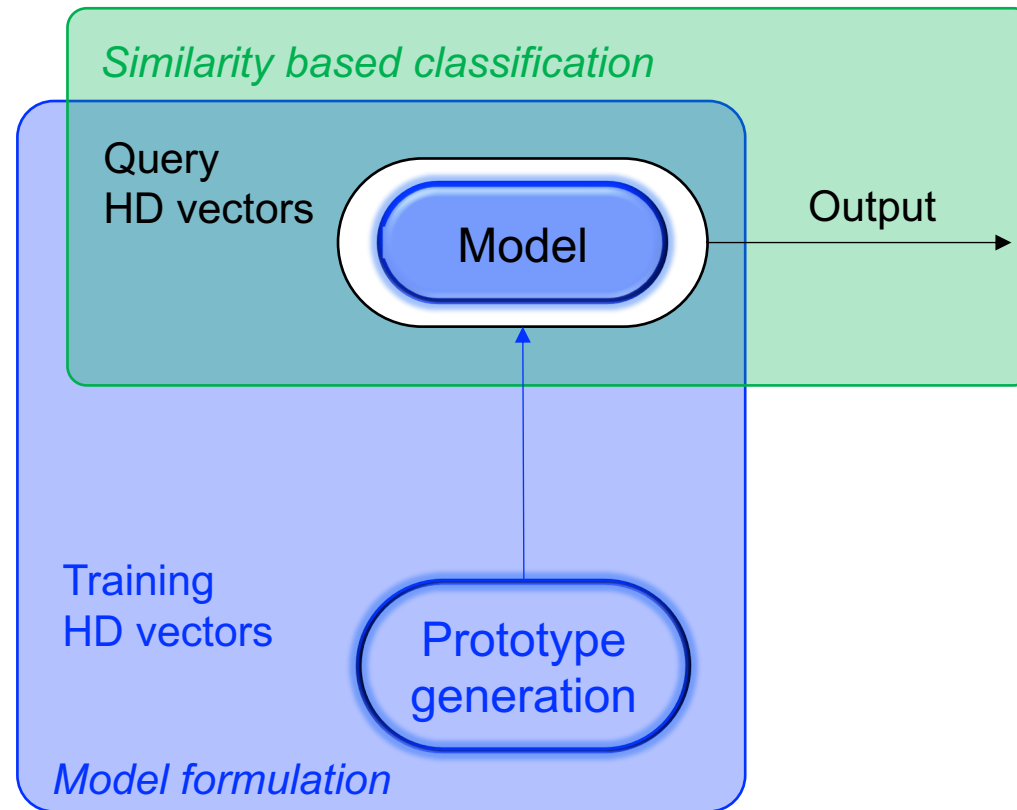
2-step sequences (bi-grams)

$$\begin{aligned} S_1 &= \rho(L) \otimes L \\ S_2 &= \rho(L) \otimes R \\ S_3 &= \rho(L) \otimes U \\ S_4 &= \rho(L) \otimes D \\ &\dots \\ S_{16} &= \rho(D) \otimes D \end{aligned}$$



# Classification with HD computing/VSAs

## 3. Model formulation and classification

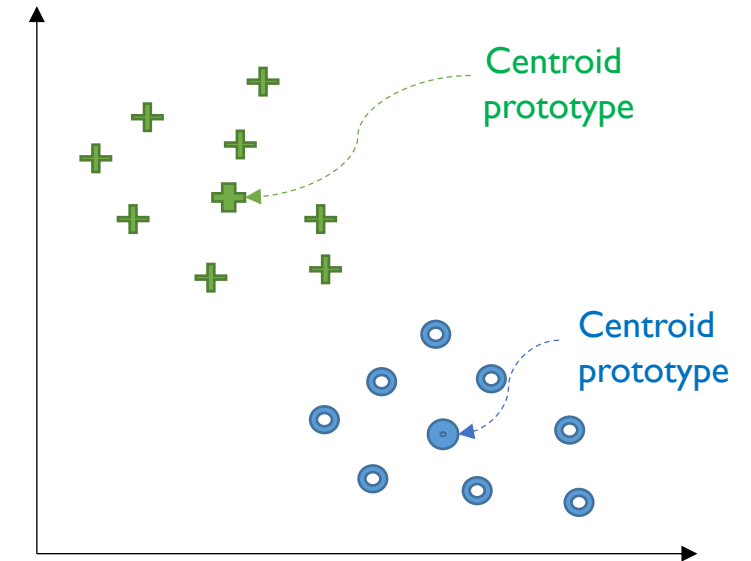


# Model formulation

- Class prototype HD vector generation.
- Multiple approaches:
  - **Centroid based prototype generation**
  - **One-shot prototype generation**
  - **Learning Vector Quantization (LVQ) family of representations**
  - Multiple prototypes per class
  - Storing models in superposition

# Class prototype generation (centroid computation)

- A prototype HD vector for each class is formed by computing the class centroid.
- For binary and bipolar HD vectors, this amounts to finding the majority of each element across all training examples.



\*This 2-dimensional depiction is for illustrative purposes.

Class **+** training examples

1	1	-1	1	-1	-1	1	1	-1	-1
1	-1	-1	1	-1	-1	1	1	-1	-1
...									
1	-1	-1	-1	1	-1	1	-1	-1	1

Class **o** training examples

1	1	1	1	1	-1	1	1	1	-1
1	1	1	-1	-1	-1	1	1	-1	-1
...									
1	-1	-1	-1	1	-1	1	-1	1	1



+

[.]

+

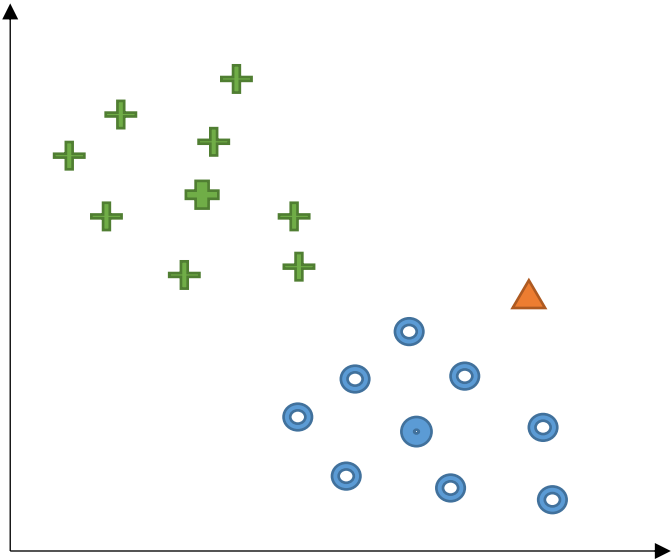
[.]

Model

Class labels		Prototype HD vectors																			
➔	Label 	<table border="1"><tr><td>1</td><td>-1</td><td>-1</td><td>1</td><td>-1</td><td>-1</td><td>1</td><td>1</td><td>-1</td><td>-1</td></tr></table>										1	-1	-1	1	-1	-1	1	1	-1	-1
1	-1	-1	1	-1	-1	1	1	-1	-1												
➔	Label 	<table border="1"><tr><td>1</td><td>1</td><td>1</td><td>-1</td><td>1</td><td>-1</td><td>1</td><td>1</td><td>1</td><td>-1</td></tr></table>										1	1	1	-1	1	-1	1	1	1	-1
1	1	1	-1	1	-1	1	1	1	-1												



# Classification

- Query HD vectors are formed using the same encoding as the training ones.
- Classification takes place by finding the nearest neighboring prototype to this query HD vector.
- Akin to a nearest centroid classifier in ML  
([https://en.wikipedia.org/wiki/Nearest\\_centroid\\_classifier](https://en.wikipedia.org/wiki/Nearest_centroid_classifier))



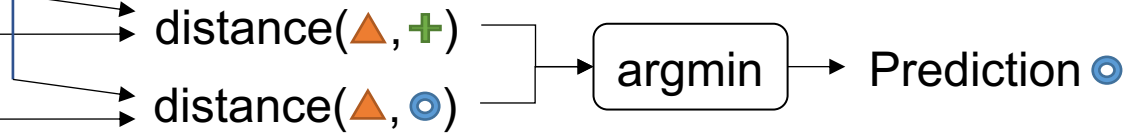
\*This 2-dimensional depiction is for illustrative purposes.

## Model

Class labels	Prototype HD vectors										
Label 	<table><tr><td>1</td><td>-1</td><td>-1</td><td>1</td><td>-1</td><td>-1</td><td>1</td><td>1</td><td>-1</td><td>-1</td></tr></table>	1	-1	-1	1	-1	-1	1	1	-1	-1
1	-1	-1	1	-1	-1	1	1	-1	-1		
Label 	<table><tr><td>1</td><td>1</td><td>1</td><td>-1</td><td>1</td><td>-1</td><td>1</td><td>1</td><td>1</td><td>-1</td></tr></table>	1	1	1	-1	1	-1	1	1	1	-1
1	1	1	-1	1	-1	1	1	1	-1		

## Query HD vector

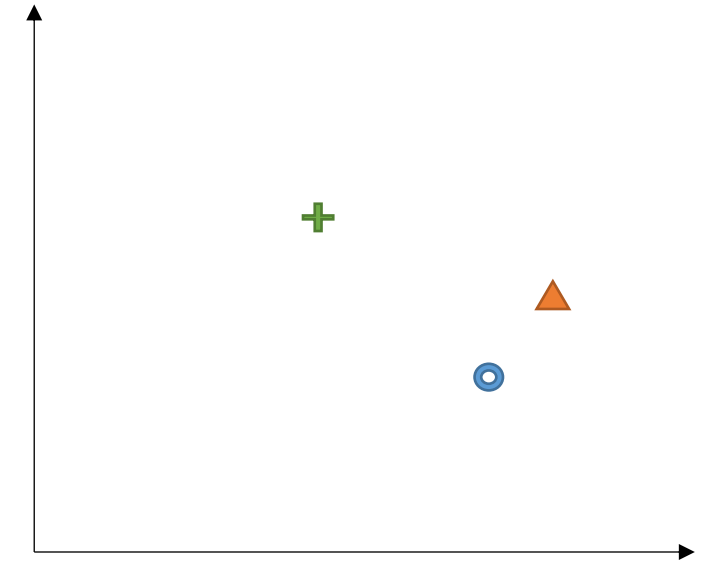
-1	1	1	-1	1	-1	1	1	1	-1
----	---	---	----	---	----	---	---	---	----



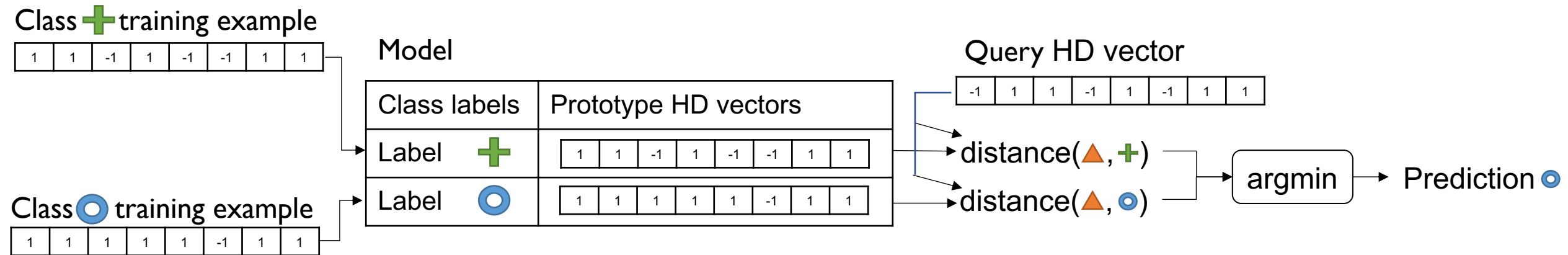


# One-shot classification

- Single training sample constitutes the class prototype.
- Classification proceeds as usual.
- Works well with HD computing/VSAs because of information-rich representations enabled by HD vectors + encoding.



\*This 2-dimensional depiction is for illustrative purposes.



# Learning Vector Quantization approaches

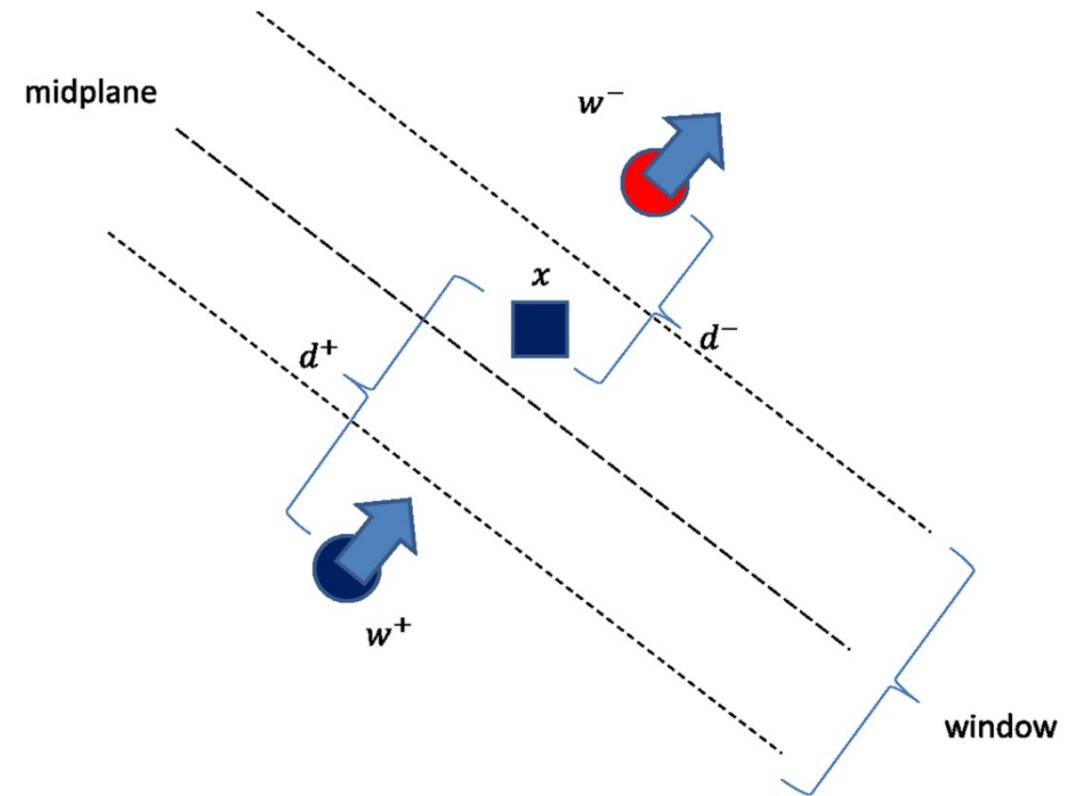
- Prototypes represent class regions.
- These prototypes are learned iteratively.

Heuristic approach:

$$w^+ = w^+ - \alpha(x - w^+)$$

$$w^- = w^- + \alpha(x - w^-)$$

Learning rate



# Learning Vector Quantization approaches

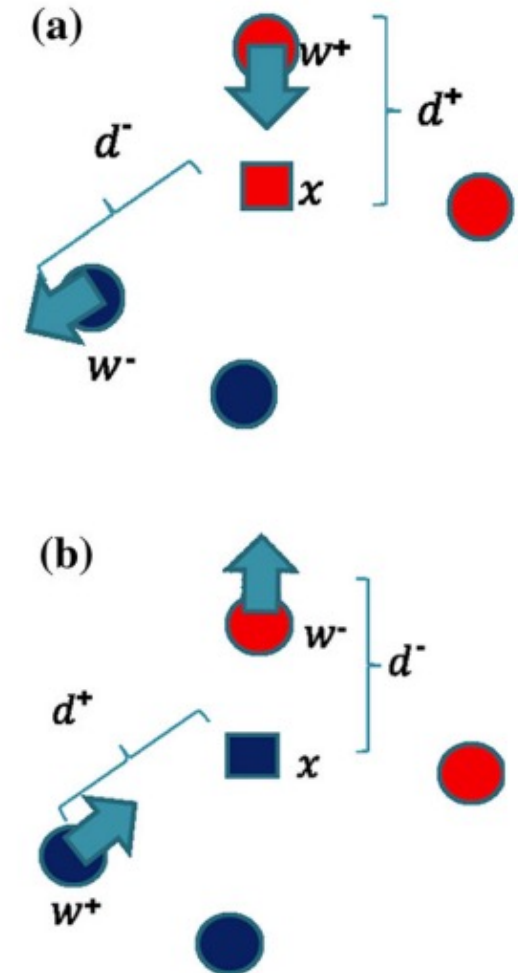
Margin maximalization approaches (e.g. Generalized LVQ):

Define a cost function from which the learning rule is derived via gradient descent.

$$w^+ = w^+ - 2\alpha \frac{\delta g}{\delta \mu} \mu^- (x - w^+)$$
$$w^- = w^- + 2\alpha \frac{\delta g}{\delta \mu} \mu^+ (x - w^-)$$

Cost function

Relative distance difference

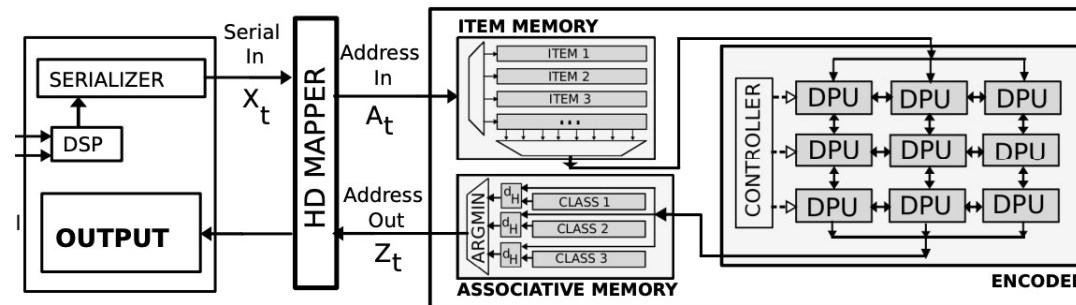


# Classification with HD computing/VSAs

## 4. Performance trade-offs

# Energy efficiency

- Departure from traditional von Neumann architectures with inefficient memory exchanges.
- Enables bit-wise operations when using Binary Spatter Codes model.
- Robustness under low SNR and sparsity can enable resource efficiency.
- Will be discussed in detail by Mohamed in Module II: Hardware implementations.

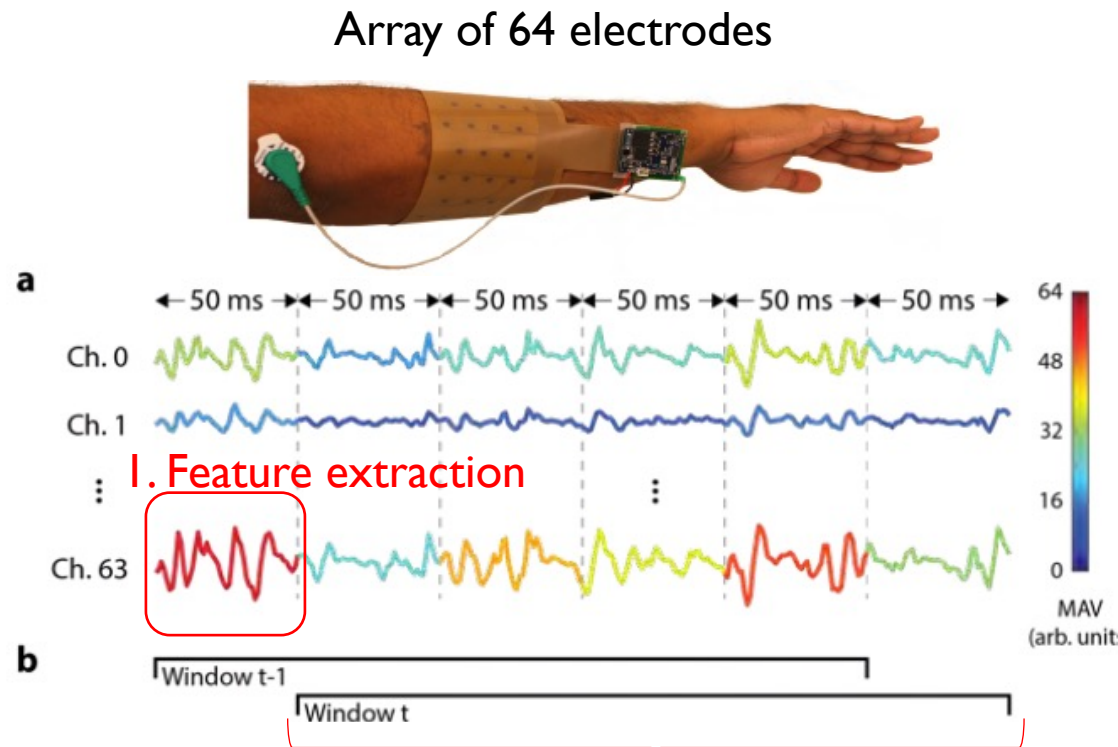


Datta, Sohum, et al. "A programmable hyper-dimensional processor architecture for human-centric IoT." *IEEE Journal on Emerging and Selected Topics in Circuits and Systems* 9.3 (2019): 439-452.

# Application examples

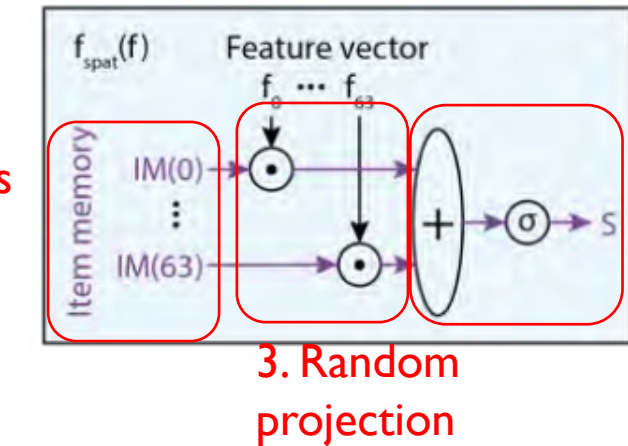
ExG signal classification, Robotics

# EMG-based hand gesture recognition



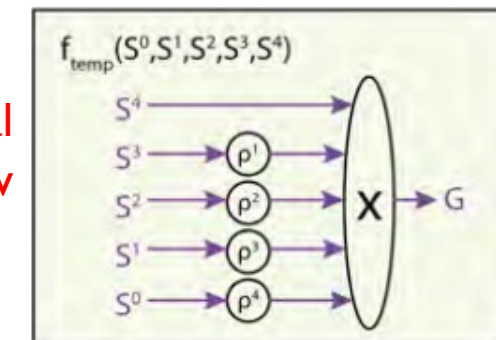
2. HD vectors for each electrode ID

HD mapping and Spatial encoding



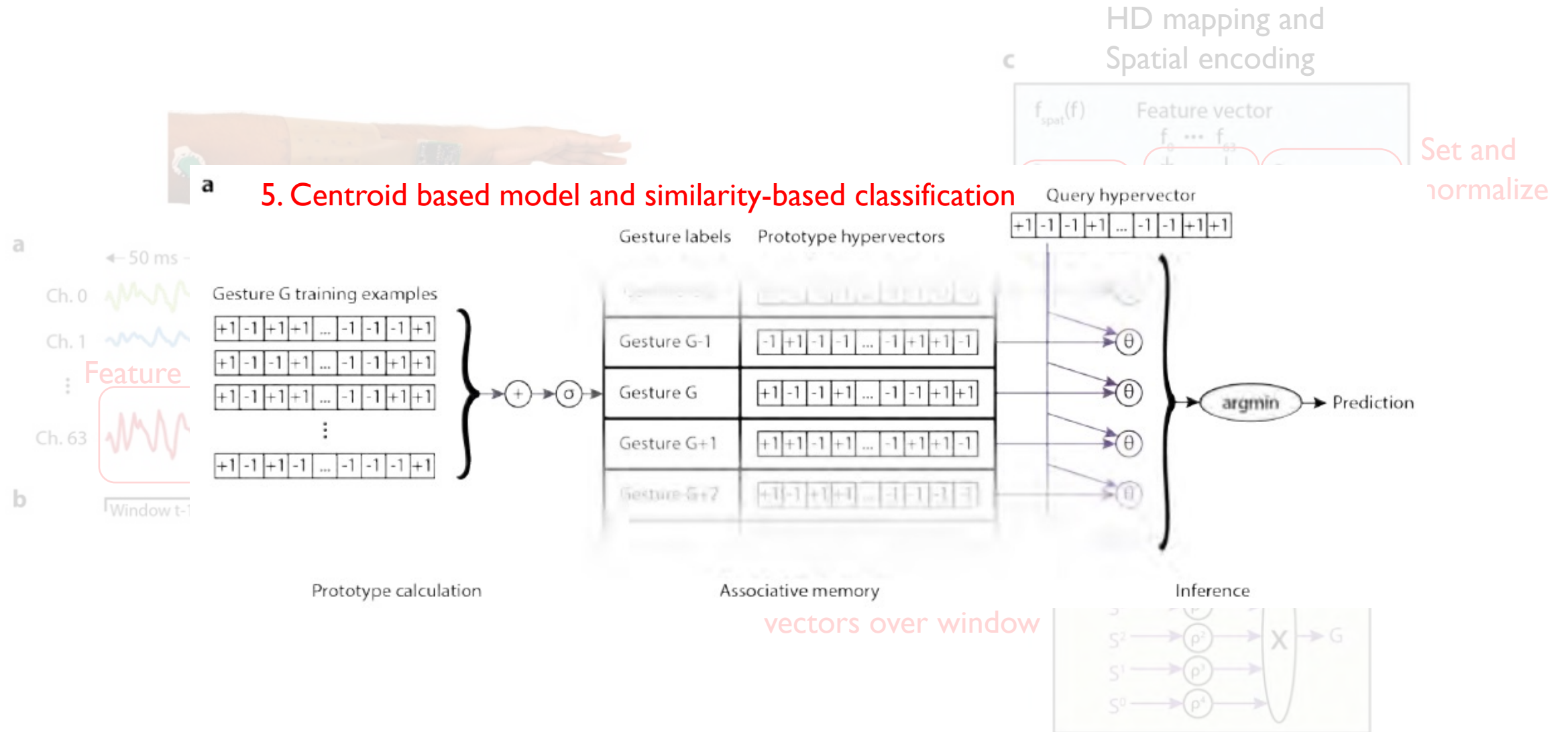
4. Spatial encoding (set of scaled vectors + normalization)

Temporal encoding



5. Sequence of spatial vectors over window

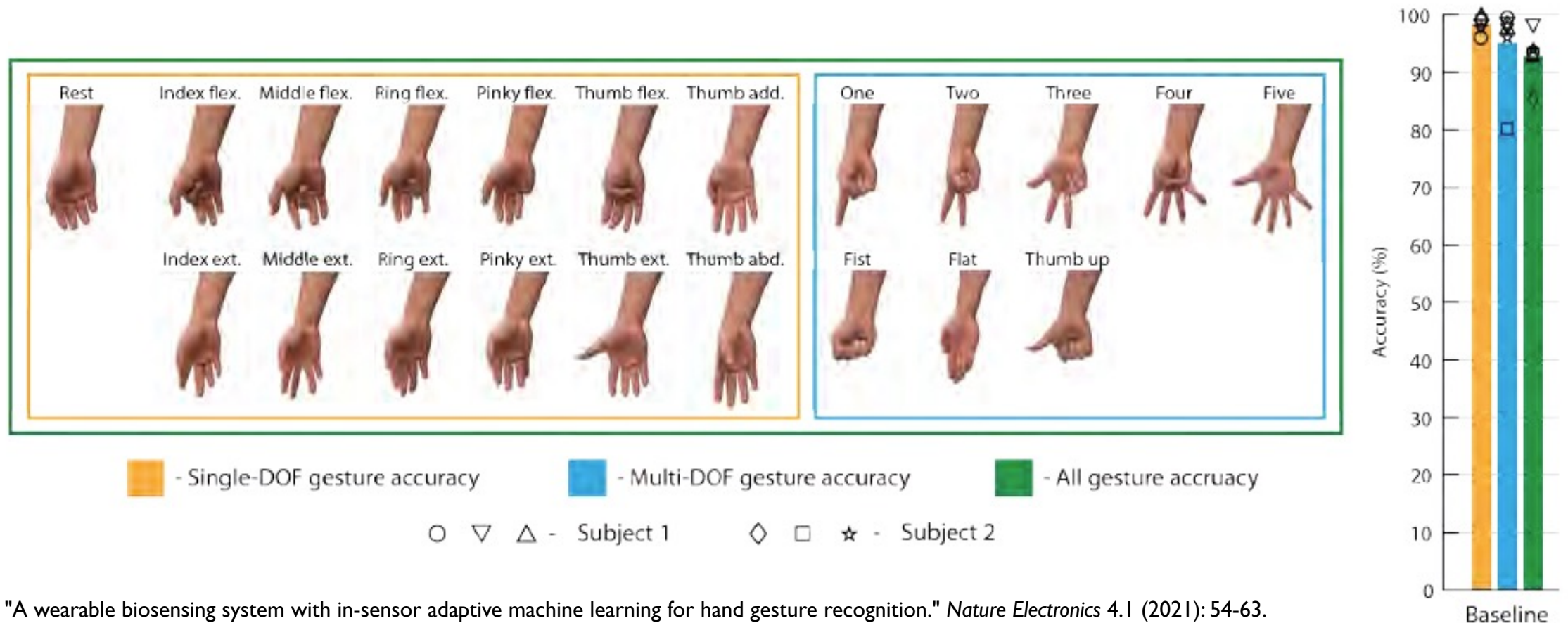
# EMG-based hand gesture recognition





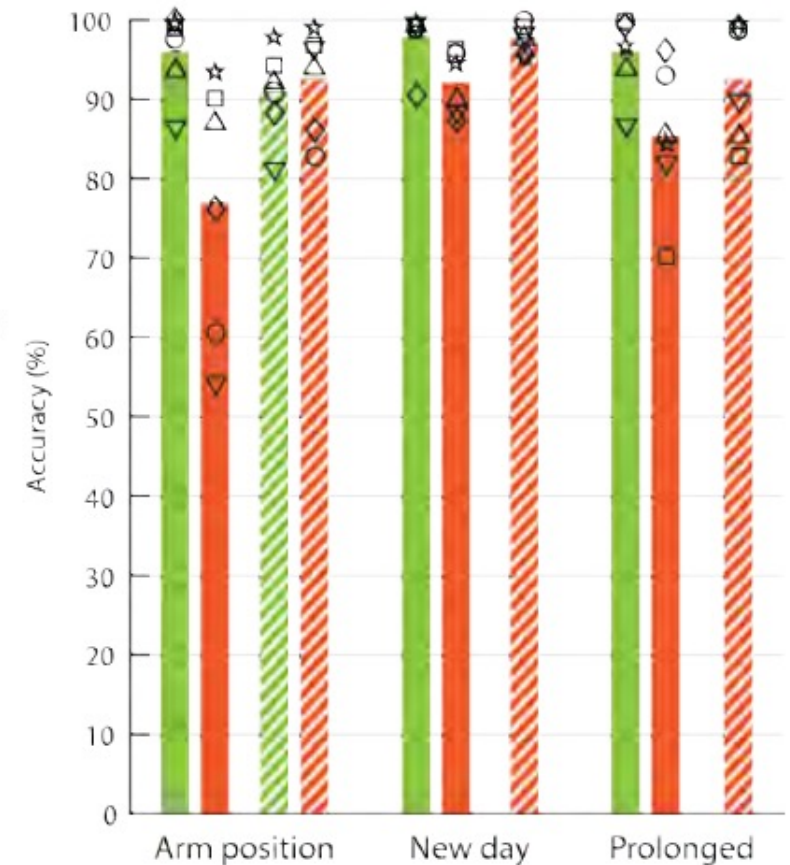
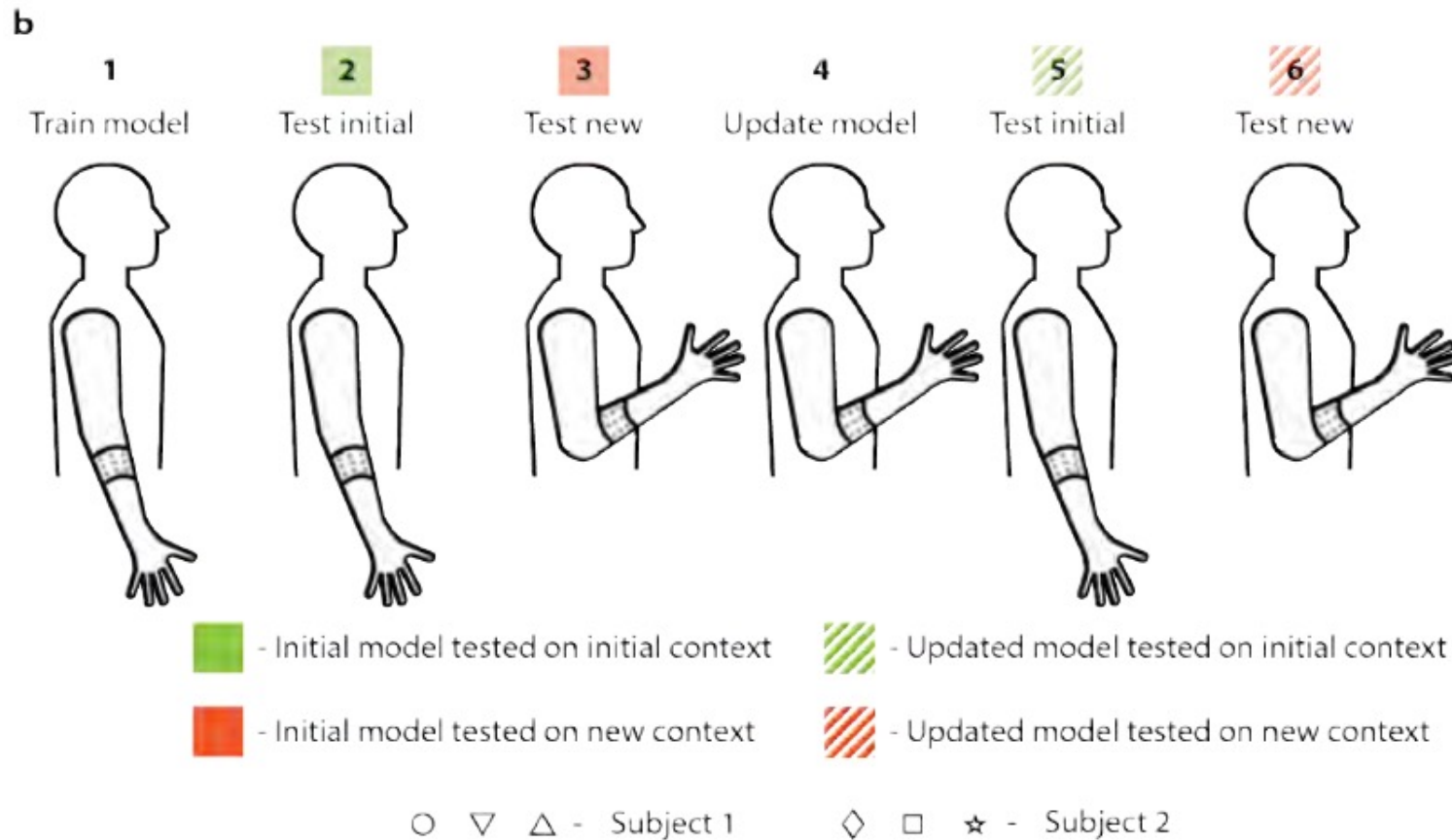
# EMG-based hand gesture recognition

- Results: high accuracy and robustness to variations from different users.



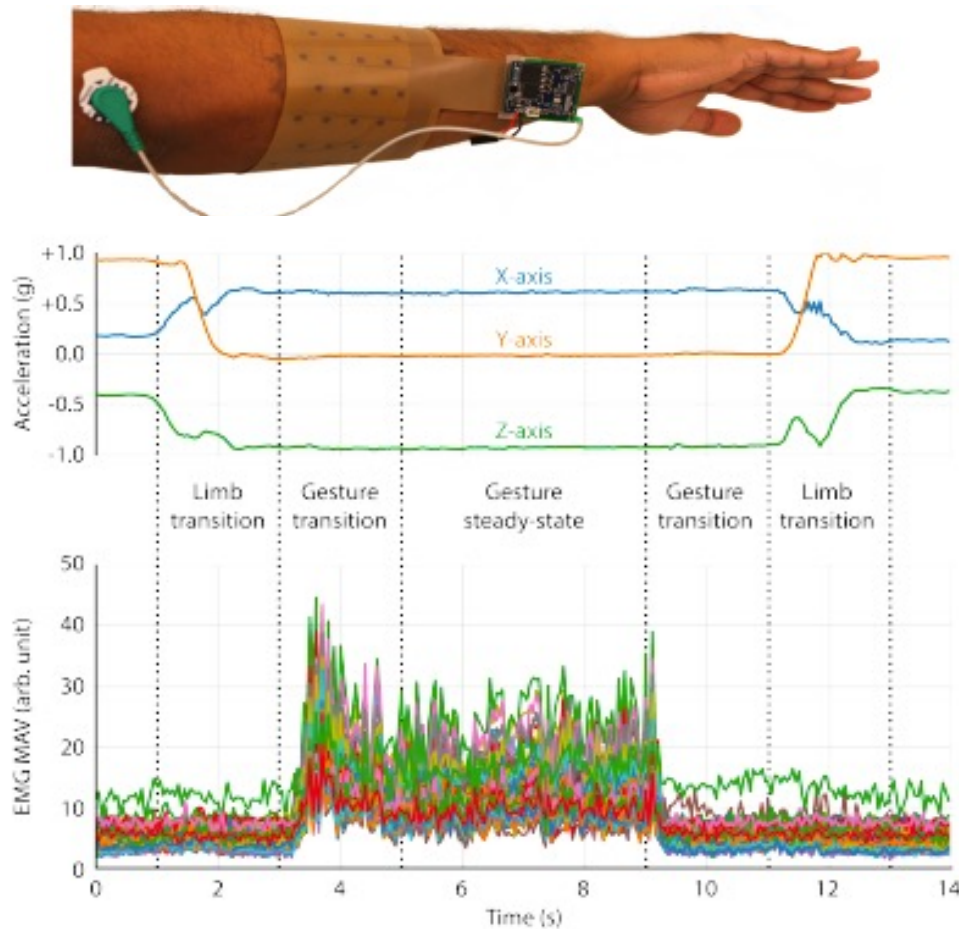
# EMG-based hand gesture recognition

- Adaptive learning without significant overhead.

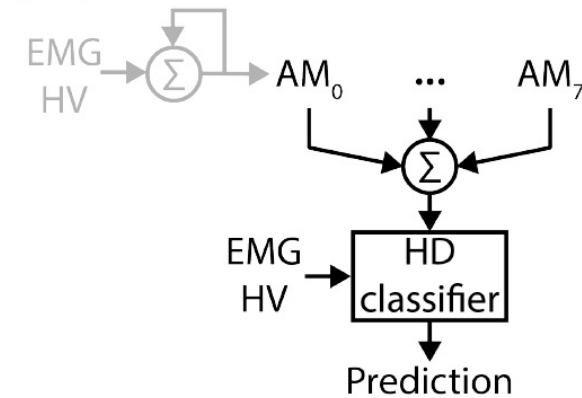


# EMG-based hand gesture recognition

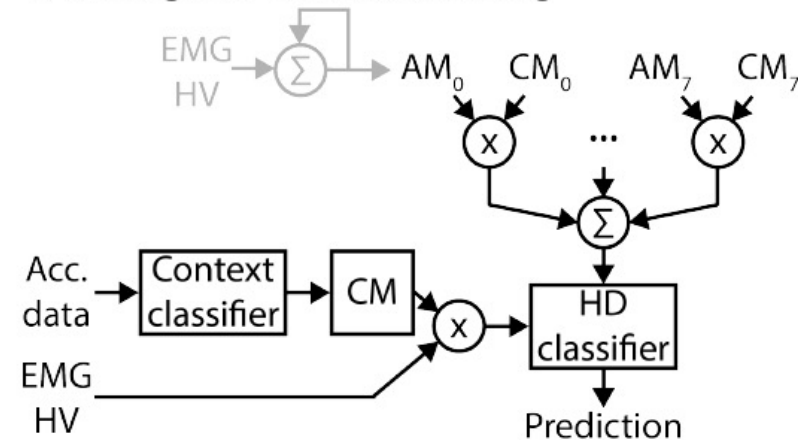
- Sensor fusion and storing context-specific models in superposition.



a) Direct superposition

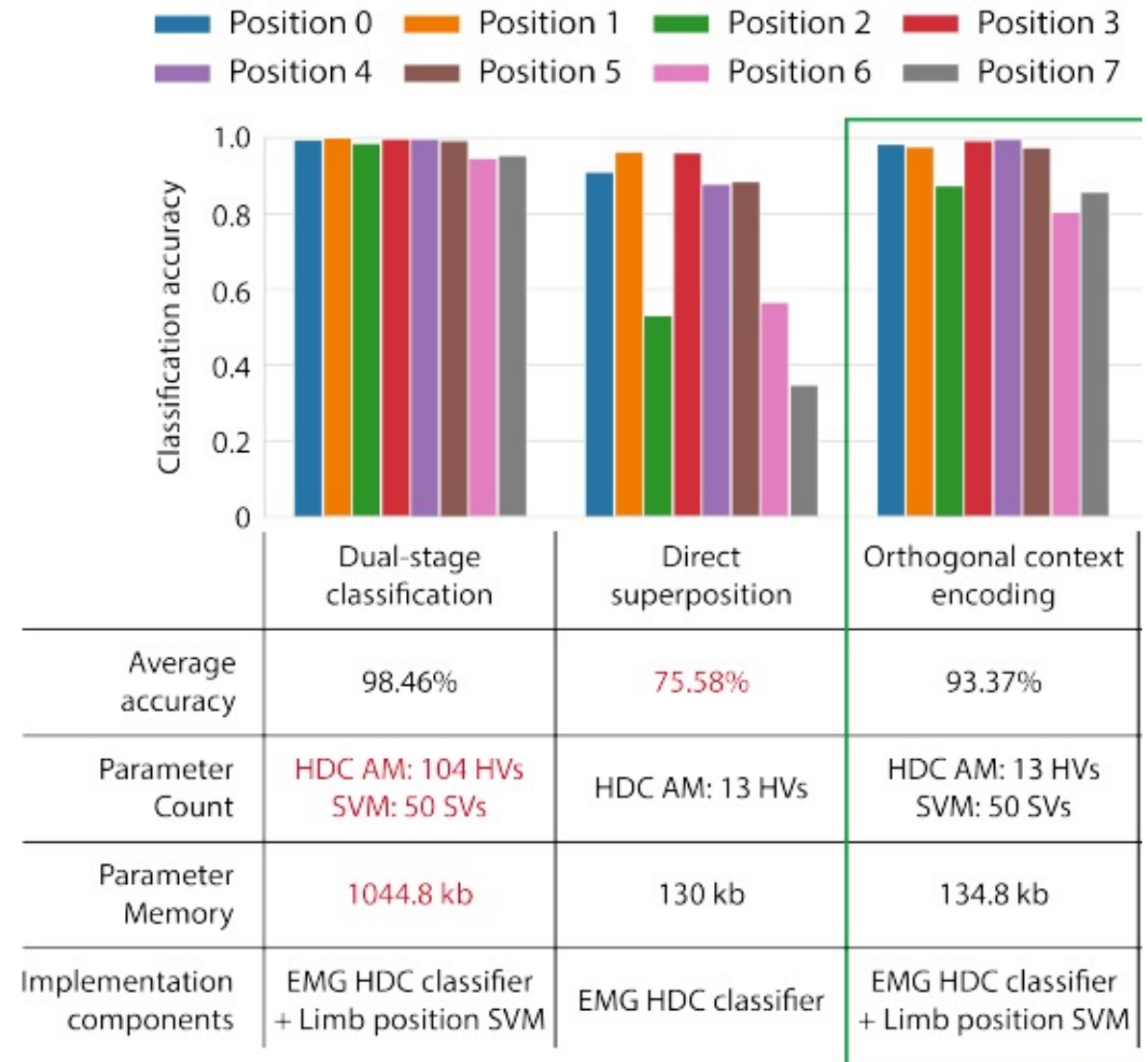


c) Orthogonal context encoding



# EMG-based hand gesture recognition

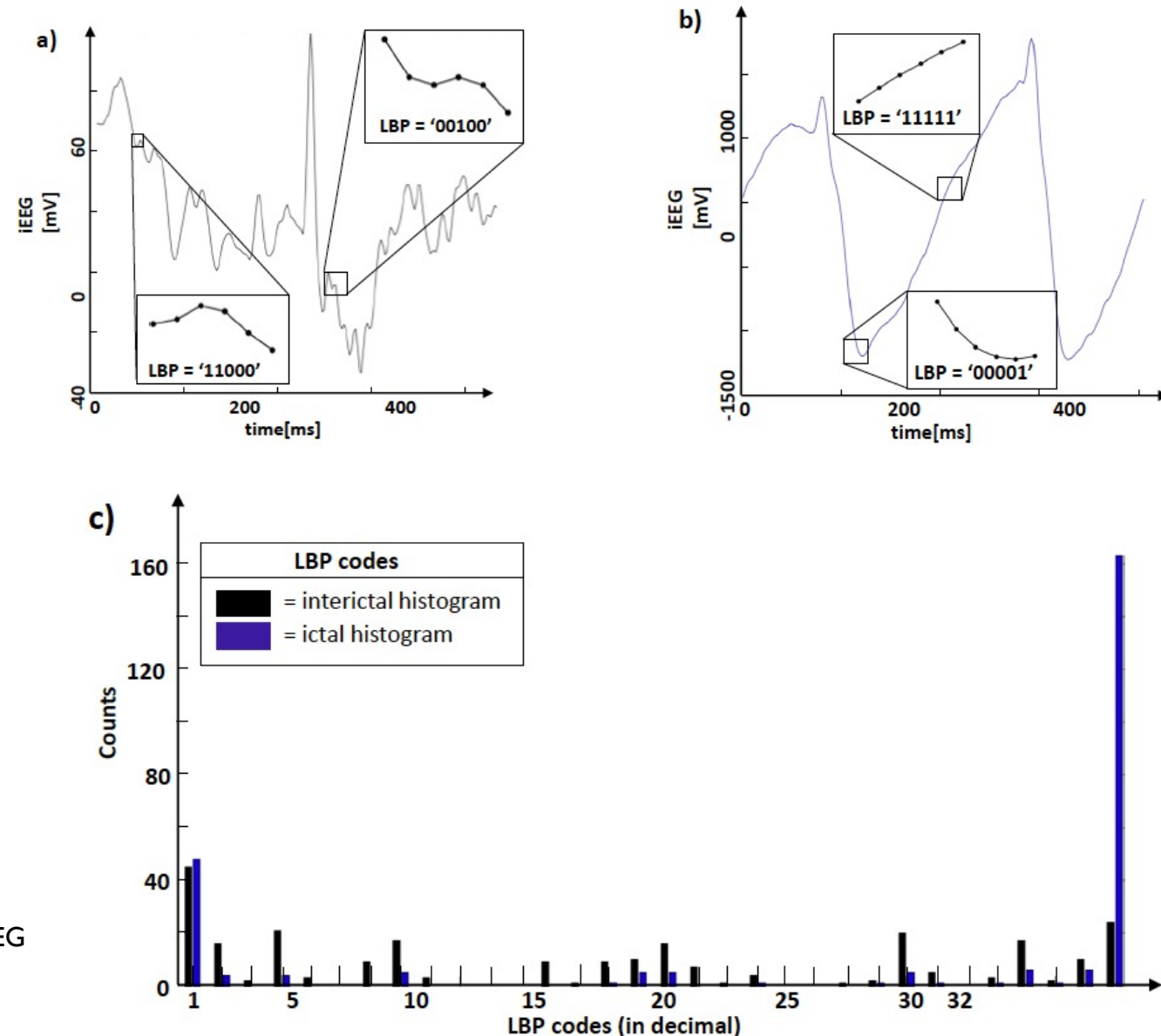
- Results: the proposed orthogonal context encoding improves accuracy with respect to direct superposition and is significantly more efficient than dual-stage classification.





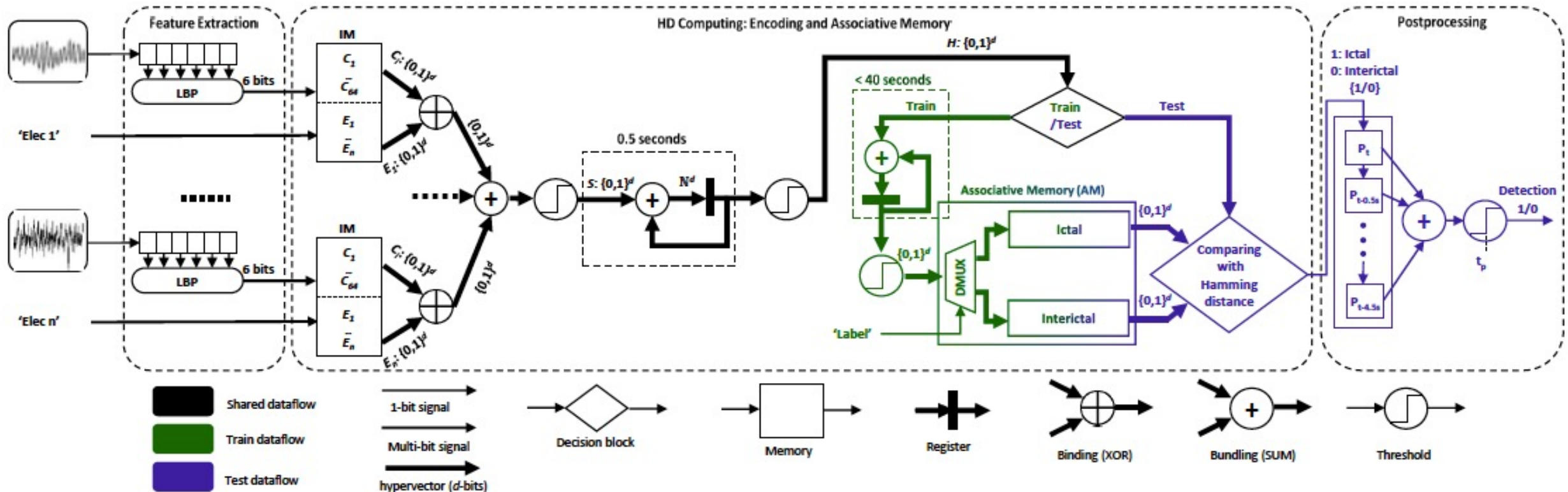
# iEEG-based Seizure Detection

- Interictal (precede seizure) and ictal (during seizure) states of brain activity have distinct pattern frequency distributions.
- These patterns can be captured through short bit strings or local binary patterns (LBPs).
- The two brain states can be identified by comparing the frequency distributions.



# iEEG-based Seizure Detection

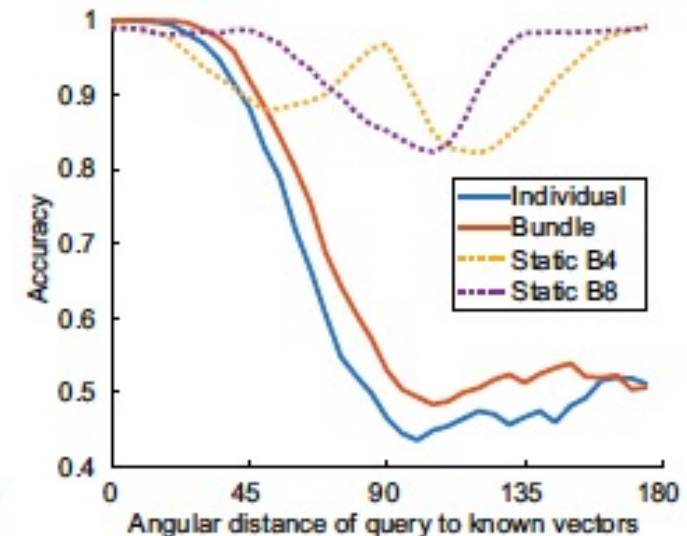
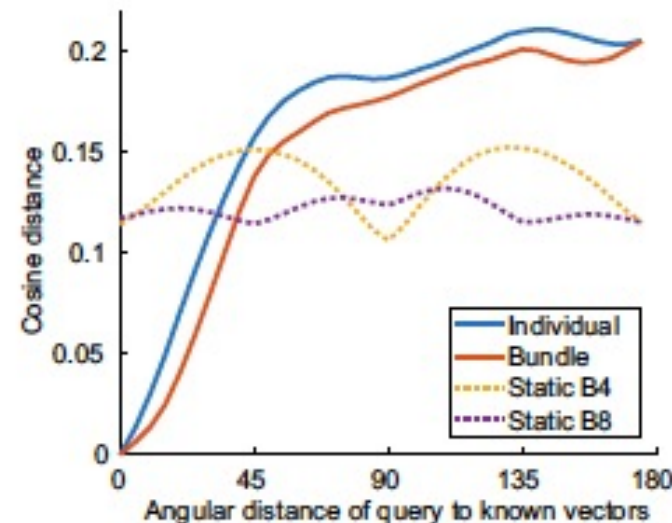
- HD computing/VSAs can be used to represent the LBP and encode histograms over a pre-determined window. Similarity metrics are used to infer whether the current brain state is ictal or interictal.



# Robotics (vision)

## Object recognition from multiple viewpoints

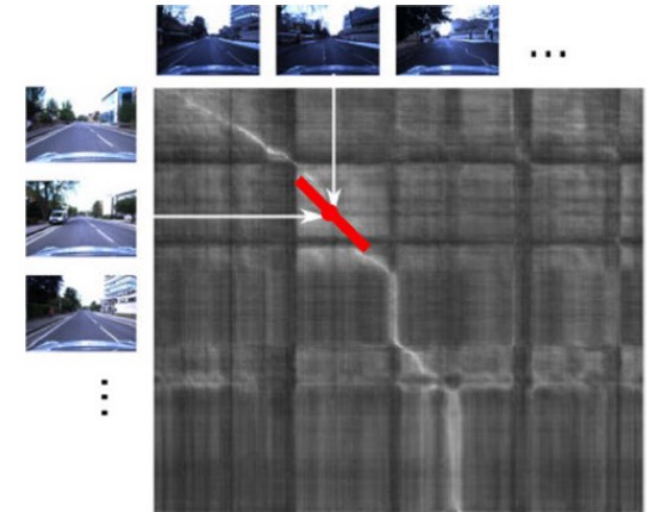
- Exploit bundling to combine multiple viewpoints in superposition.
- Then the comparison of a query vector to all views can be made with a single vector comparison.
- Straightforwardly update the representation of an object during online execution.



# Robotics (vision)

## Sequence processing for place recognition

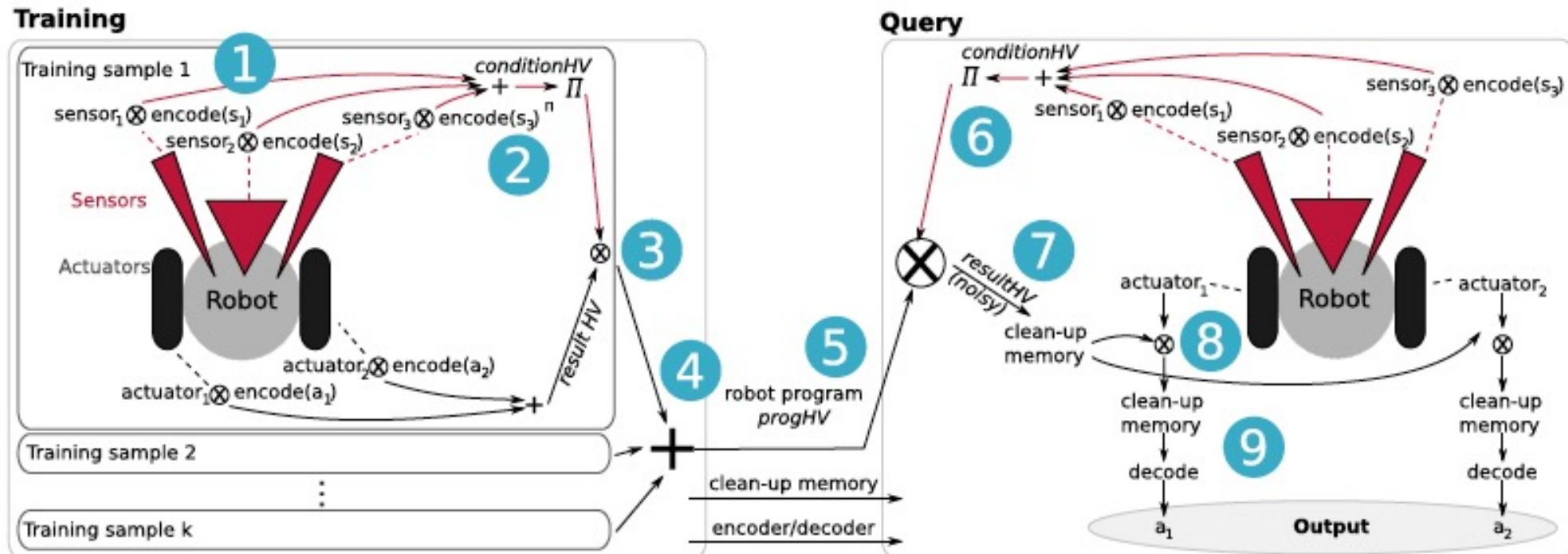
- Individual scenes are represented by binding their vector representation to their current position.
- Sequences of scenes are formed by bundling a set of the representations above.
- Prototype encodings are then compared to query ones to infer the scene.
- This representation is robust even through seasonal changes.





# Robotics

- Learning and recall of reactive behavior:
  - Learn from demonstrations a representation that encodes sensor-action pairs.
  - This behavior can be resembled at run-time by extracting the action corresponding to the current sensor value through an unbinding operation.



# Other applications of HD computing/VSAs

- Survey papers:

## Module 9 (10/27): Solving classification problems

- Focus paper: Rahimi, Kanerva, Benini, Rabaey: [Efficient Biosignal Processing Using Hyperdimensional Computing: Network Templates for Combined Learning and Classification of ExG Signals](#)
- Further recommended reading:
  - Ge, Parhi: [Classification using Hyperdimensional Computing: A Review](#)
  - Neubert, Schubert, Protzel: [An Introduction to Hyperdimensional Computing for Robotics](#)
  - Joshi, Halseth, Kanerva. [Language Geometry Using Random Indexing](#)
  - Rahimi, Benatti, Kanerva, Benini, Rabaey: [Hyperdimensional Biosignal Processing: A Case Study for EMG-based Hand Gesture Recognition](#)
  - Kleyko, Rahimi, Rachkovskij, Osipov, Rabaey: [Classification and Recall with Binary Hyperdimensional Computing: Tradeoffs in Choice of Density and Mapping Characteristic](#)

- Comprehensive list of VSA/HD computing works



This page lists VSAs/Hyperdimensional Computing publications in the chronological order

We strive to keep this collection up-to-date but [please let us know](#) if some publications are currently missing and should be added to this list. We will take care of updating it.

Search	1988	2021	Type	+	Author(s)	▼	⋮
Venue	+	Keywords	▼				
1 - 442 / 442							
pairwise	Author(s)	Title	Venue	Pages	Keywords		
2021	Poduval P., Zou Z., Najafi H., Homayoun H., Imani M.	<a href="#">StochHD: Stochastic Hyperdimensional System for Efficient and Robust Learning from Raw Data</a>	ACM/ESDA/IEEE Design Automation Conference (DAC)	pp. 1-6	Machine Learning, Hardware		
2021	Halawani Y., Kilani D., Hassan E., Tesfai H., Saleh H., Mohammad B.	<a href="#">RRAM-Based CAM Combined With Time-Domain Circuits for Hyperdimensional Computing</a>	Scientific Reports	vol. 11, pp. 1-11	Hardware		
2021	Thomas A., Dasgupta S., Rosing T.	<a href="#">A Theoretical Perspective on Hyperdimensional Computing</a>	Journal of Artificial Intelligence Research	vol. 72, pp. 215-249			
2021	Eggimann M., Rahimi A., Benini L.	<a href="#">A 5 <math>\mu</math>W Standard Cell Memory-based Configurable Hyperdimensional Computing Accelerator for Always-on Smart Sensing</a>	IEEE Transactions on Circuits and Systems I: Regular Papers	vol. 68, no. 10, pp. 4116-4128			

# Questions?

Thank you!