1. Why high dimensions? What is the purpose of using high-dimensional vectors? Why might it make sense to select vectors randomly? Explain your answer in about a paragraph.

2. Plotting pseudo-orthogonality. For binary vectors \( \{0,1\}^N \), let \( d(x,y) \), the normalized Hamming between two vectors \( x, y \), be defined as:

\[
\text{dist}_{\text{Ham}}(x,y) = \frac{1}{N} |x - y| = \frac{1}{N} (|x_1 - y_1| + \cdots + |x_N - y_N|)
\]

The distribution of \( \text{dist}_{\text{Ham}}(x,y) \) for i.i.d. random binary vectors will look something like this:

(from Kanerva, *Sparse Distributed Memory*, p. 20)

Plot the distribution of cosine similarity for i.i.d. random bipolar vectors \( \{-1,+1\}^N \) in each of the following cases: \( N = 10^1, 10^2, 10^3, \) and \( 10^4 \). You can obtain the plot by first randomly drawing several thousand pairs of \( x \) and \( y \), calculating their cosine similarity, and fitting the normal distribution to the histogram of the observed cosine similarities. Do you see the results, which resemble the figure above?
3. Implementing basic HDC/VSA operations.
(a) For bipolar \{-1,+1\} vectors, recreate the following three functions corresponding to the basic operations of HDC/VSA:

```python
def bind(vector1, vector2):
    #your code here
    return bound_vector

def bundle(vector1, vector2):
    #your code here
    return bundled_vector

def permutation(vector1):
    #your code here
    return permuted_vector
```

Note: For now, assume that the bundling operation is simply component-wise addition without any normalization.

(b) Demonstrate that the bundling operation is similarity-preserving, whereas the binding and permutation operations are not.

(c) Assume that your codebook includes 5 random bipolar vectors \{a,b,c,d,e\}. Form the compositional vector \( z \) of the following form \( z = a + a + b + b + c \).
Plot the histogram of cosine similarities between \( z \) and \( \{a,b,c,d,e\} \) that is averaged over several random initializations of the codebook. What regalities do you observe?