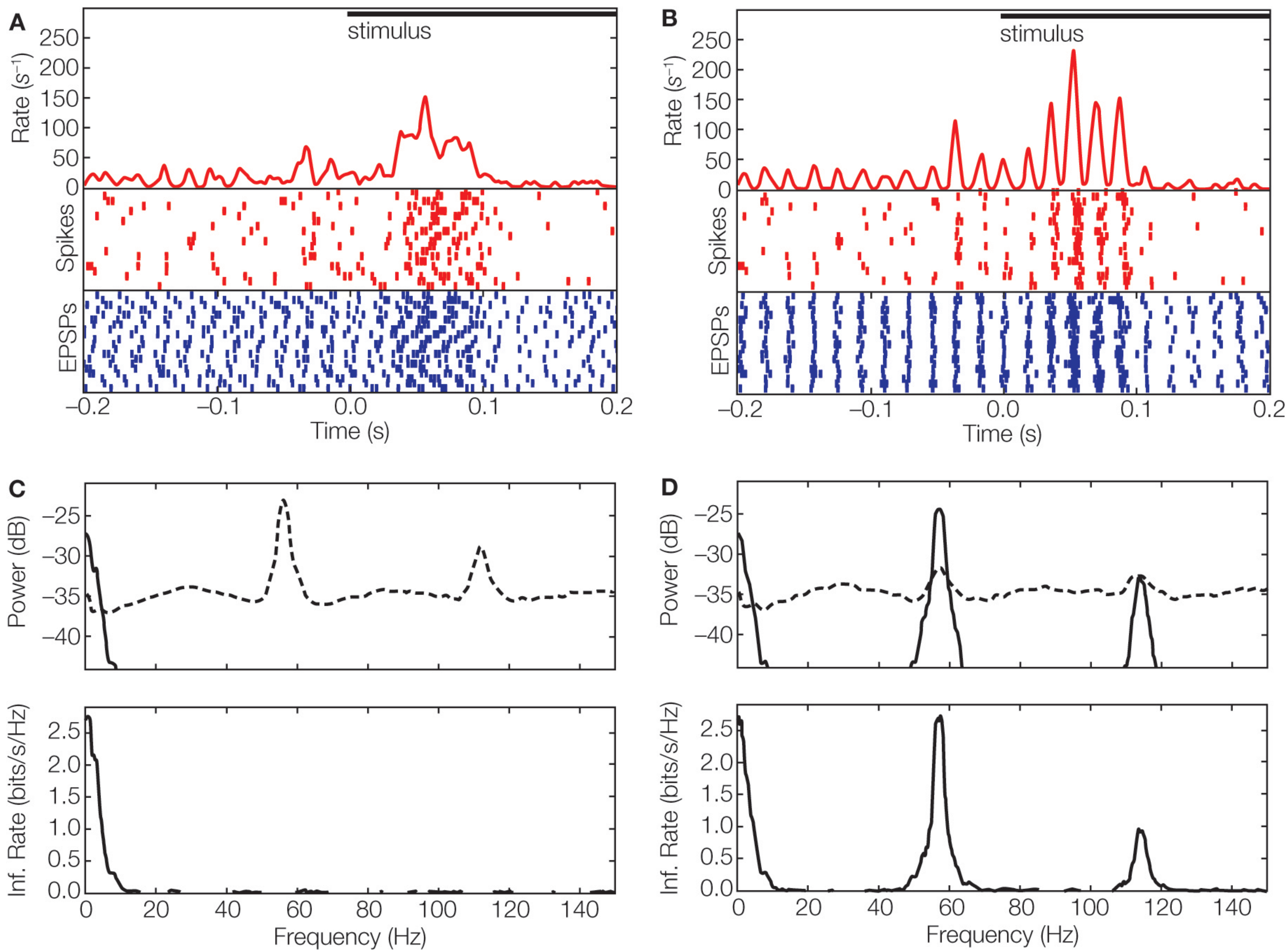
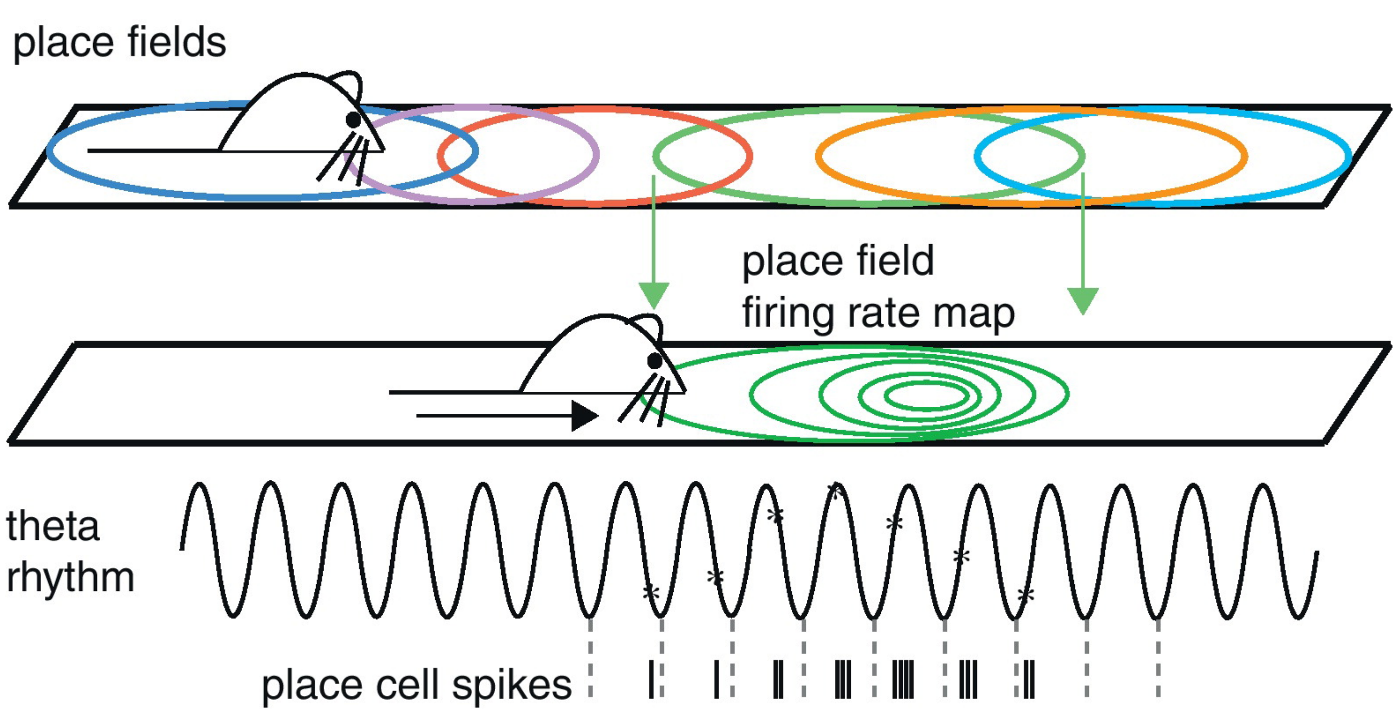
Computing with oscillations and waves

Retinal oscillations carry information to cortex

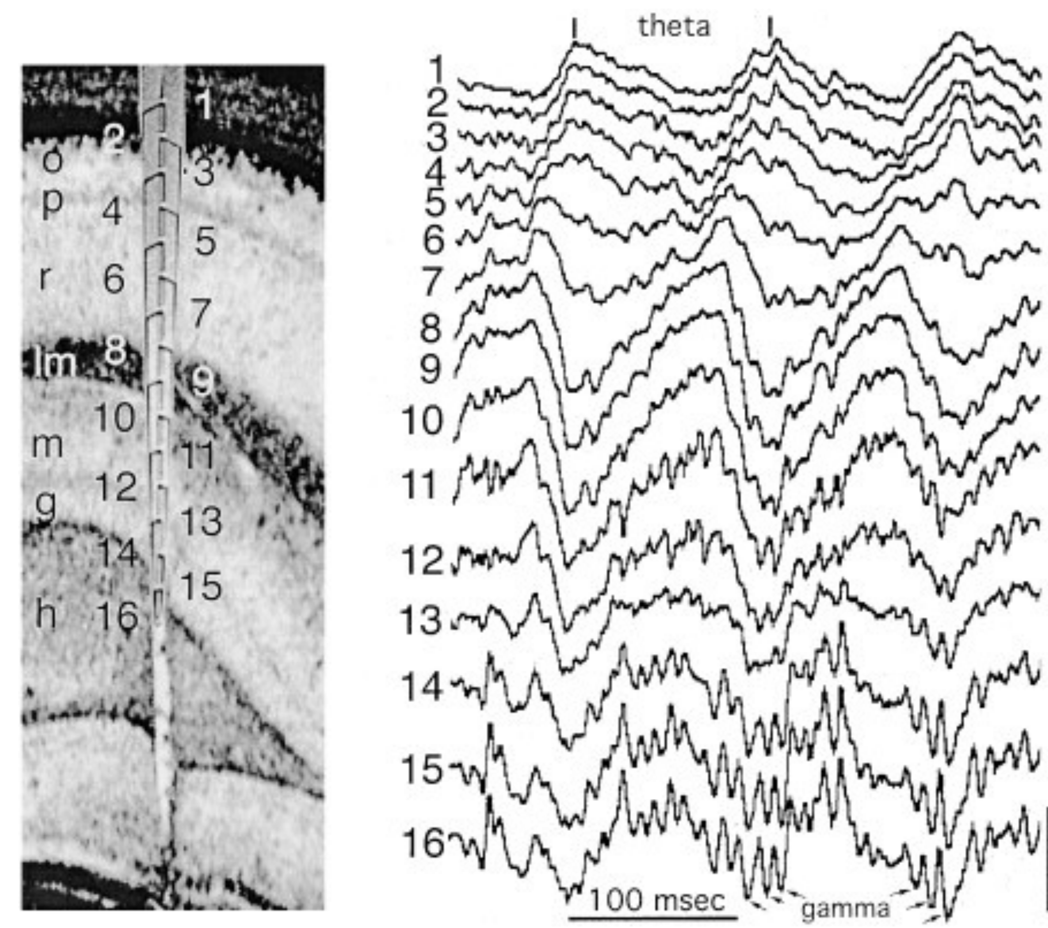
(Koepsell, Wang, Hirsch & Sommer 2009)



The Hippocampal Theta Rhythm
(Agarwal, Sommer & Buzsaki 2013)

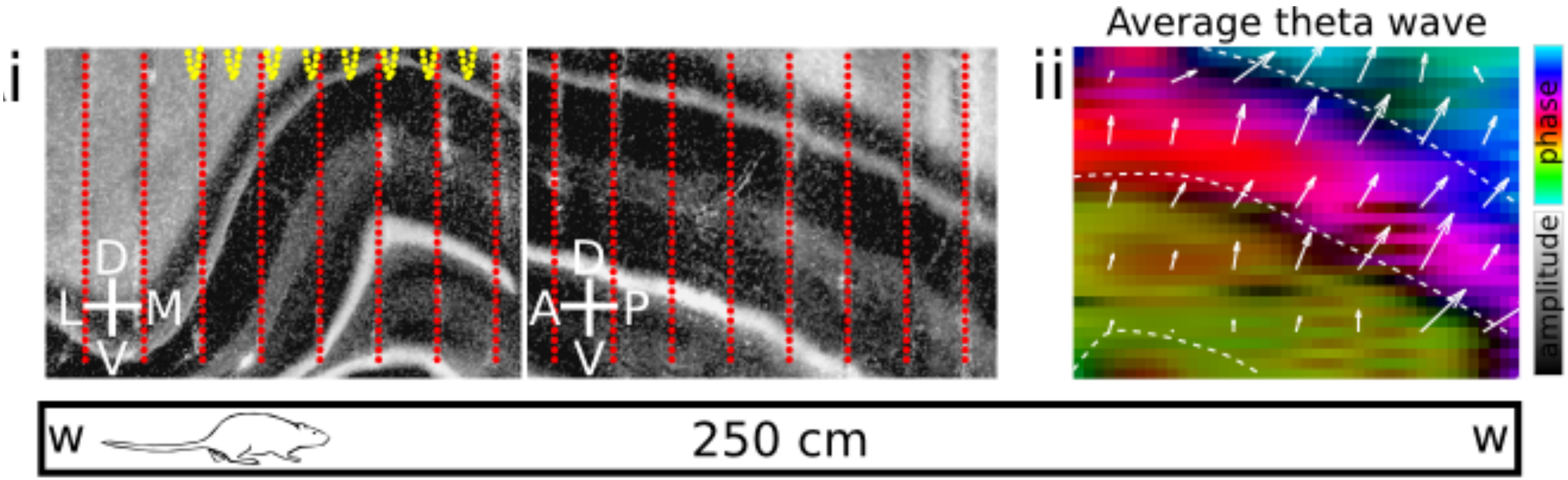


Foster & Knierim 2012

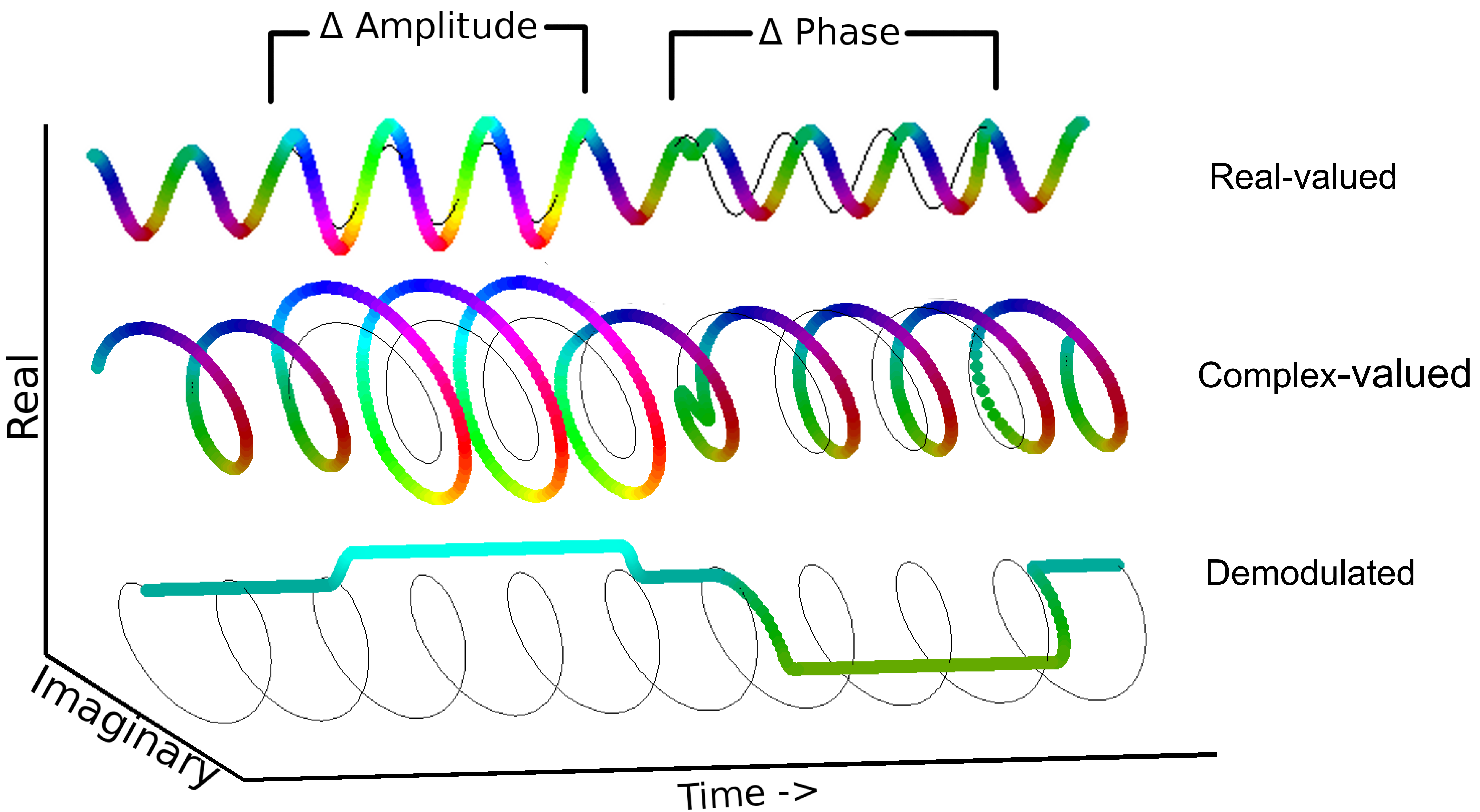


Buzsaki 2002

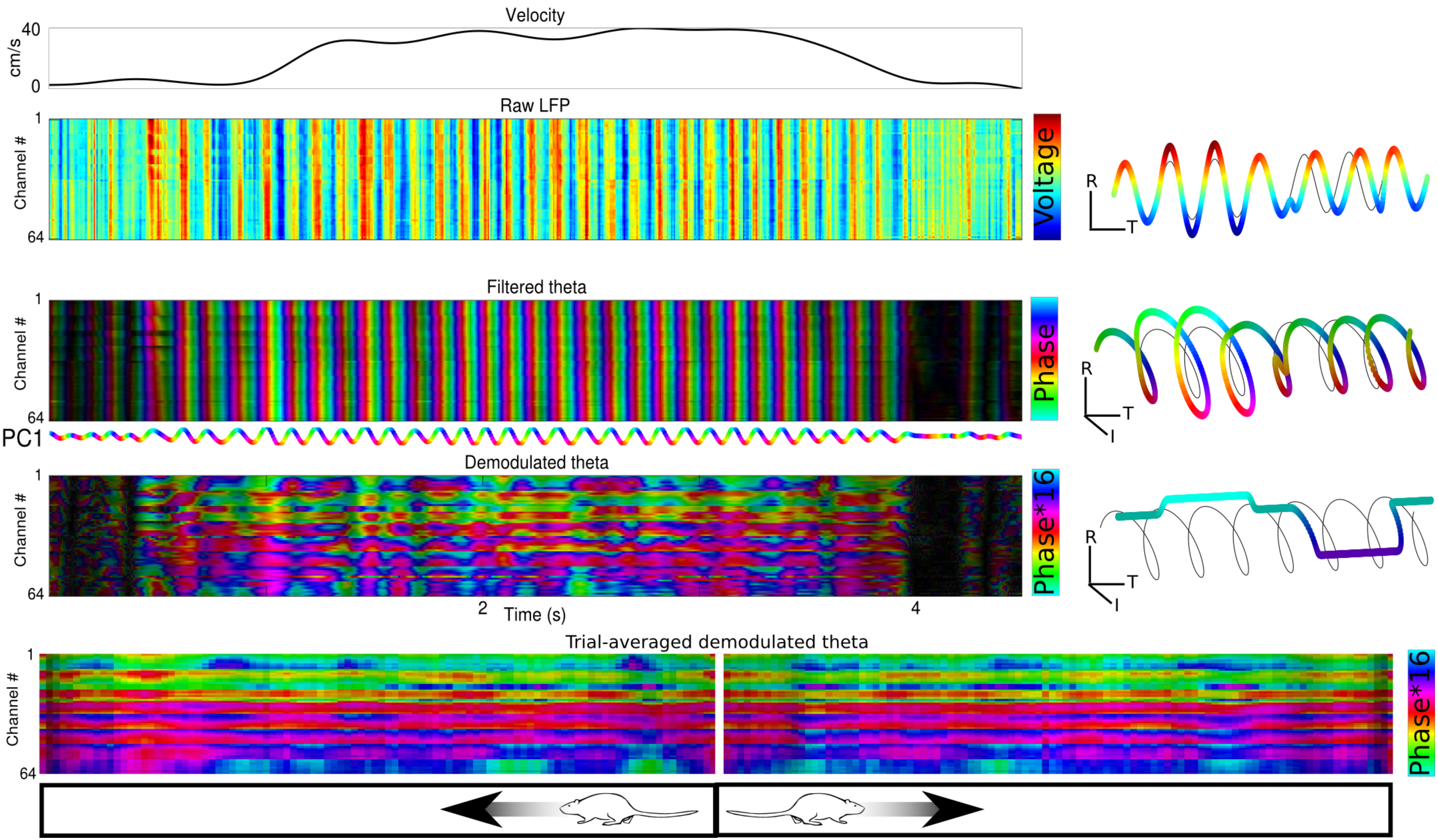
Experimental Setup



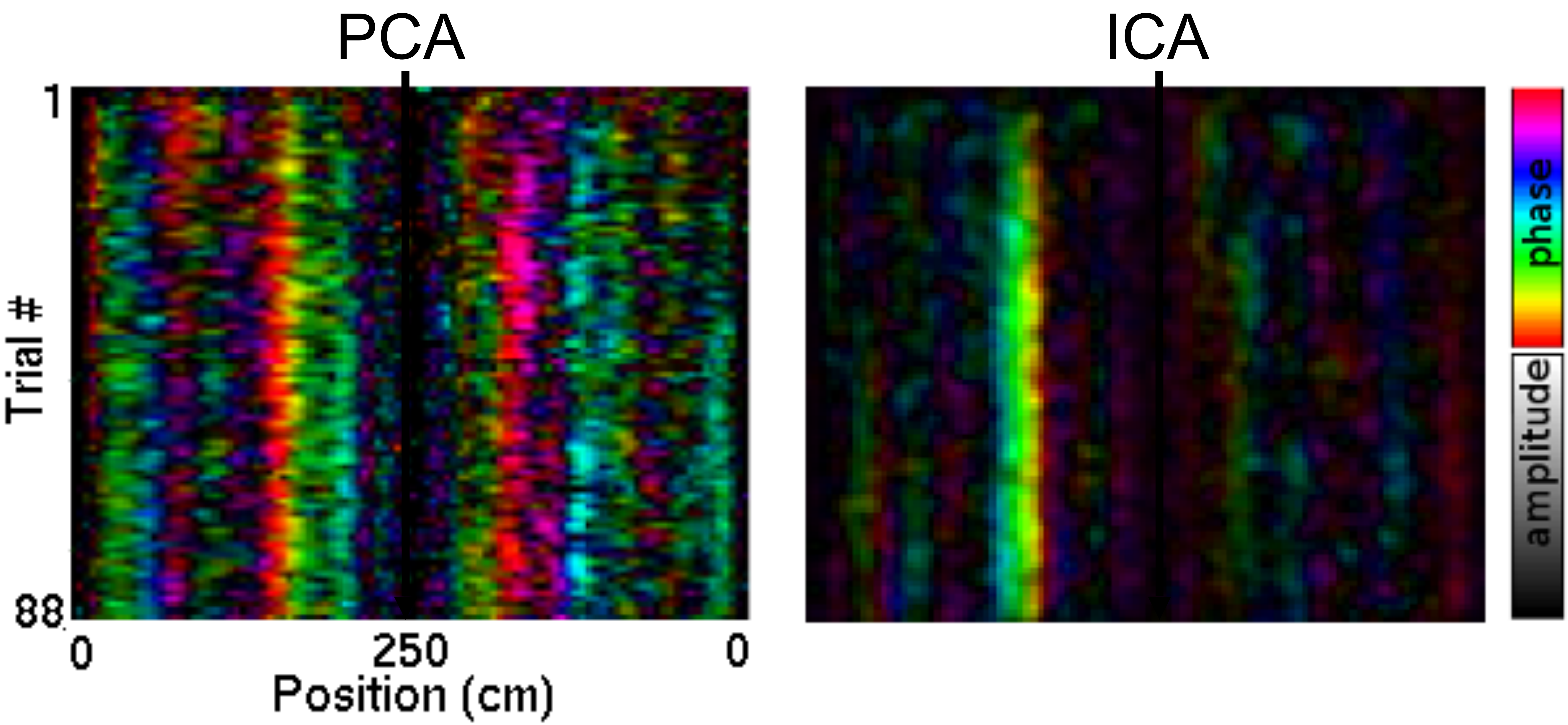
Oscillatory descriptors



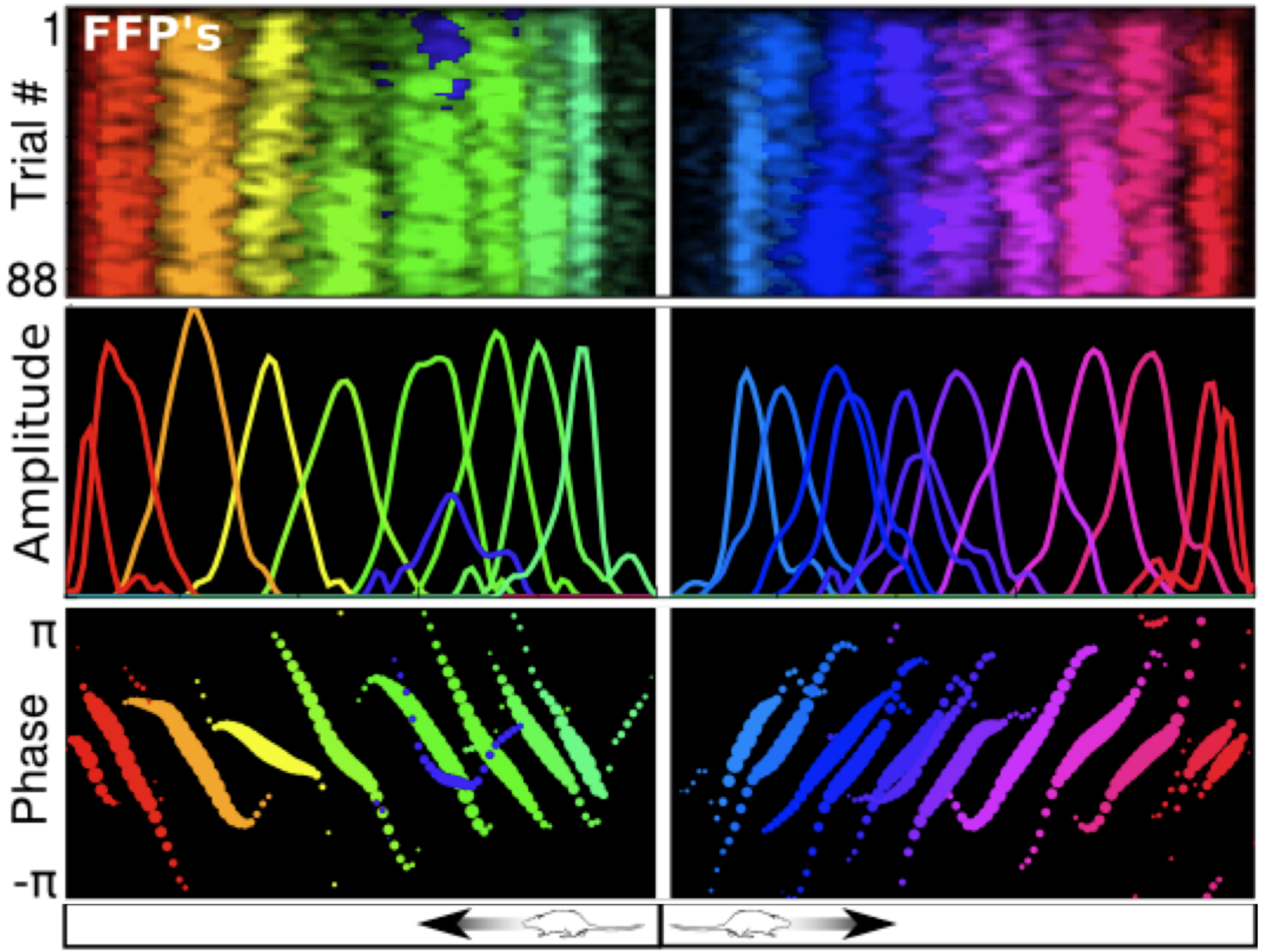
Demodulation reveals behaviorally relevant signal components



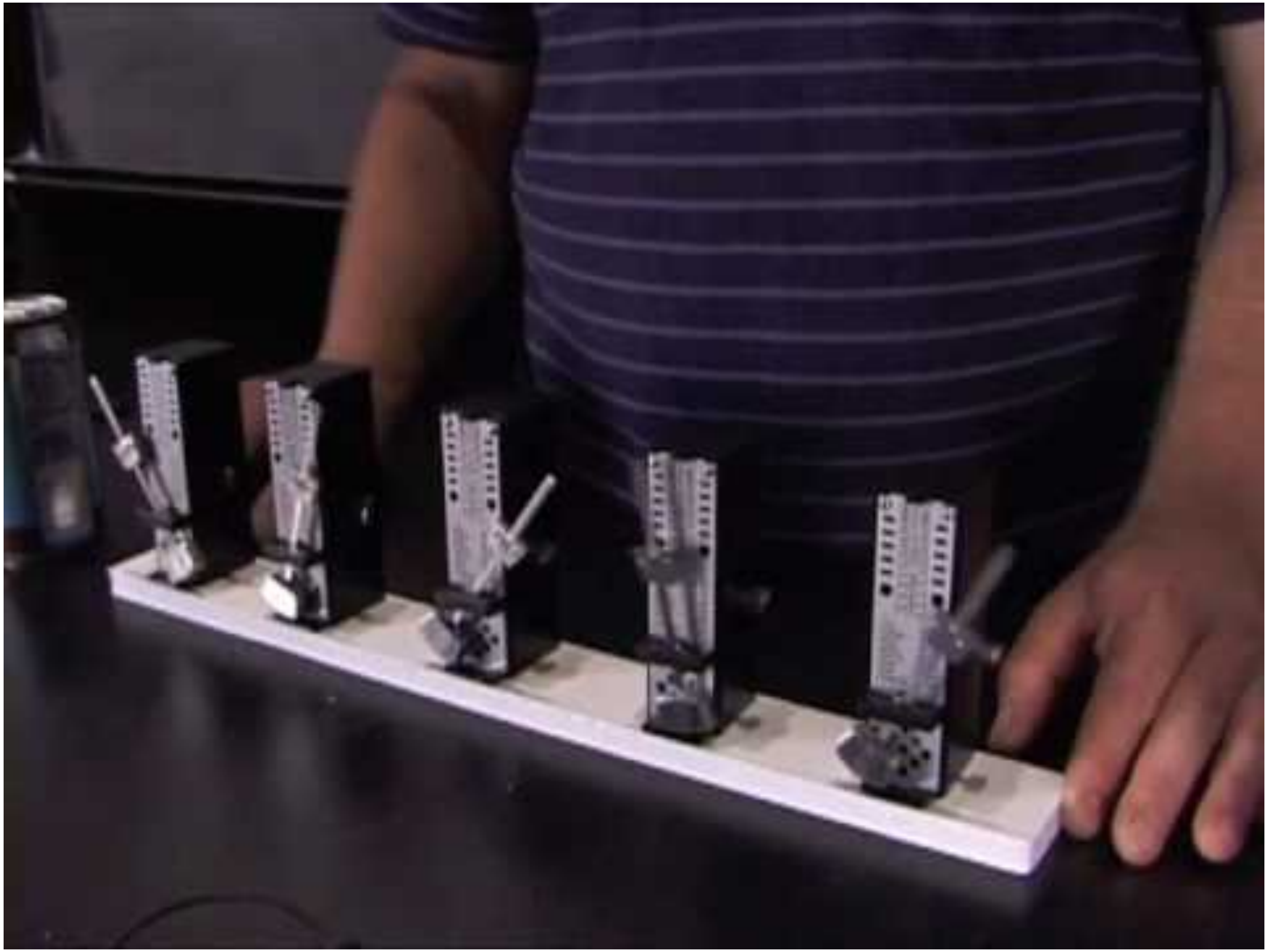
ICA reveals ‘place components’



FFP's tile space



Coupled oscillator models

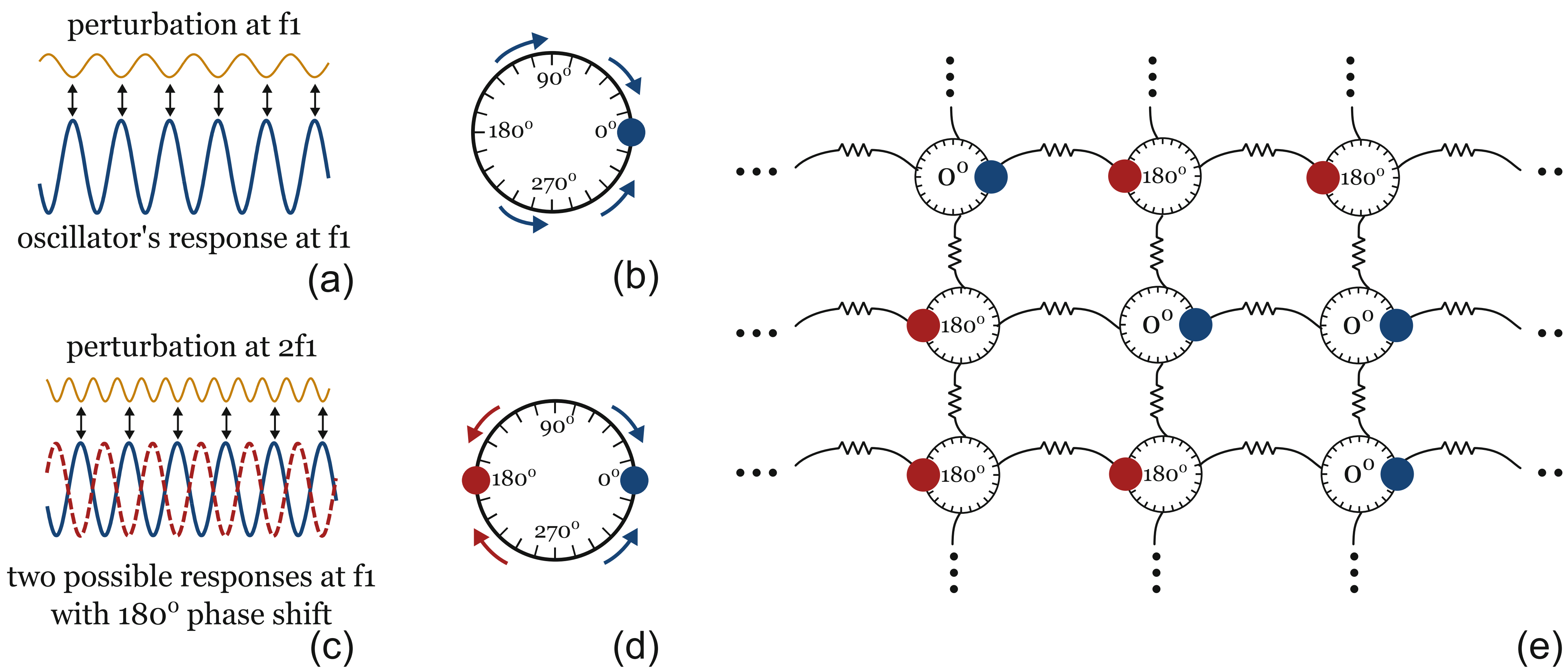


Kuramoto model

$$\dot{\theta}_i = \omega_i + K \sum_j \sin(\theta_j - \theta_i)$$

Oscillation-based Ising machines for solving combinatorial optimization problems
(Wang & Roychowdhury 2019)

$$\min \quad H \triangleq -\sum_{1\le i<j\le n} J_{ij}s_i s_j - \sum_{i=1}^{n} h_i s_i, \quad \text{such that} \quad s_i \in \{-1, \ +1\}$$



Lyapunov function and dynamics

$$E(\vec{\phi}(t)) = -K \cdot \sum_{i,j,\ i \neq j} J_{ij} \cdot \cos(\phi_i(t) - \phi_j(t)) - K_s \cdot \sum_{i=1}^{n} \cos\left(2\phi_i(t)\right)$$

$$\frac{d}{dt}\phi_i(t) = -K \cdot \sum_{j=1,\ j \neq i}^{n} J_{ij} \cdot \sin(\phi_i(t) - \phi_j(t)) - K_s \cdot \sin(2\phi_i(t))$$

Implementation

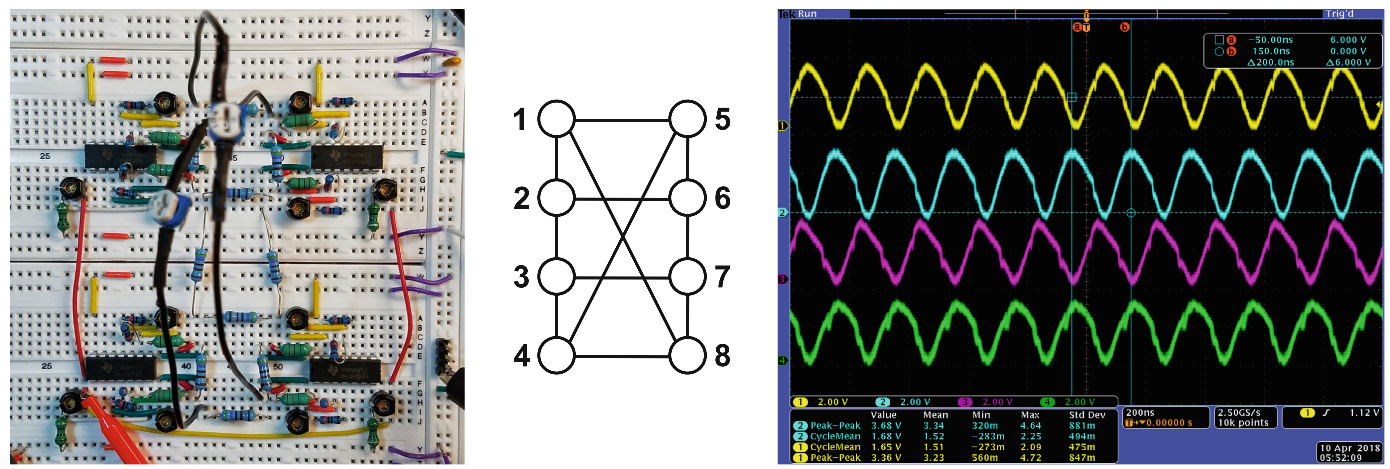


Fig. 8. A simple oscillator-based Ising machine solving size-8 cubic graph MAX-CUT problems: (a) breadboard implementation with 8 CMOS LC oscillators; (b) illustration of the connections; (c) oscilloscope measurements showing waveforms of oscillator 1∼4.

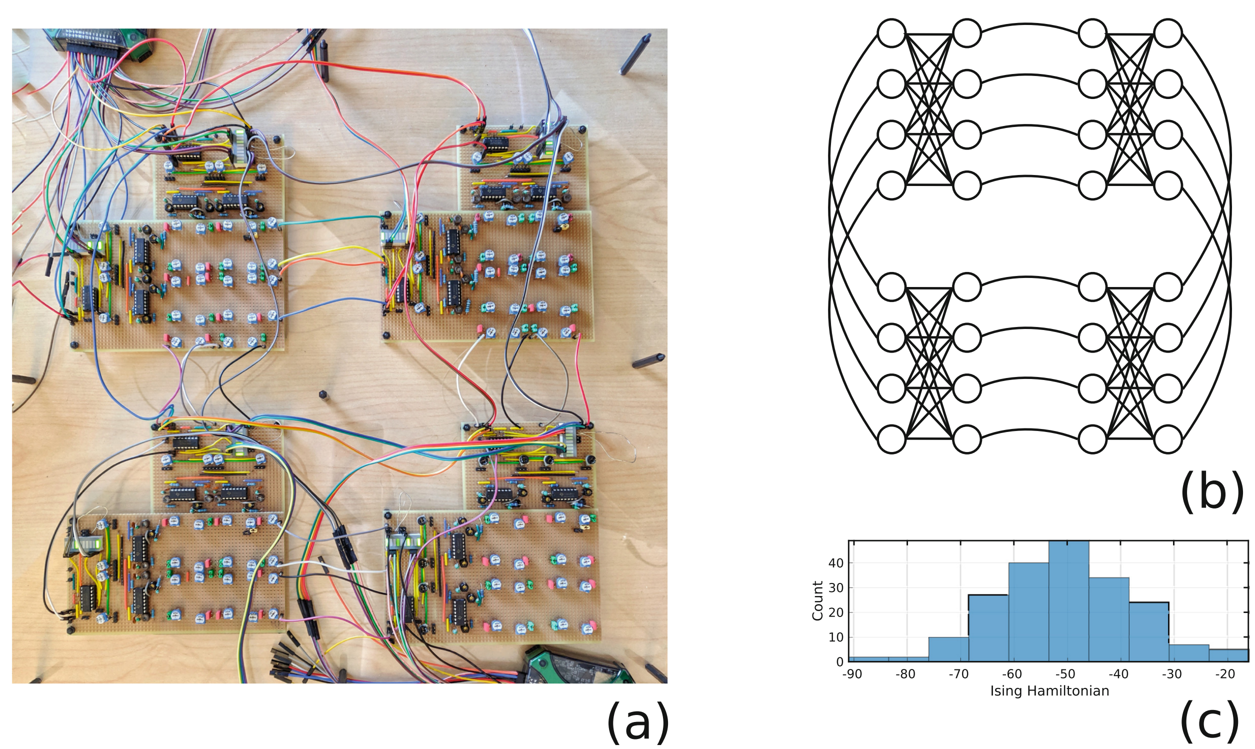


Fig. 9. A size-32 oscillator-based Ising machine: (a) photo of the implementation on perfboards; (b) illustration of the connectivity; (c) a typical histogram of the energy values achieved in 200 runs on a random size-32 Ising problem; the lowest energy level is -88 and is achieved once in this case.

Graph cut

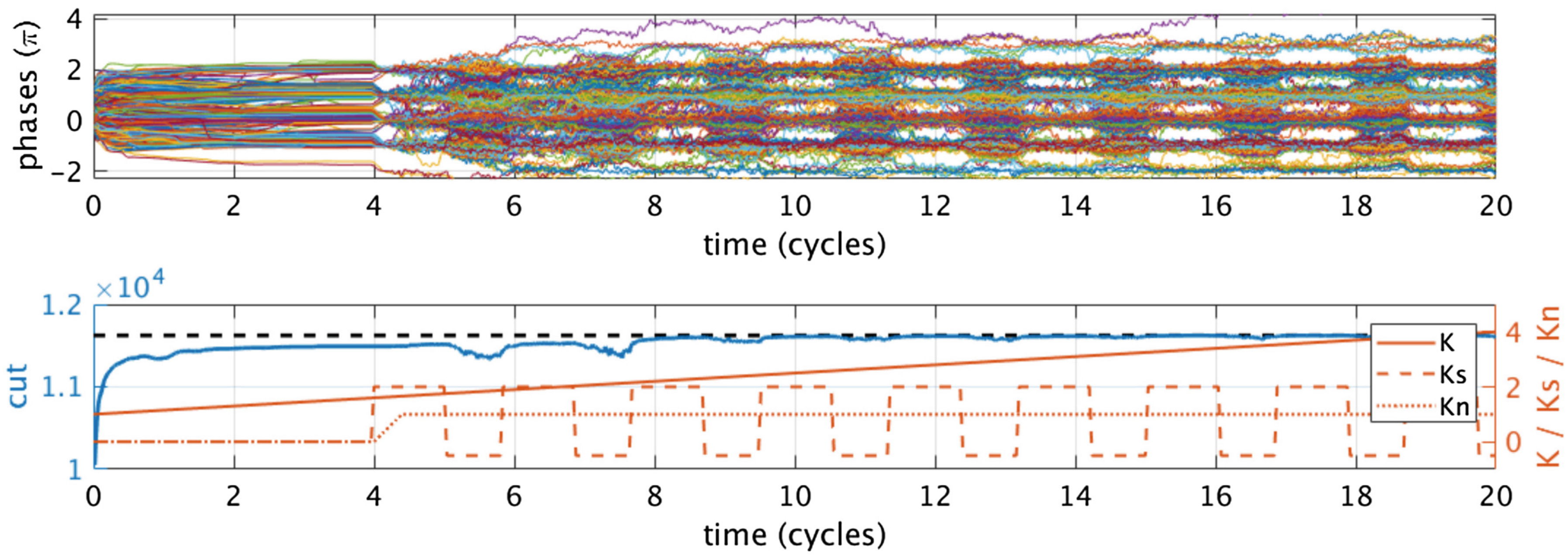
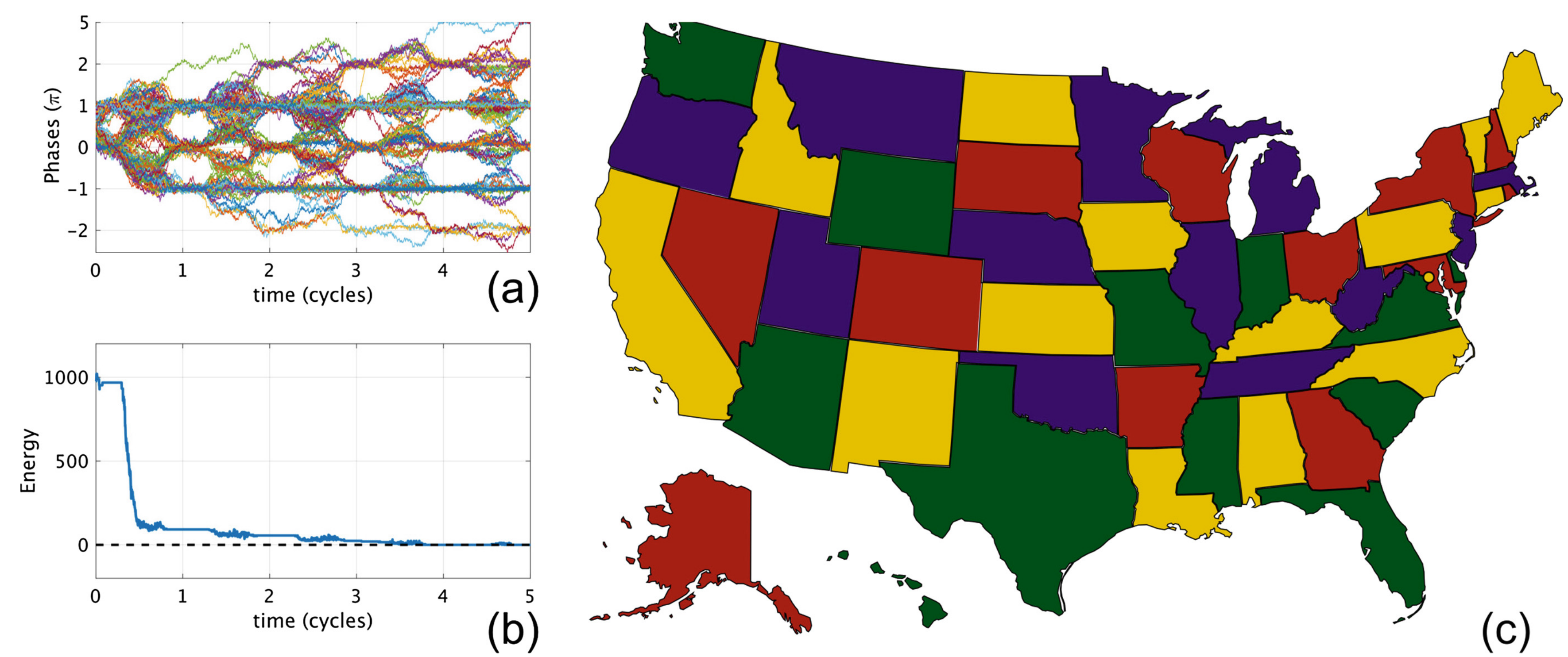


Fig. 10. Coupled oscillators solving MAX-CUT benchmark problem G1 [43] to its best-known cut size 11624.

Graph coloring



Threshold phasor associative memory (TPAM)

(Frady & Sommer 2019)

Energy function

$$E(\mathbf{z}) = -\frac{1}{2}\sum_{ij} W_{ij}\, z_i\, z_j^{*} + \Theta\|\mathbf{z}\|_1 \qquad z_i = |z_i|e^{j\phi_i}$$

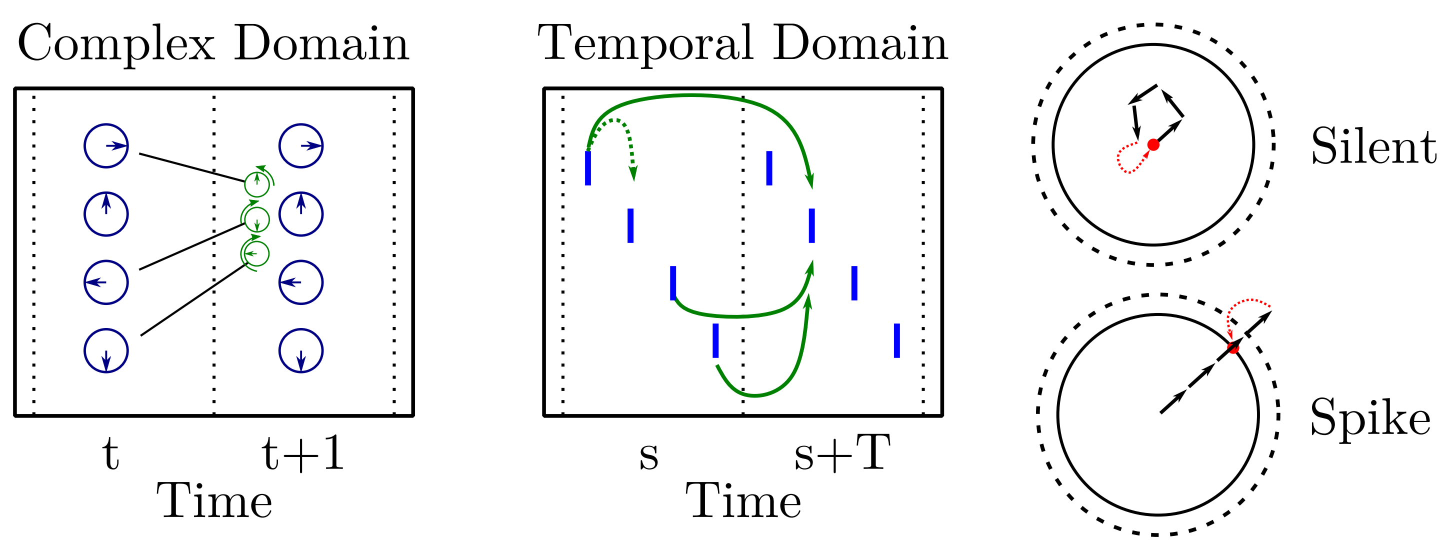
Dynamics

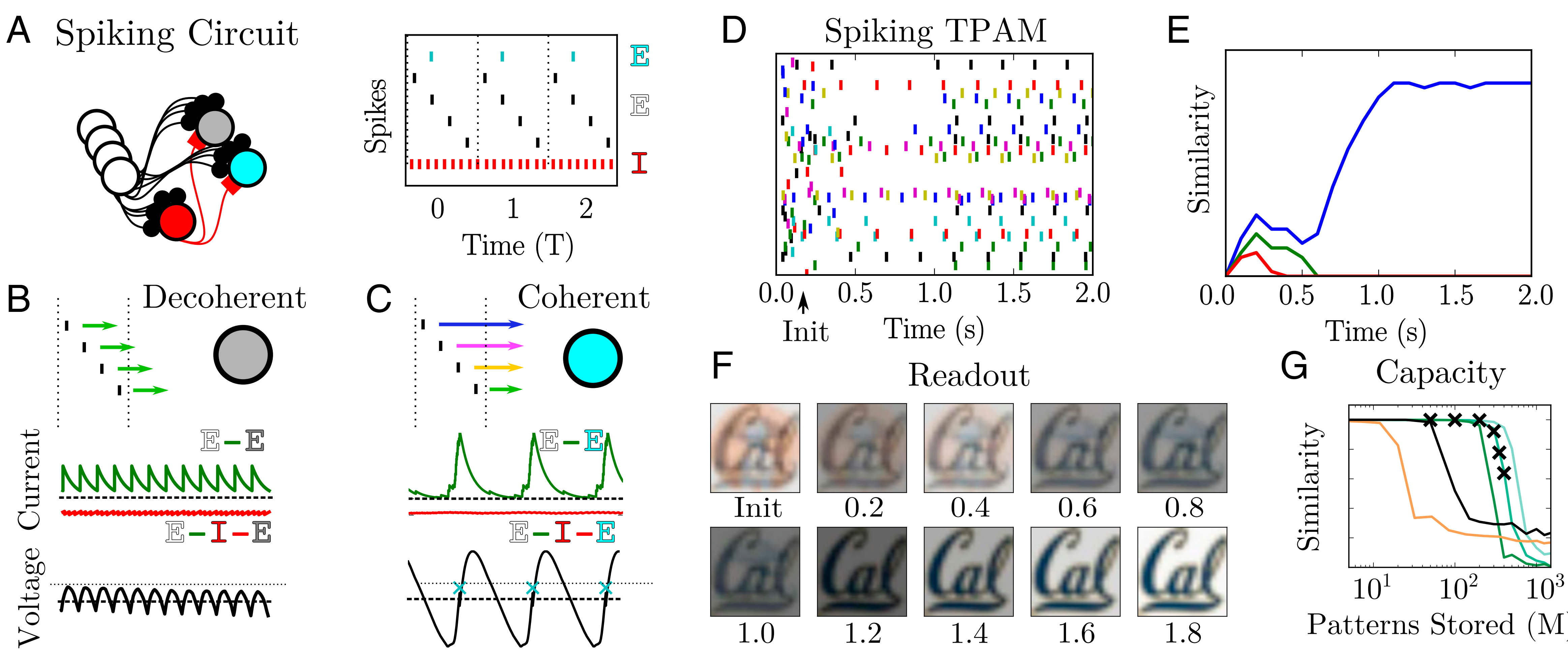
$$u_i(t) = \sum_j W_{ij}\, z_j(t)$$

$$z_i(t+1) = g(u_i(t), \Theta(t)) := \frac{u_i(t)}{|u_i(t)|} H(|u_i(t)| - \Theta(t))$$

$$\Theta(t) = \theta \sum_i |z_i(t)| = \theta|\mathbf{z}(t)|$$

Implementation in spiking neurons





Physics successfully implements Lagrange multiplier optimization

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Contributed by Eli Yablonovitch, August 25, 2020 (sent for review July 27, 2020; reviewed by Thomas Kailath and Stanley Osher)

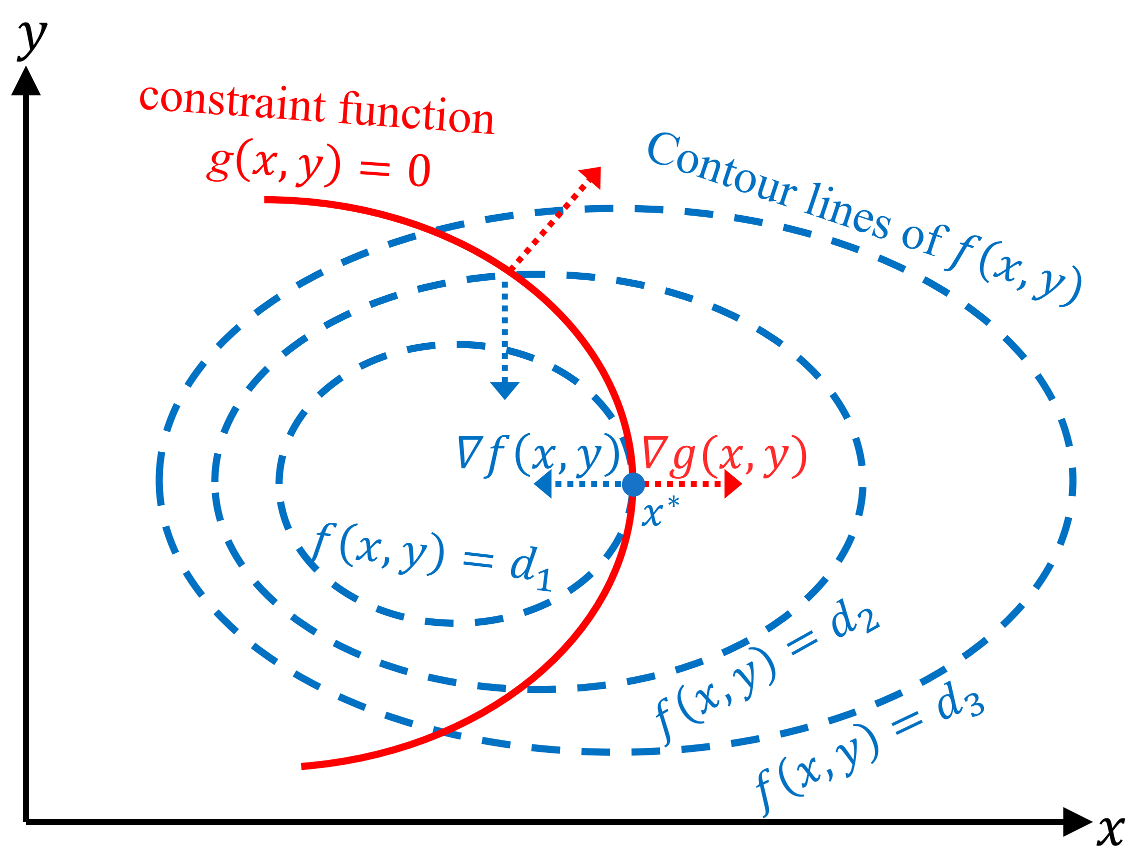
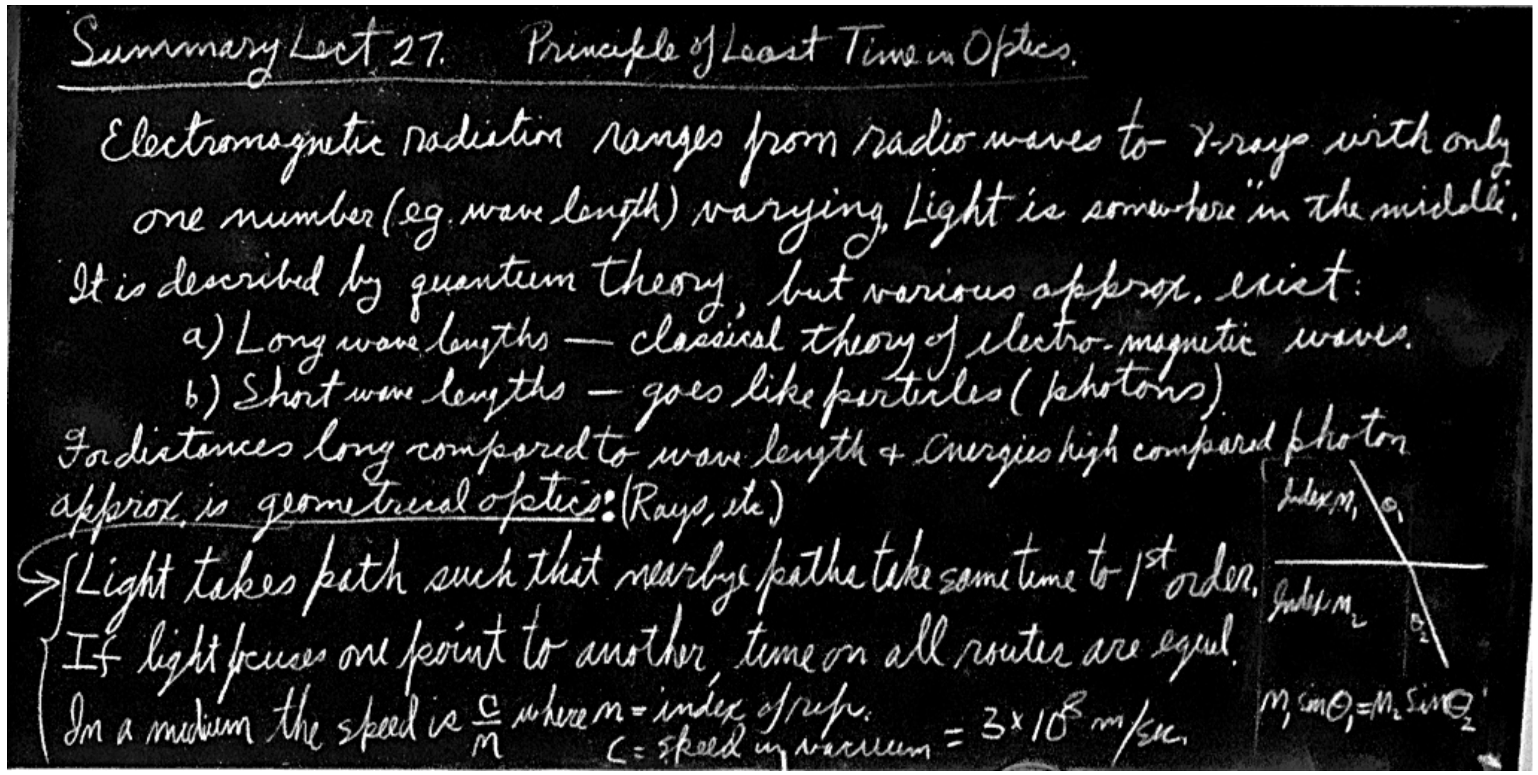


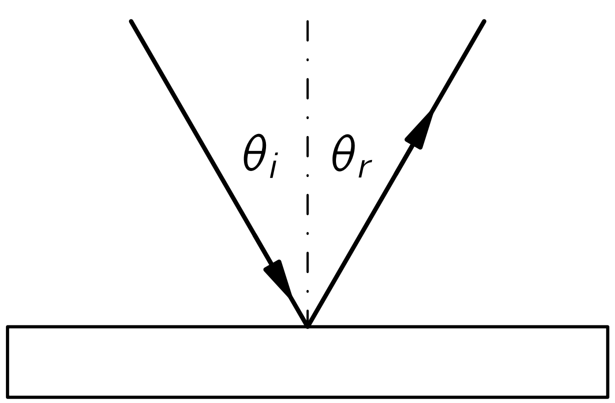
Fig. 6. Maximization of function $f(x, y)$ subject to the constraint $g(x, y) = 0$. At the constrained local optimum, the gradients of f and g, namely $\nabla f(x, y)$ and $\nabla g(x, y)$, are parallel.

Fermat's principle of least-time

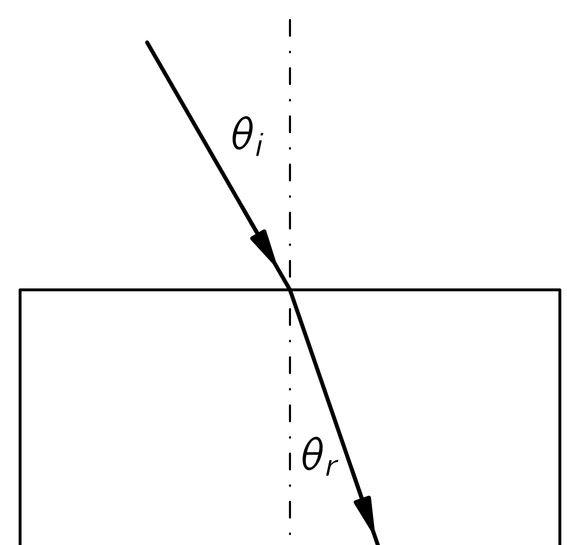
(Feynman lectures, chapter 26)



reflection



refraction



26–3Fermat's principle of least time

Now in the further development of science, we want more than just a formula. First we have an observation, then we have numbers that we measure, then we have a law which summarizes all the numbers. But the real *glory* of science is that *we can find a way of thinking* such that the law is *evident*.

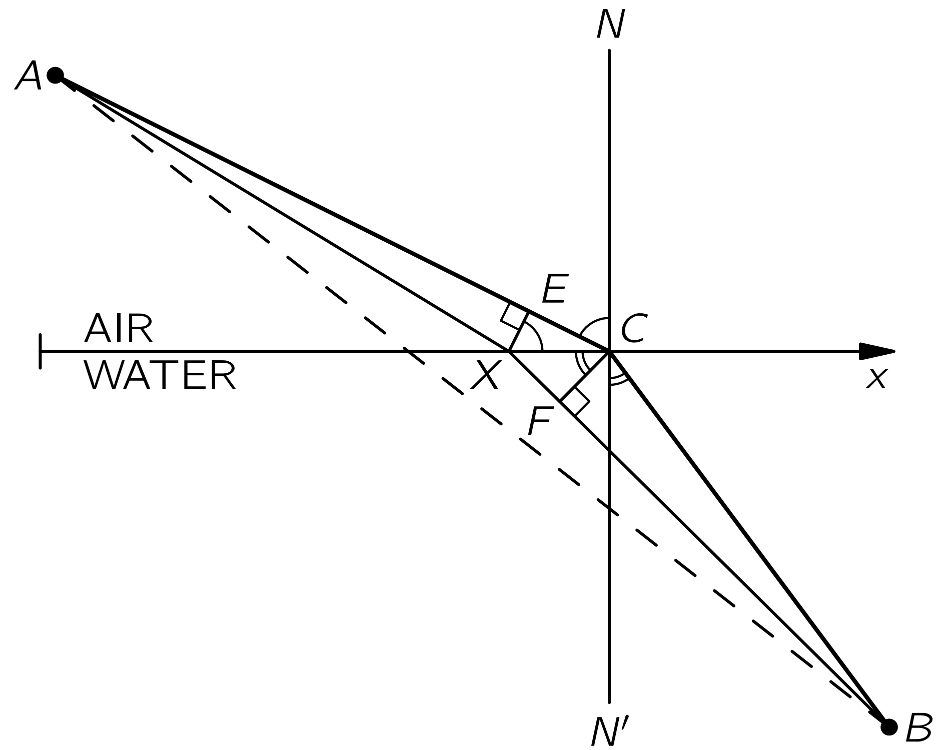
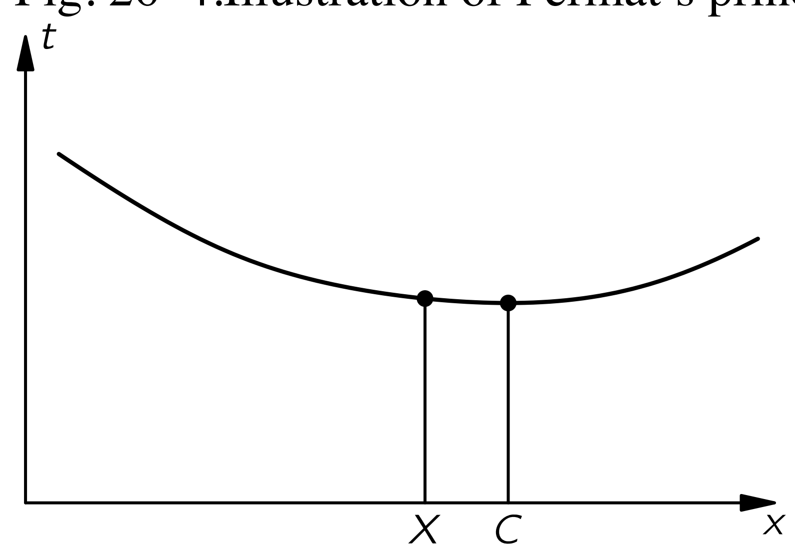
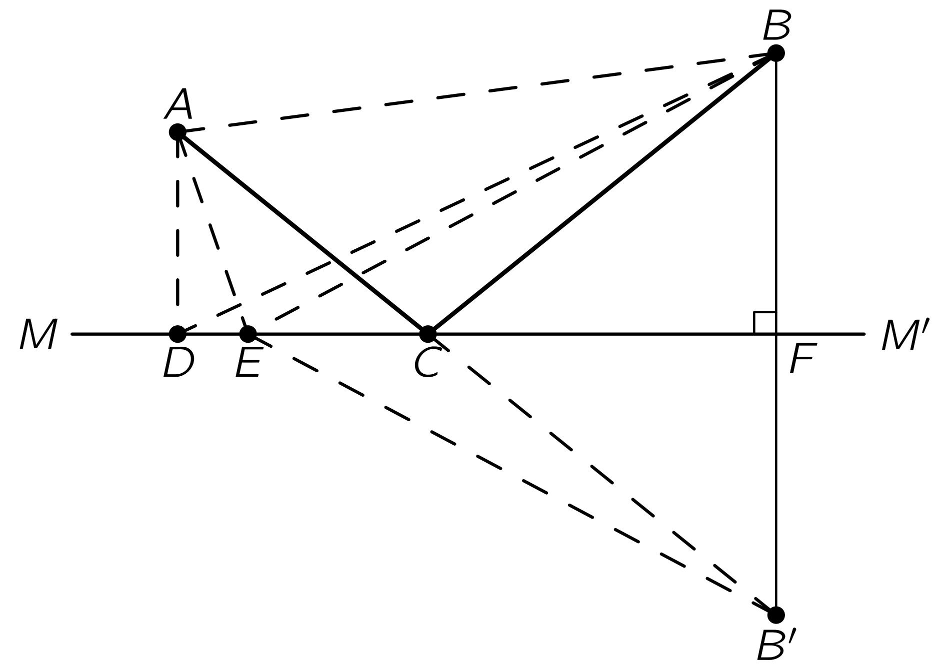


Fig. 26–4.Illustration of Fermat's principle for refraction.



Fermat's principle of least time

light decides which is the shortest time, or the extreme one, and chooses that path. But *what* does it do, *how* does it find out? Does it *smell* the nearby paths, and check them against each other? The answer is, yes, it does, in a way.



Take any path and find the time for that path; then make a complex number, or draw a little complex vector, $\rho e^{i\theta}$, *whose angle θ is proportional to the time*. The number of turns per second is the frequency of the light. Now take another path; it has, for instance, a different time, so the vector for it is turned through a different angle—the angle being always proportional to the time. Take *all* the available paths and add on a little vector for each one; then the answer is that the chance of arrival of the photon is proportional to the square of the length of the final vector, from the beginning to the end!

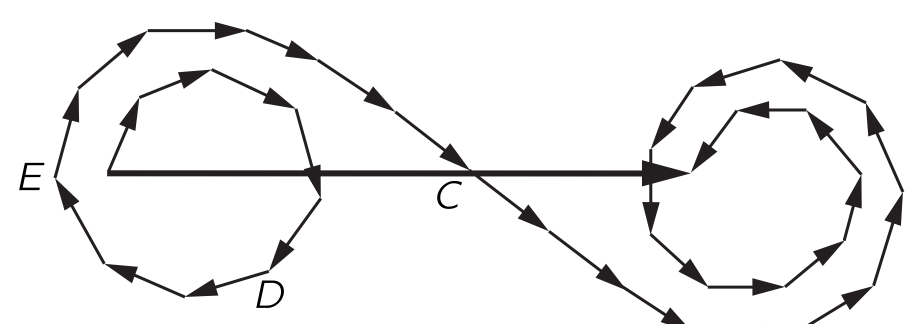


Fig. 26–14.The summation of probability amplitudes for many neighboring paths.