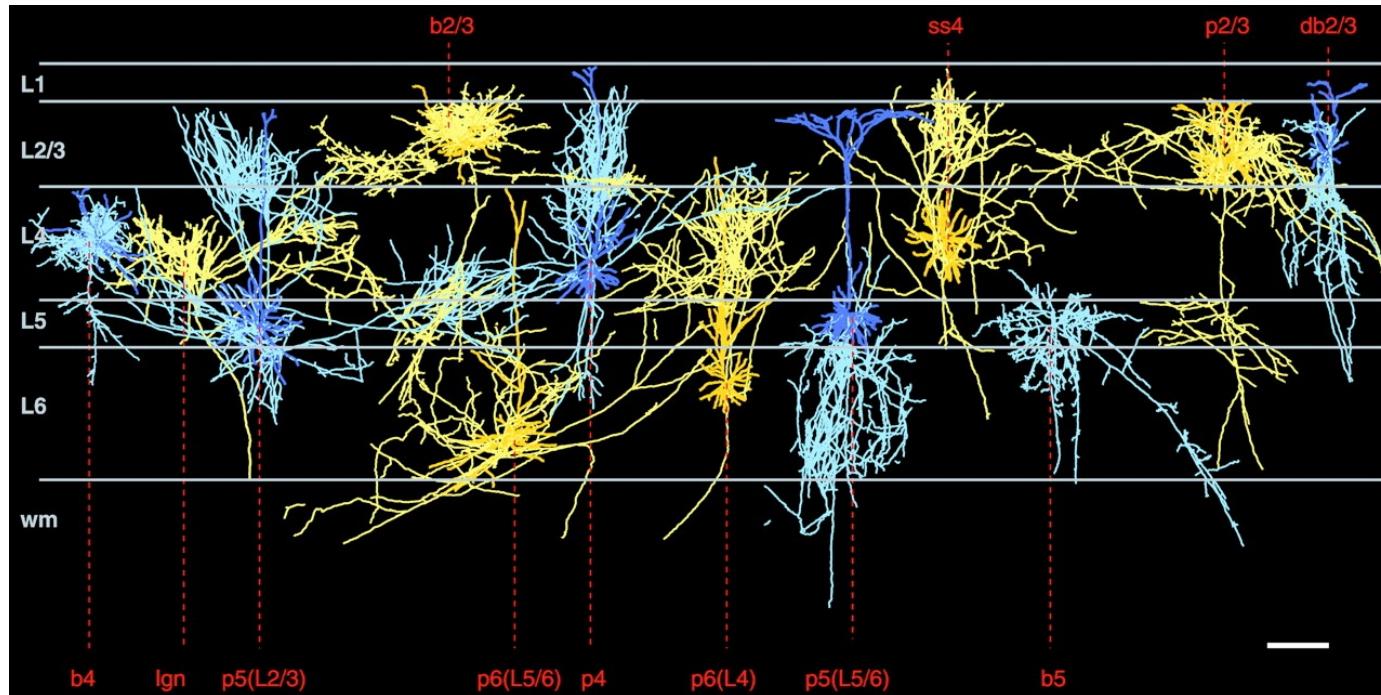
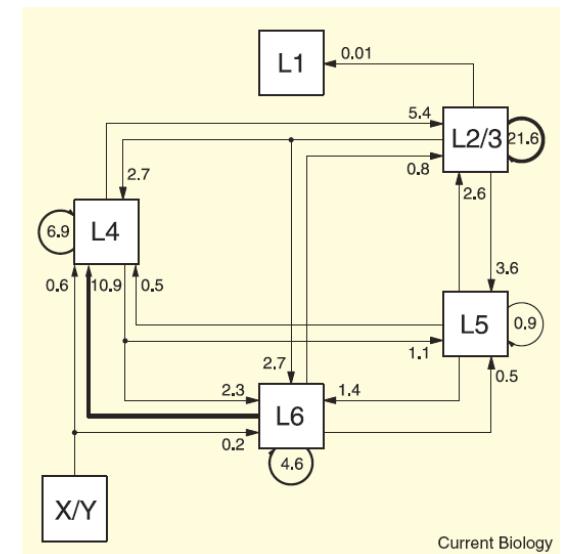


Attractor neural networks

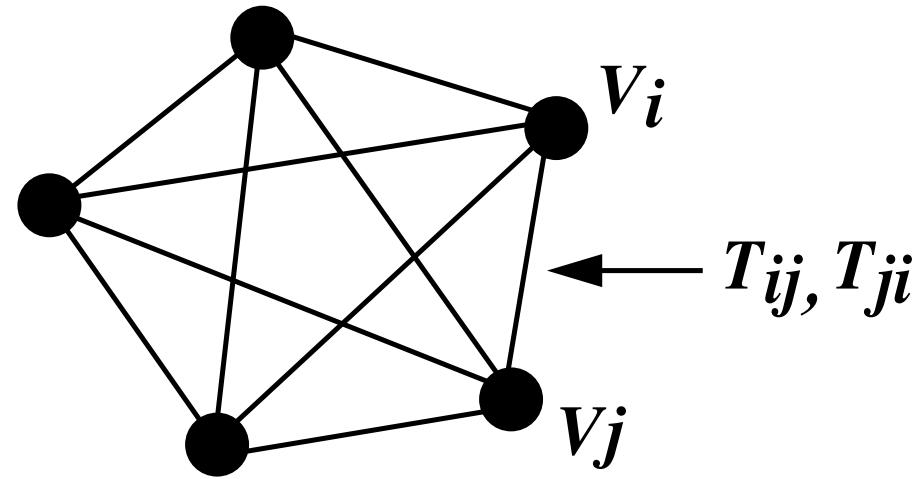
Laminar organization and ‘canonical microcircuit’



(Binzegger, Douglas & Martin, 2007)



(Douglas and Martin, 2007)



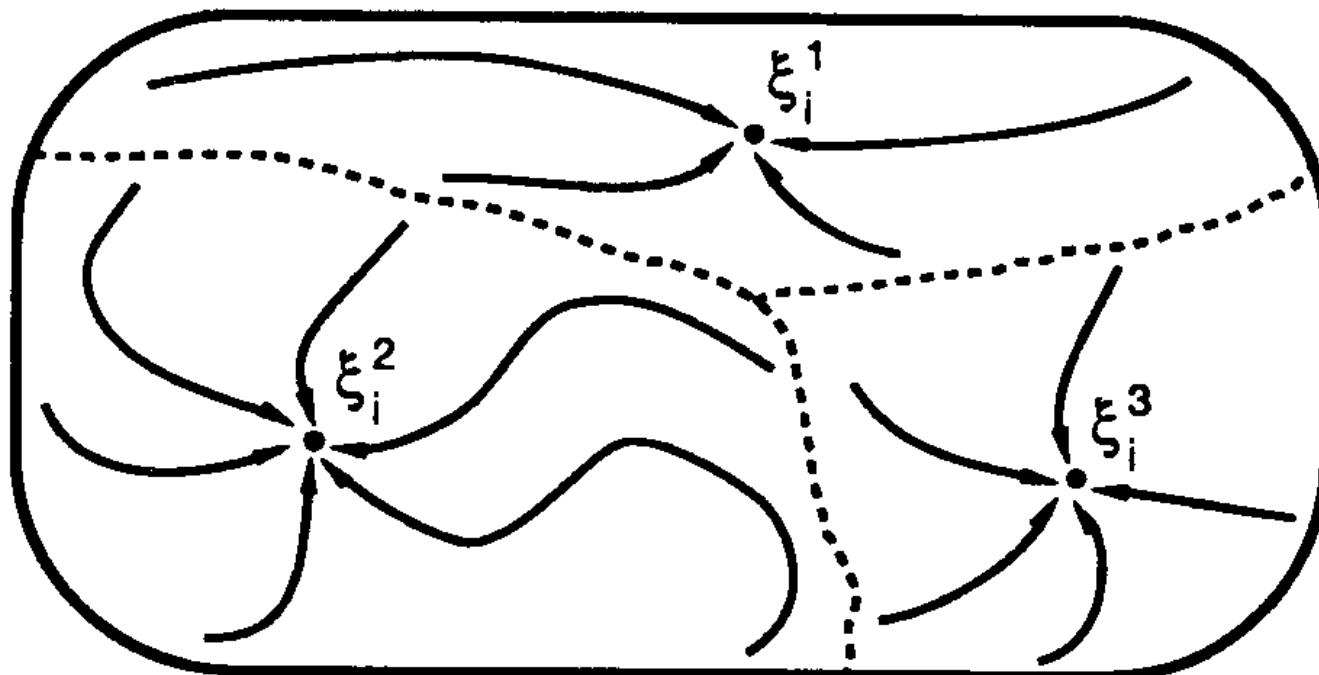
$$U_i = \sum_j T_{ij} V_j$$

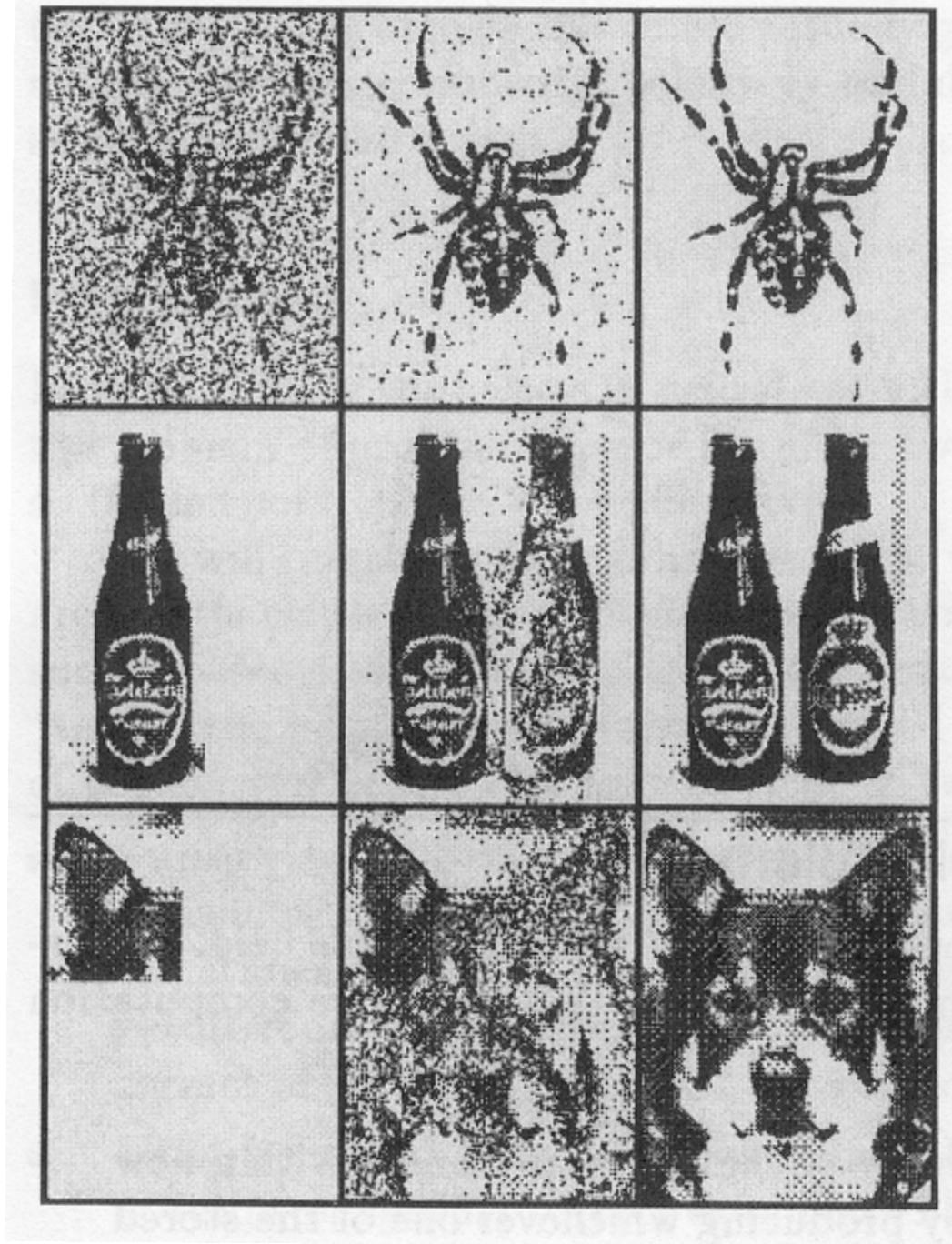
Dynamics:

$$V_i = \text{sign}(U_i)$$

Energy function: $E = -\frac{1}{2} \sum_{i,j \neq i} T_{ij} V_i V_j$

Basins of attraction





input → recall

Outer-product (Hebb) rule

$$\begin{aligned} T_{ij} &= \sum_{\alpha} P_i^{(\alpha)} P_j^{(\alpha)} \\ &= P_i^{(1)} P_j^{(1)} + P_i^{(2)} P_j^{(2)} + P_i^{(3)} P_j^{(3)} + \dots \end{aligned}$$

or $\mathbf{T} = \mathbf{P}^{(1)} \mathbf{P}^{(1)T} + \mathbf{P}^{(2)} \mathbf{P}^{(2)T} + \mathbf{P}^{(3)} \mathbf{P}^{(3)T} + \dots$

Thus

$$\begin{aligned} \mathbf{U} &\cong (\mathbf{P}^{(1)} \mathbf{P}^{(1)T} + \mathbf{P}^{(2)} \mathbf{P}^{(2)T} + \mathbf{P}^{(3)} \mathbf{P}^{(3)T} + \dots) \mathbf{V} \\ &= \mathbf{P}^{(1)} (\mathbf{P}^{(1)} \cdot \mathbf{V}) + \mathbf{P}^{(2)} (\mathbf{P}^{(2)} \cdot \mathbf{V}) + \mathbf{P}^{(3)} (\mathbf{P}^{(3)} \cdot \mathbf{V}) + \dots \end{aligned}$$

$$\mathbf{V} = \text{sgn}(\mathbf{U})$$

Capacity vs. error rate

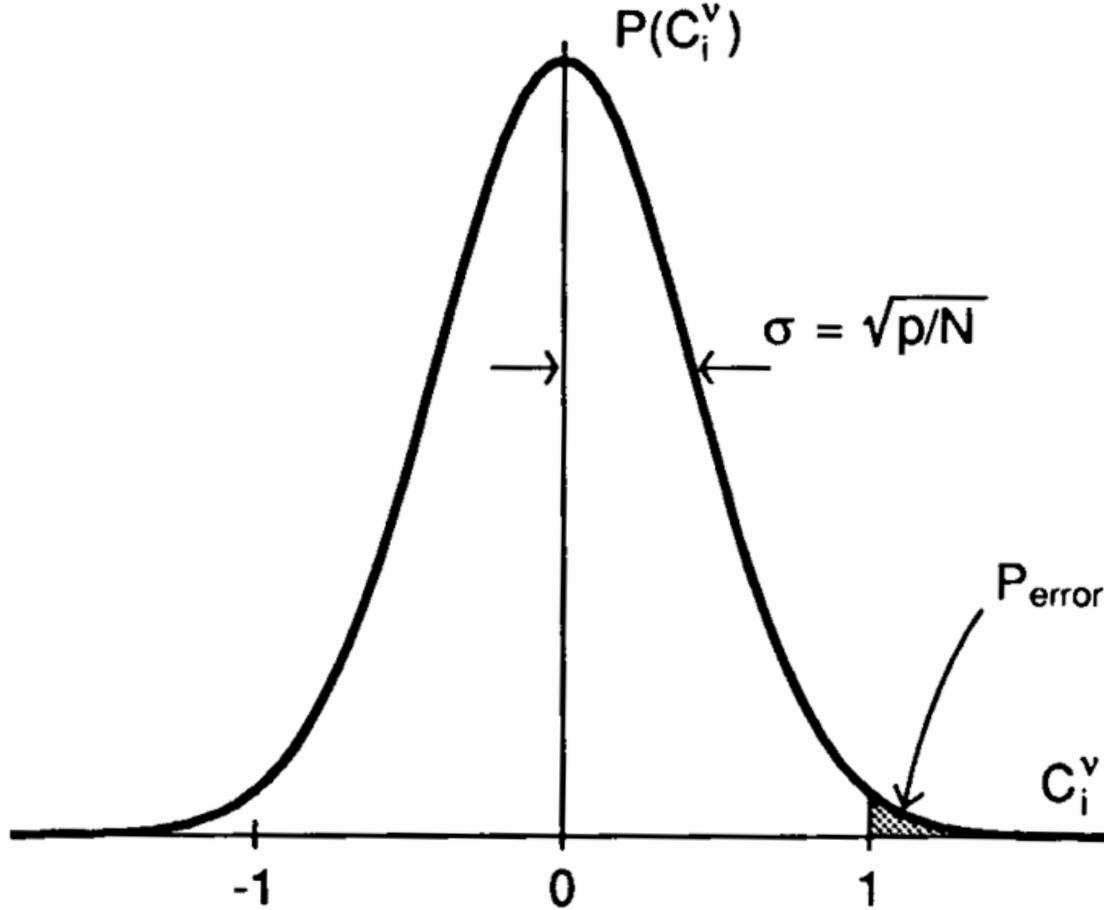
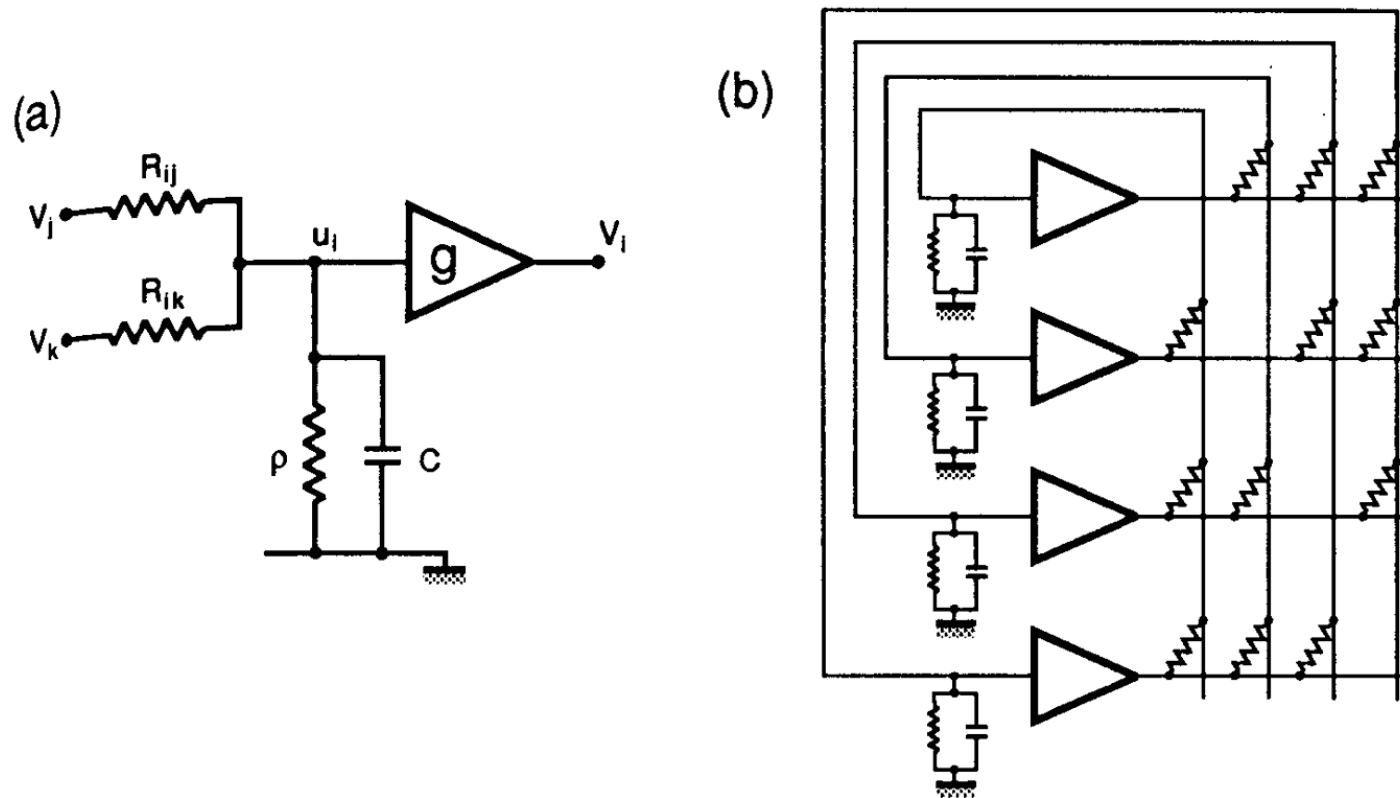


TABLE 2.1 Capacities

P_{error}	p_{\max}/N
0.001	0.105
0.0036	0.138
0.01	0.185
0.05	0.37
0.1	0.61

Hopfield network with analog units

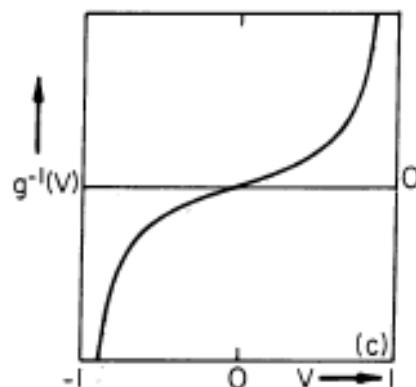
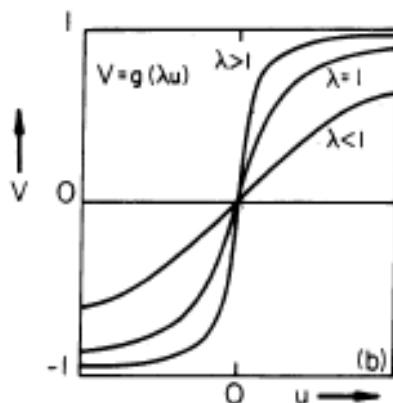
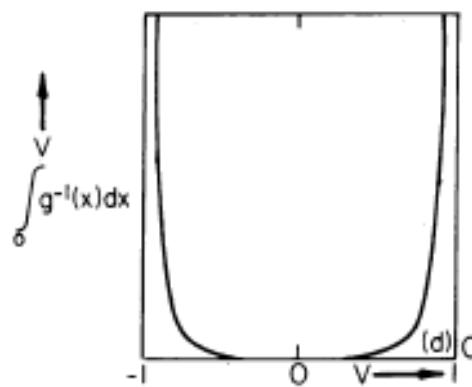
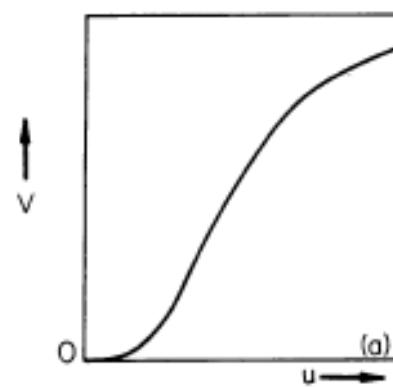


$$\tau \dot{u}_i + u_i = \sum_{j \neq i} T_{ij} V_j + I_i$$

$$V_i = g(u_i)$$

Lyapunov function

$$E = -\frac{1}{2} \sum_i \sum_{j \neq i} T_{ij} V_i V_j + \sum_i \int_0^{V_i} g^{-1}(V) dV - \sum_i V_i I_i$$



From Lyapunov function to dynamics

$$E = -\frac{1}{2} \sum_i \sum_{j \neq i} T_{ij} V_i V_j + \sum_i \int_0^{V_i} g^{-1}(V) dV - \sum_i V_i I_i$$

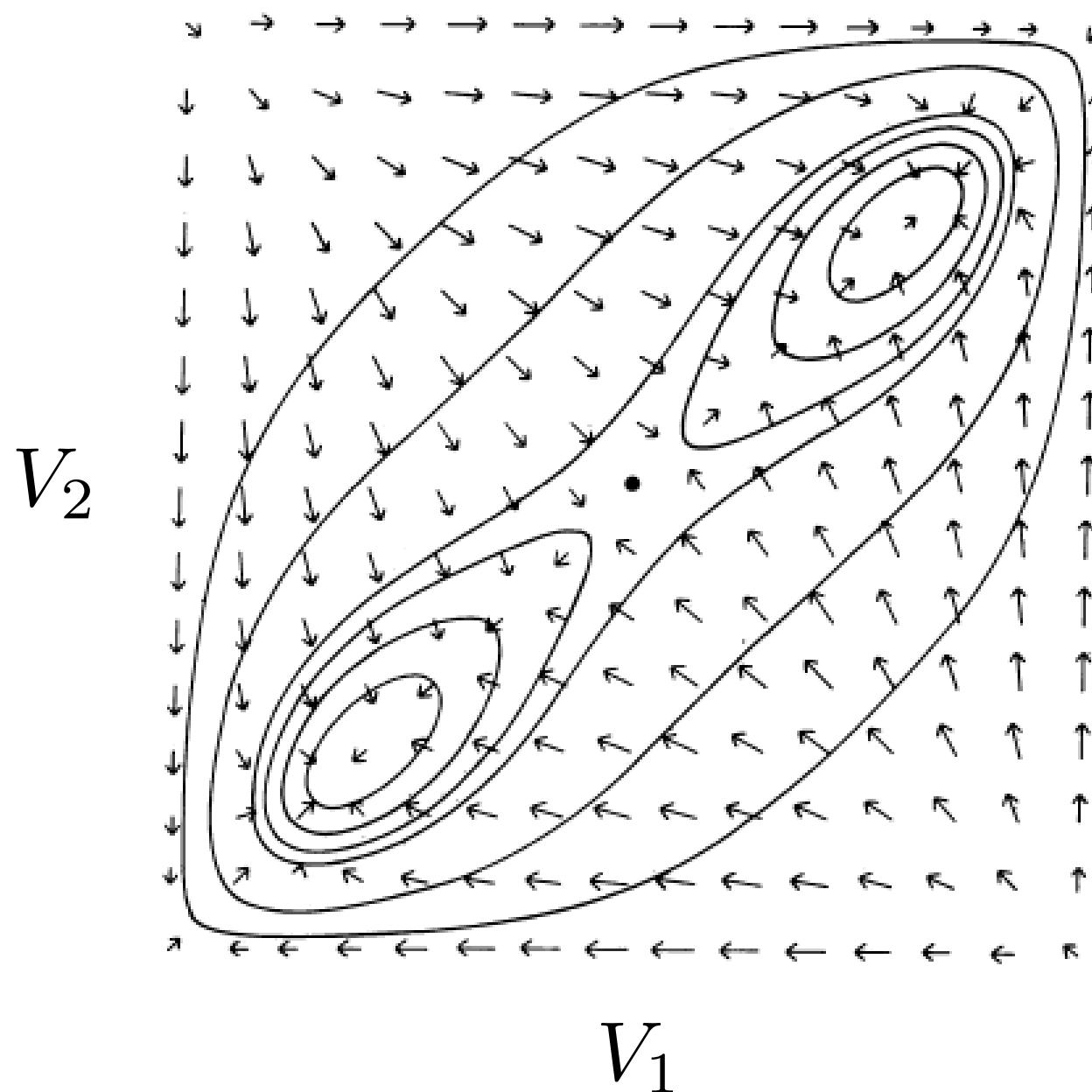
$$\frac{\partial E}{\partial V_k} = - \sum_{j \neq k} T_{kj} V_j + g^{-1}(V_k) - I_k$$

Let $u_i = g^{-1}(V_i) \Rightarrow V_i = g(u_i)$

$$\dot{u}_i \propto -\frac{\partial E}{\partial V_i} = \sum_{j \neq i} T_{ij} V_j + I_i - u_i$$

Thus $\dot{E} = \frac{\partial E}{\partial V} \frac{\partial V}{\partial u} \dot{u} < 0$

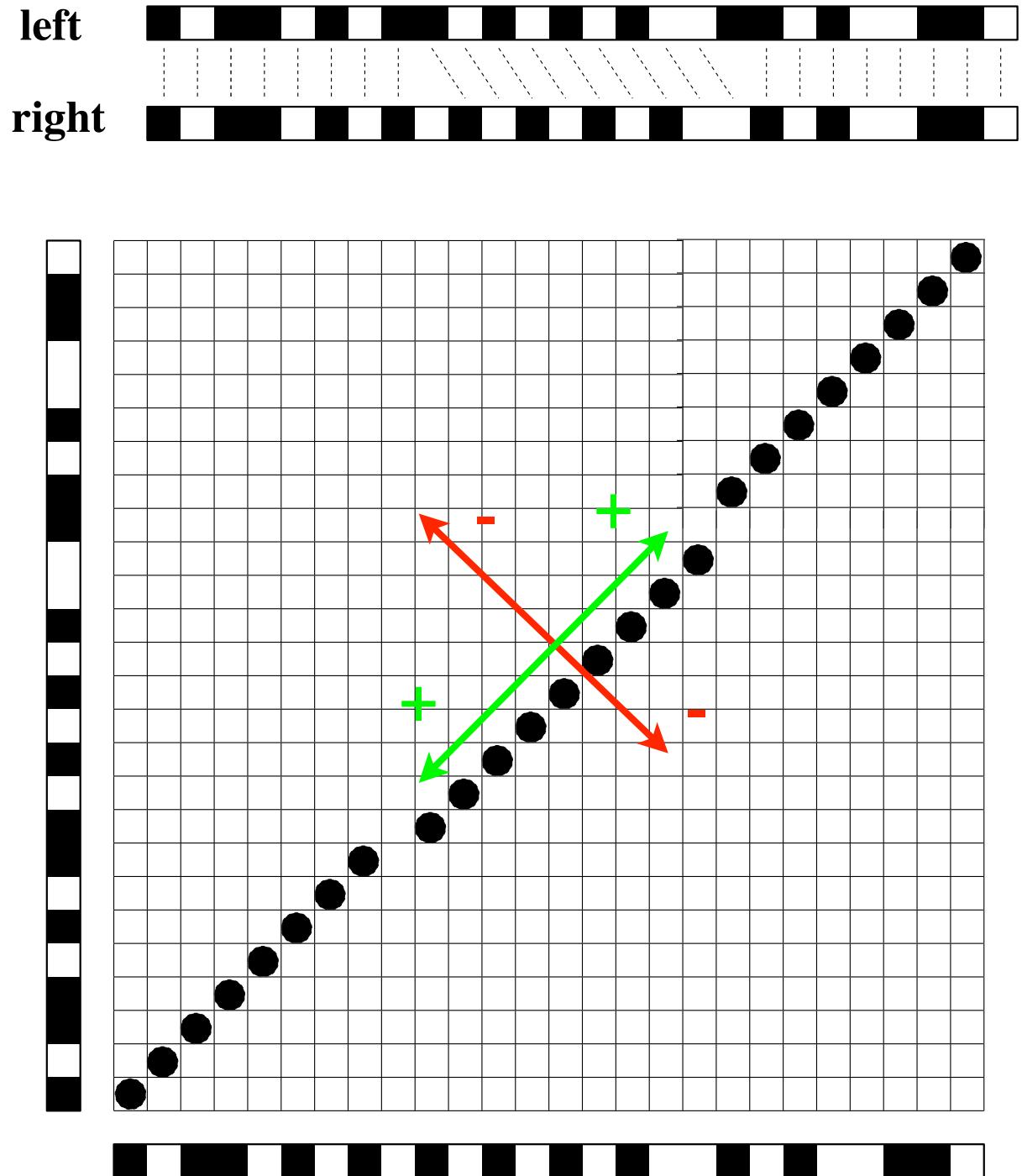
State space





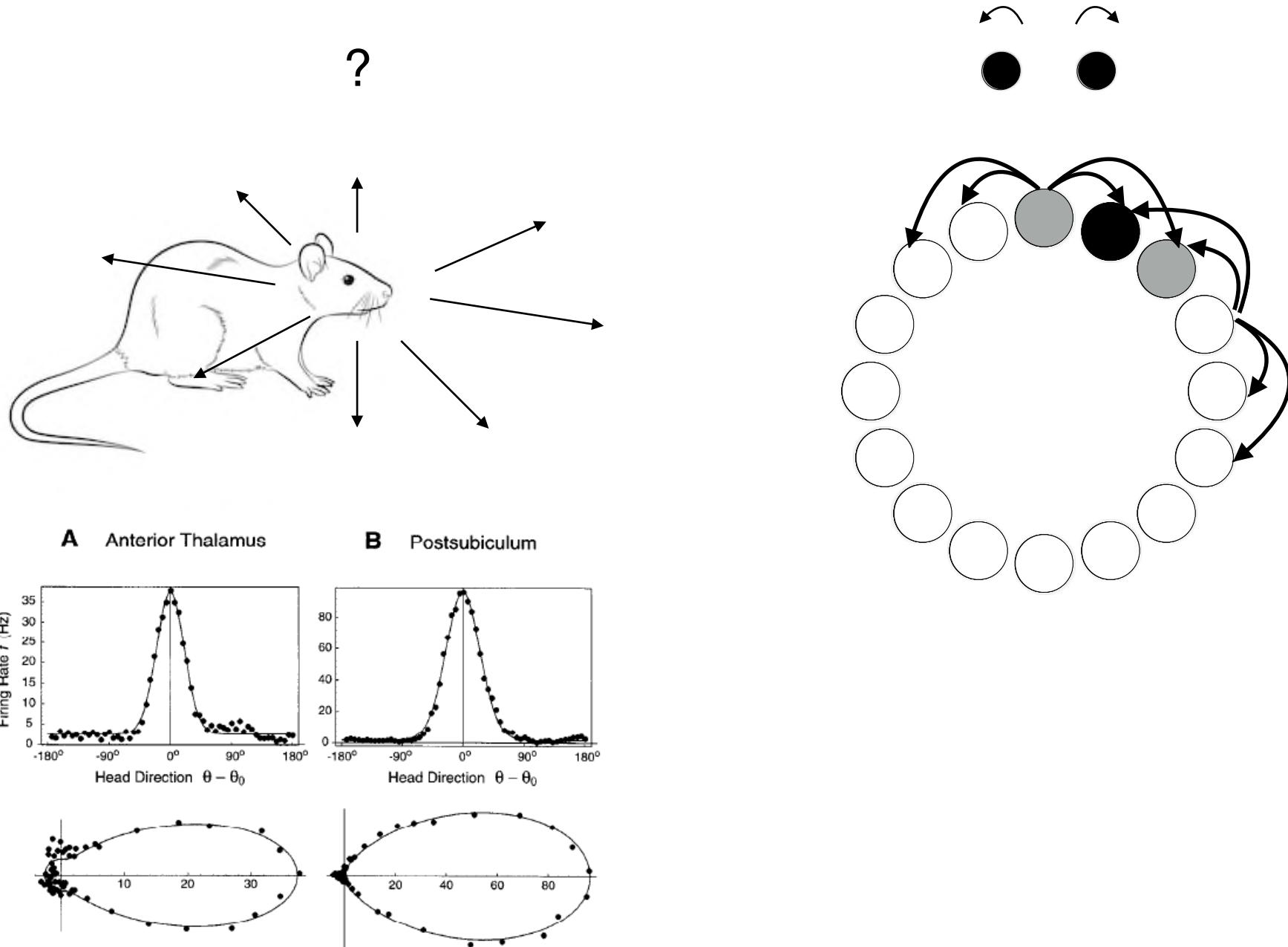


Marr-Poggio stereo algorithm (Marr & Poggio 1976)



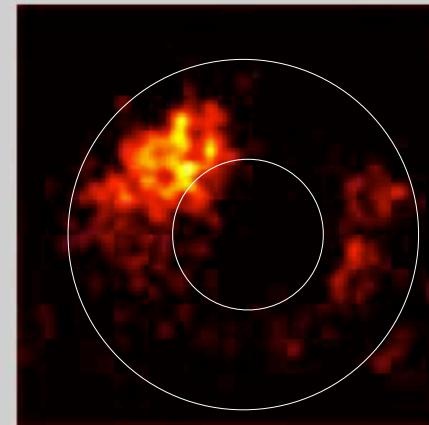
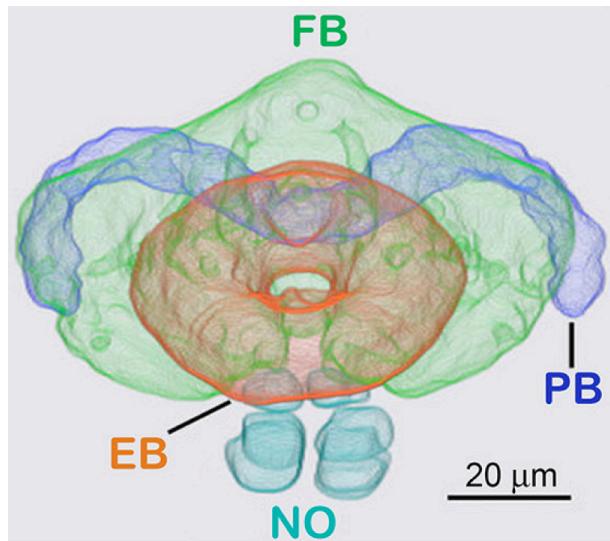
‘Bump circuits’ and ring attractors (Zhang, Sompolinsky, Seung and others)

Head-direction cells



Head-direction cells in ellipsoid body of Drosophila

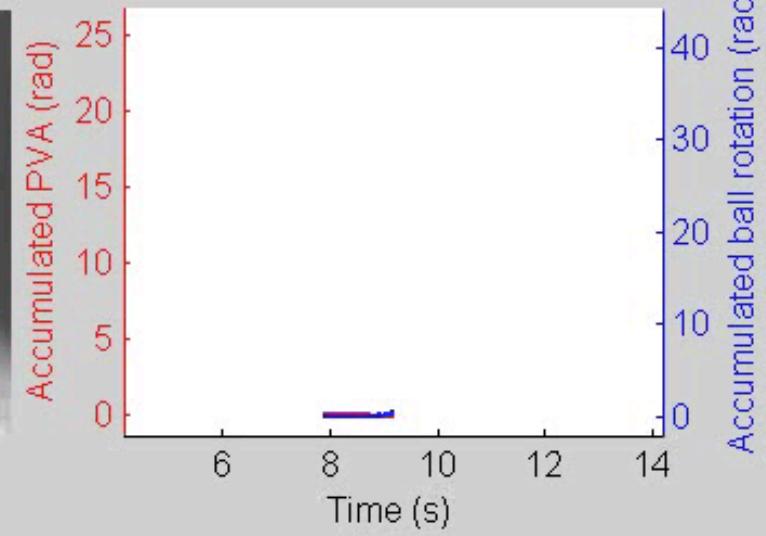
(Seelig & Jayaraman 2015)



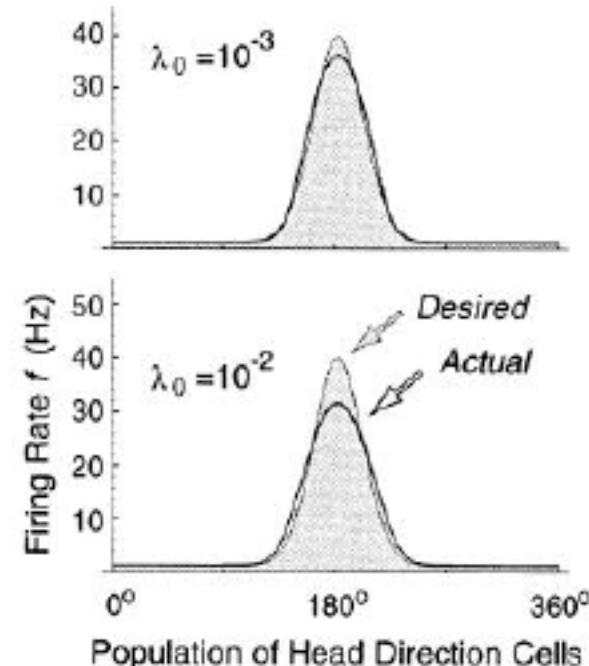
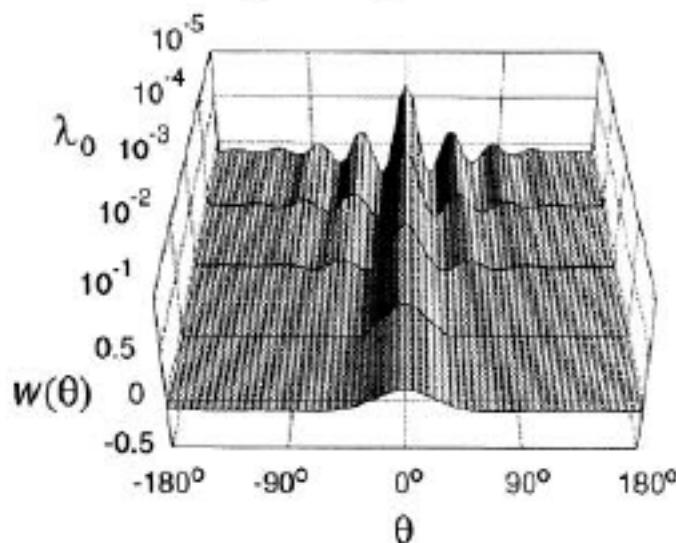
Ellipsoid body activity
(calcium imaging)



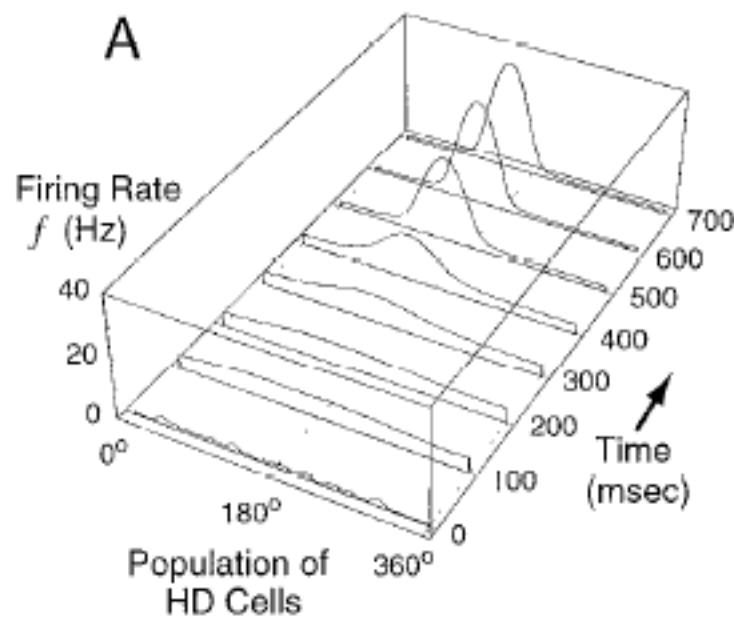
Decoded vs. actual head dir.



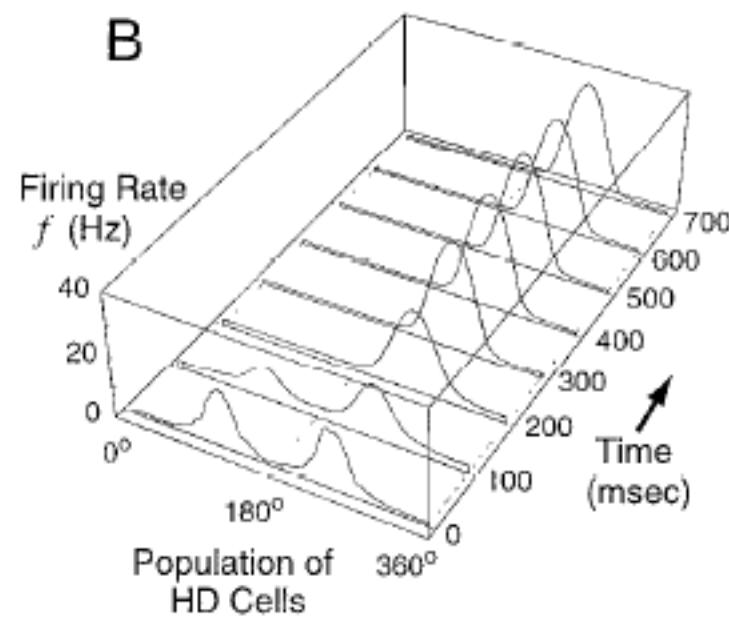
B Weight Regularization



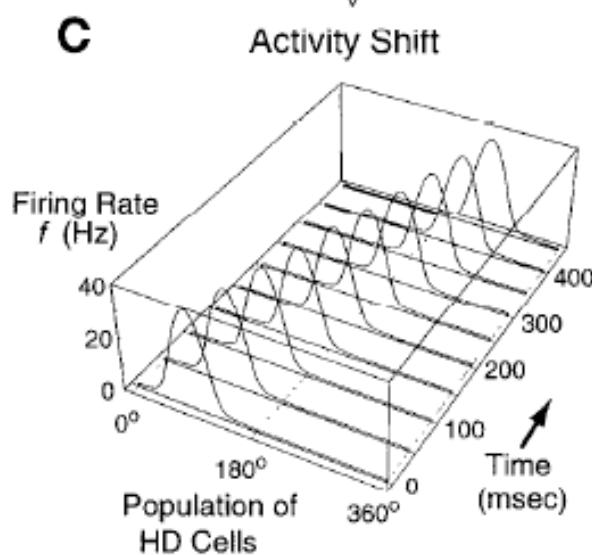
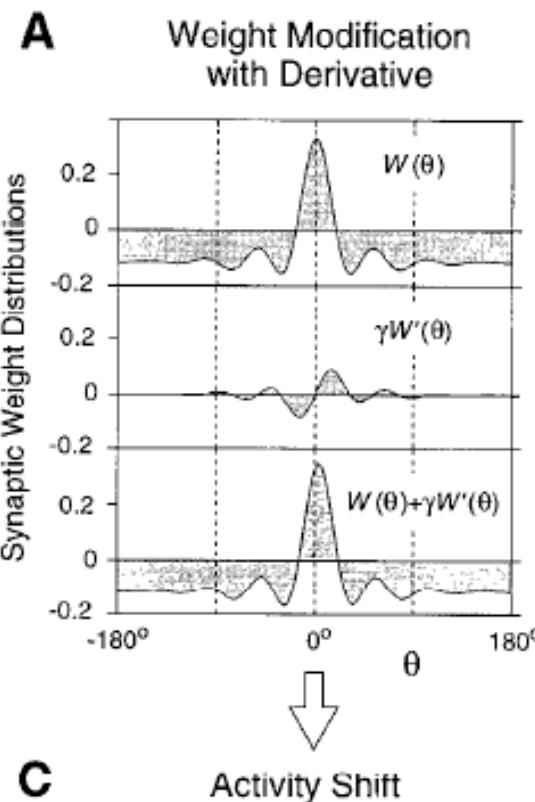
A



B

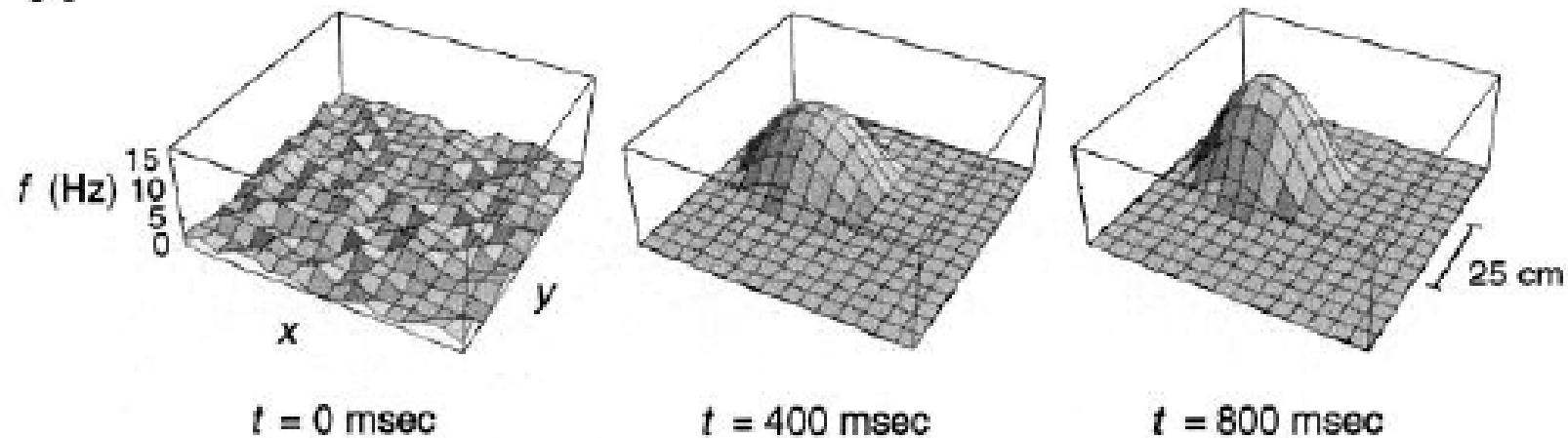


Shifting the bump



2D bumps

A



B

