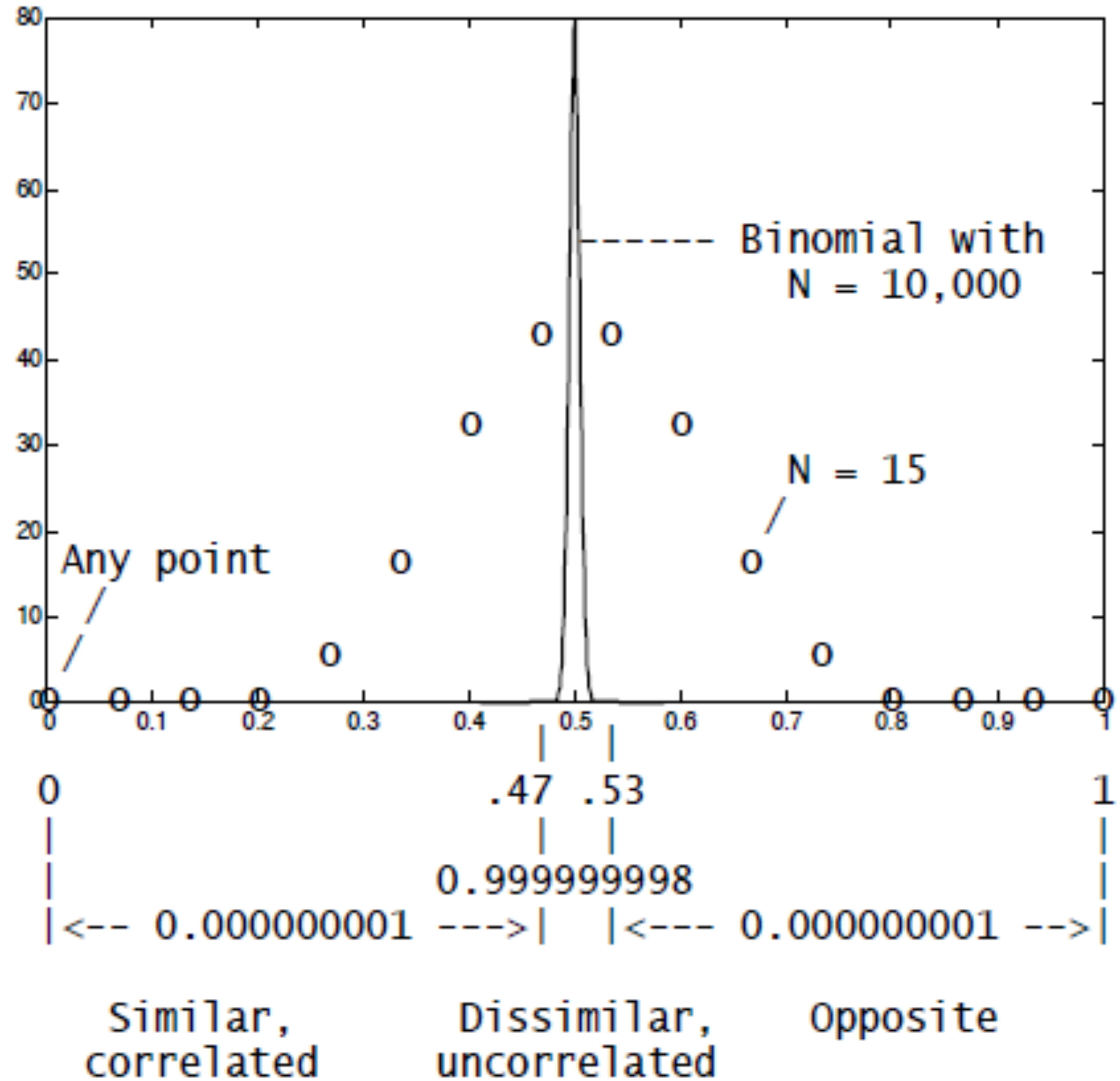
Today’s lecture

- Orthogonality
- Review of HD algebra
- Visual working memory/attention
- Factorization
- Visual analogies (Spencer/Navneedh)

Random HD vectors are nearly orthogonal



normalized Hamming distance

Computing with high-dimensional vectors

Concepts, variables, attributes are represented as high-dimensional vectors (e.g., 10,000 bits)

Three fundamental operations:

unbinding

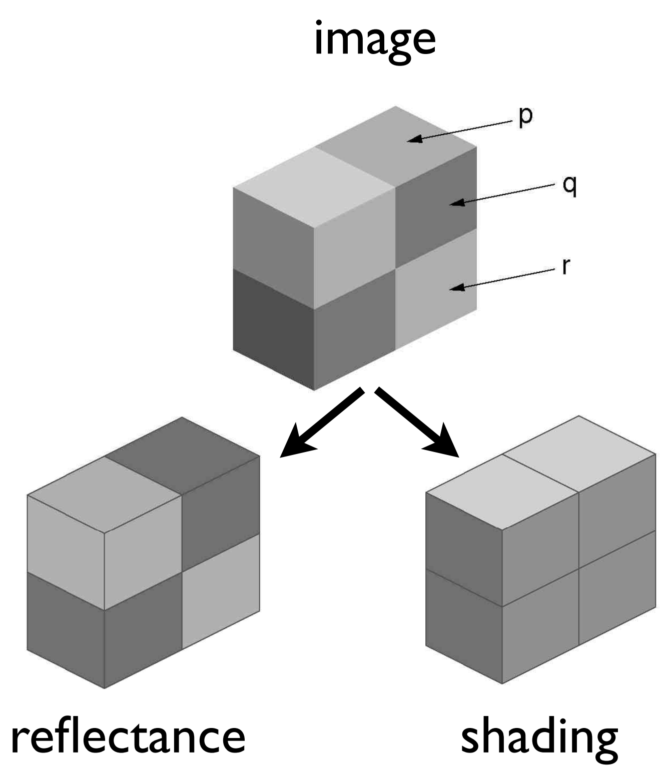
- multiplication (binding) $\mathbf{t} = \mathbf{c} \odot \mathbf{x} \quad \mathbf{x} = \mathbf{c}^{-1} \odot \mathbf{t}$
- addition (combining) $\mathbf{s} = [\mathbf{x} + \mathbf{y} + \ldots]$ or $\mathbf{s} = [\mathbf{c} \odot \mathbf{x} + \mathbf{d} \odot \mathbf{y} + \ldots]$
- permutation (sequencing) $\rho(\mathbf{x})$

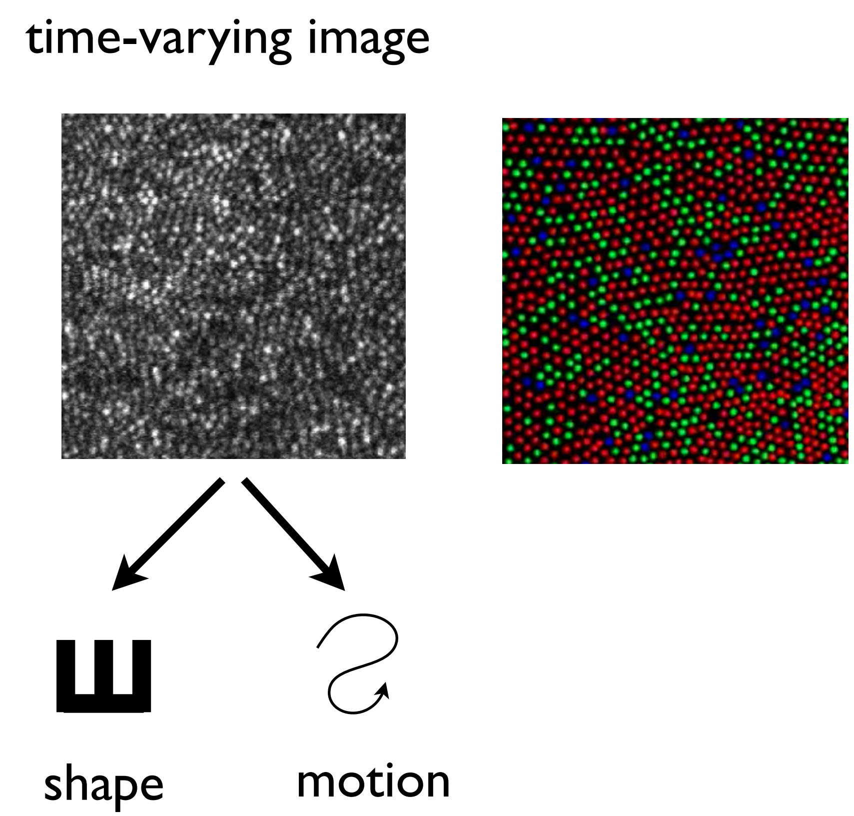
Approximates a *field*

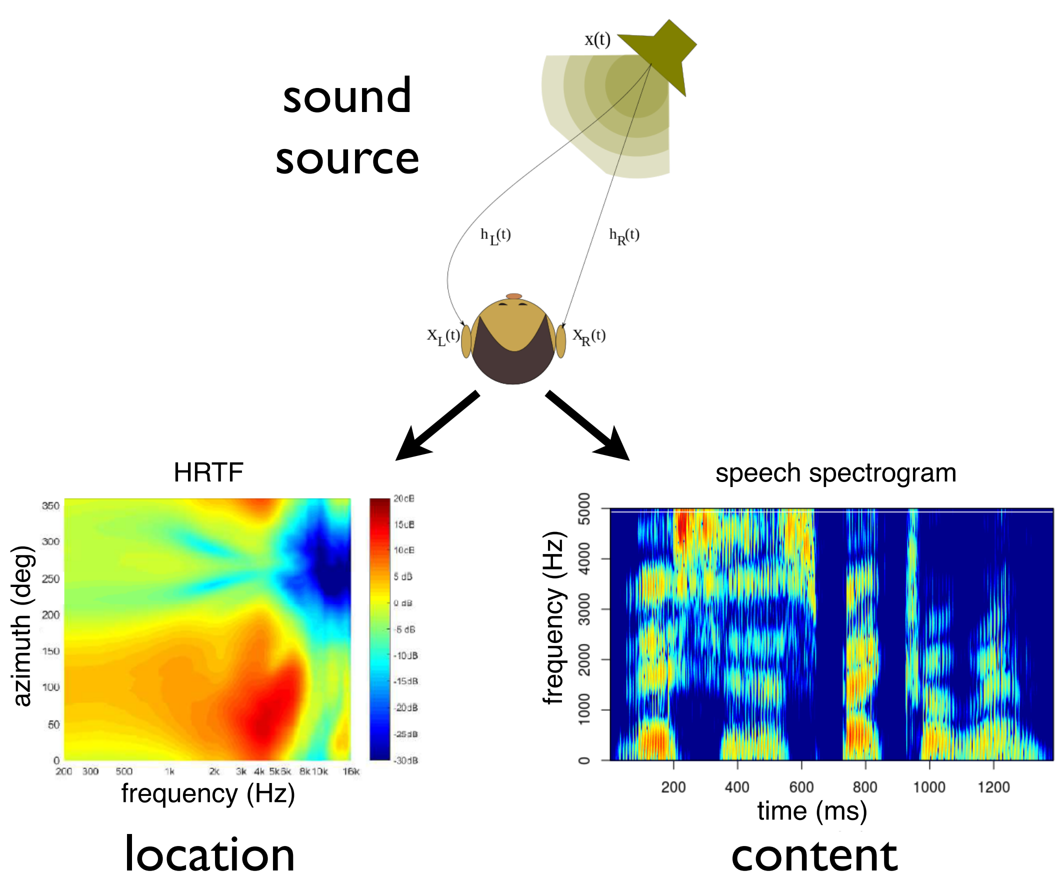
Kanerva P (2009) Hyperdimensional Computing: An Introduction to Computing in Distributed Representation with High-Dimensional Random Vectors. *Cognitive Computing, 1*: 139-159.

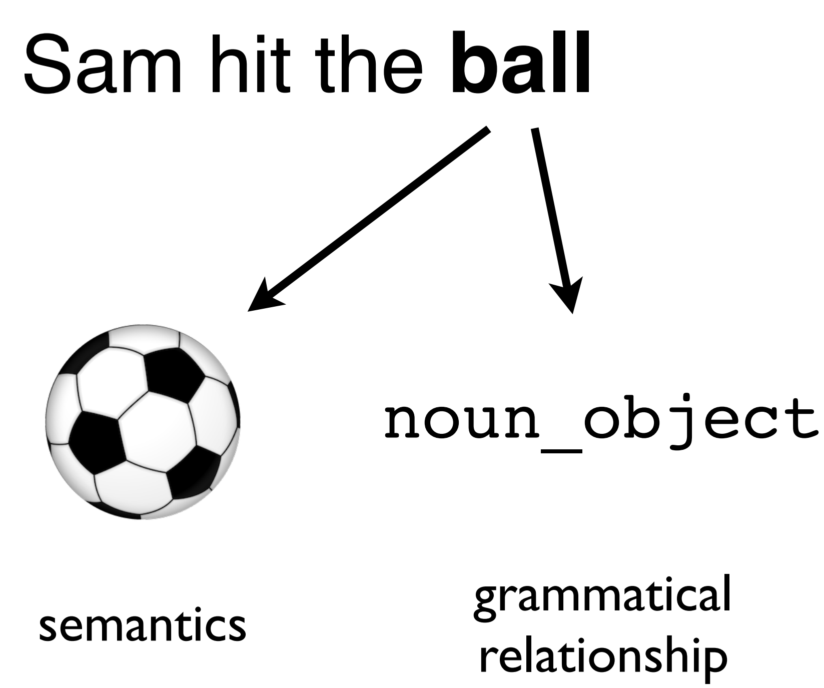
Factorization

Factorization is central to perception and cognition









Resonator Networks for factorizing HD vectors

Let $\mathbf{b} = \mathbf{x} \otimes \mathbf{y} \otimes \mathbf{z}$

$$\mathbf{x} \in \mathbb{X} := \{\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n\}$$
$$\mathbf{y} \in \mathbb{Y} := \{\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_n\}$$
$$\mathbf{z} \in \mathbb{Z} := \{\mathbf{z}_0, \mathbf{z}_1, \ldots, \mathbf{z}_n\}$$

Problem: You are given $\mathbf{b}$, what are $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$?

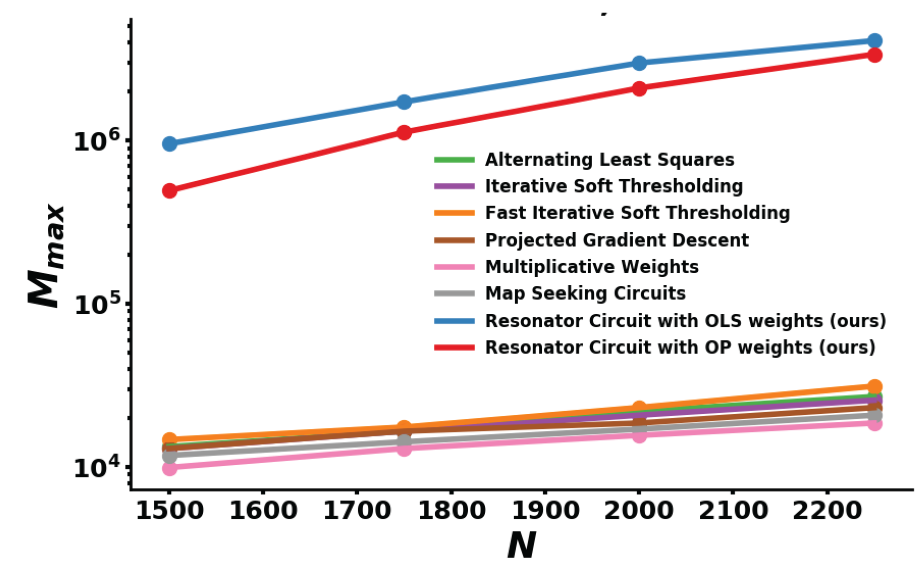
Solution: Resonate

$$\hat{\mathbf{x}}_{t+1} = g\big(\mathbf{X}\mathbf{X}^\top(\mathbf{b} \otimes \hat{\mathbf{y}}_t^{-1} \otimes \hat{\mathbf{z}}_t^{-1})\big)$$
$$\hat{\mathbf{y}}_{t+1} = g\big(\mathbf{Y}\mathbf{Y}^\top(\mathbf{b} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{z}}_t^{-1})\big)$$
$$\hat{\mathbf{z}}_{t+1} = g\big(\mathbf{Z}\mathbf{Z}^\top(\mathbf{b} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{y}}_t^{-1})\big)$$

$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \ldots & \mathbf{x}_n \\ | & | & & | \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} | & | & & | \\ \mathbf{y}_1 & \mathbf{y}_2 & \ldots & \mathbf{y}_n \\ | & | & & | \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} | & | & & | \\ \mathbf{z}_1 & \mathbf{z}_2 & \ldots & \mathbf{z}_n \\ | & | & & | \end{bmatrix}$$

$$g(x) = \mathrm{sgn}(x)$$

Combinatorial capacity exceeds competing methods by *two orders of magnitude*



Three factors, $F = 3$

Frady EP, Kent S, Olshausen BA & Sommer FT (2020) Resonator Networks,1: An efficient solution for factoring distributed representations of data structures. *Neural Computation, 32(12).*

Kent S, Frady EP, Sommer FT & Olshausen BA (2020) Resonator Networks outperform optimization methods at solving high-dimensional vector factorization. *Neural Computation, 32(12).*

Energy function?

$$(\alpha_1\beta_1\gamma_1\,\mathbf{x}_1\otimes\mathbf{y}_1\otimes\mathbf{z}_1 + \ldots + \alpha_i\beta_j\gamma_k\,\mathbf{x}_i\otimes\mathbf{y}_j\otimes\mathbf{z}_k + \ldots + \alpha_n\beta_n\gamma_n\,\mathbf{x}_n\otimes\mathbf{y}_n\otimes\mathbf{z}_n)$$

$$E = -\mathbf{b}\cdot\left(\mathbf{x}\bigotimes\mathbf{y}\bigotimes\mathbf{z}\right)$$

$$\mathbf{x} = \sum_{i=1}^{n}\alpha_i\,\mathbf{x}_i, \quad \mathbf{y} = \sum_{i=1}^{n}\beta_i\,\mathbf{y}_i, \quad \mathbf{z} = \sum_{i=1}^{n}\gamma_i\,\mathbf{z}_i$$

Energy function?

1,000,000 combinations! (n=100)

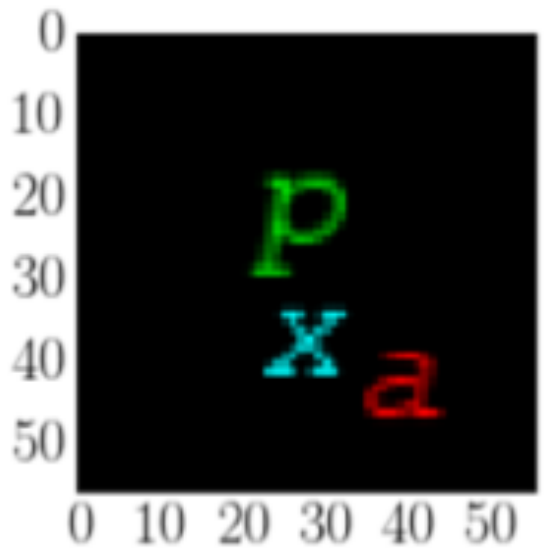
$$(\alpha_1\beta_1\gamma_1\,\mathbf{x}_1\otimes\mathbf{y}_1\otimes\mathbf{z}_1\;+\;\ldots\;+\;\alpha_i\beta_j\gamma_k\,\mathbf{x}_i\otimes\mathbf{y}_j\otimes\mathbf{z}_k\;+\;\ldots\;+\;\alpha_n\beta_n\gamma_n\,\mathbf{x}_n\otimes\mathbf{y}_n\otimes\mathbf{z}_n)$$

$$E=-\mathbf{b}\cdot\big(\mathbf{x}\otimes\mathbf{y}\otimes\mathbf{z}\big)$$

$$\mathbf{x}=\sum_{i=1}^{n}\alpha_i\,\mathbf{x}_i,\quad \mathbf{y}=\sum_{i=1}^{n}\beta_i\,\mathbf{y}_i,\quad \mathbf{z}=\sum_{i=1}^{n}\gamma_i\,\mathbf{z}_i$$

Factorization of shape, color and position

(Paxon Frady)



$\mathbf{u}^{x_i}$ = horizontal position x_i

$\mathbf{v}^{y_i}$ = vertical position y_j

$\mathbf{w}_c$ = color channel c

$$\mathbf{s} = \sum_{i,j,c} I(x_i, y_j, c)\, \mathbf{u}^{x_i}\, \mathbf{v}^{y_j}\, \mathbf{w}_c$$

Given $\mathbf{s}$, find $\mathbf{x}$, $\mathbf{y}$, $\mathbf{c}$ and $\mathbf{p}$ via resonator:

$$\hat{\mathbf{x}}_{t+1} = g(\mathbf{X}\mathbf{X}^\top (\mathbf{s} \otimes \hat{\mathbf{y}}_t^{-1} \otimes \hat{\mathbf{c}}_t^{-1} \otimes \hat{\mathbf{p}}_t^{-1}))$$ horizontal position

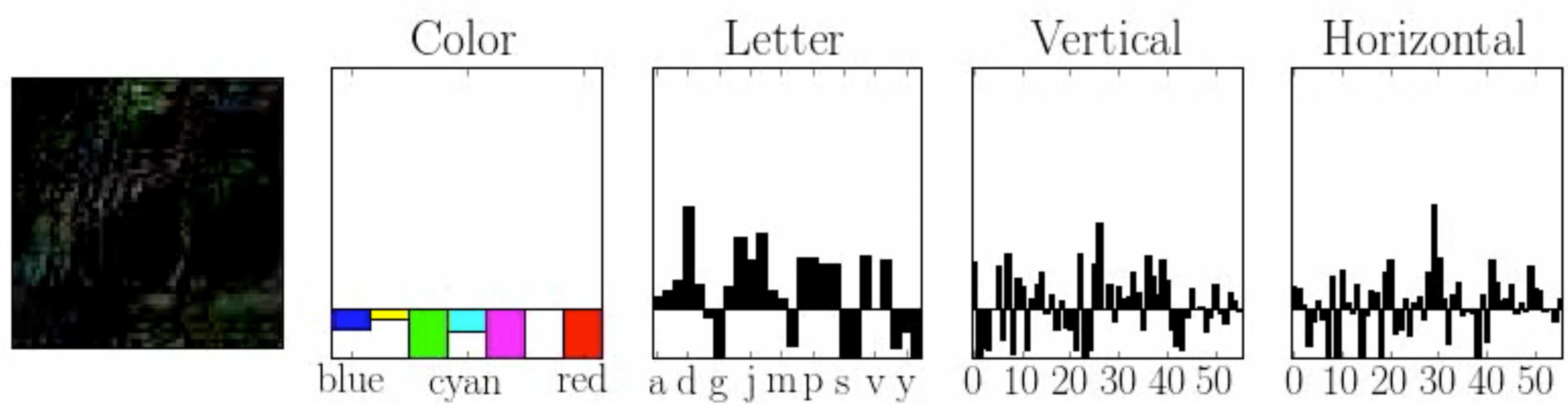
$$\hat{\mathbf{y}}_{t+1} = g(\mathbf{Y}\mathbf{Y}^\top (\mathbf{s} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{c}}_t^{-1} \otimes \hat{\mathbf{p}}_t^{-1}))$$ vertical position

$$\hat{\mathbf{c}}_{t+1} = g(\mathbf{C}\mathbf{C}^\top (\mathbf{s} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{y}}_t^{-1} \otimes \hat{\mathbf{p}}_t^{-1}))$$ color

$$\hat{\mathbf{p}}_{t+1} = g(\mathbf{P}\mathbf{P}^\top (\mathbf{s} \otimes \hat{\mathbf{x}}_t^{-1} \otimes \hat{\mathbf{y}}_t^{-1} \otimes \hat{\mathbf{c}}_t^{-1}))$$ pattern

Visual scene analysis via factorization of HD vectors

(Paxon Frady)



Other efforts

- Berkeley/Stanford EE (Rabaey, Salahuddin, Mitra, Wong) - hardware implementation, cnFET's, PCM/RRAM
- Waterloo (Eliasmith) - SPAUN
- U Maryland (Fernmuller, Aloimonos) - event-based camera robot navigation
- BMW (Mirus, Blouw, Stewart, Conradt) - vehicle position monitoring and prediction.
- VSA online seminar series: https://sites.google.com/ltu.se/vsaonline/winter-2021
- Website: https://www.hd-computing.com