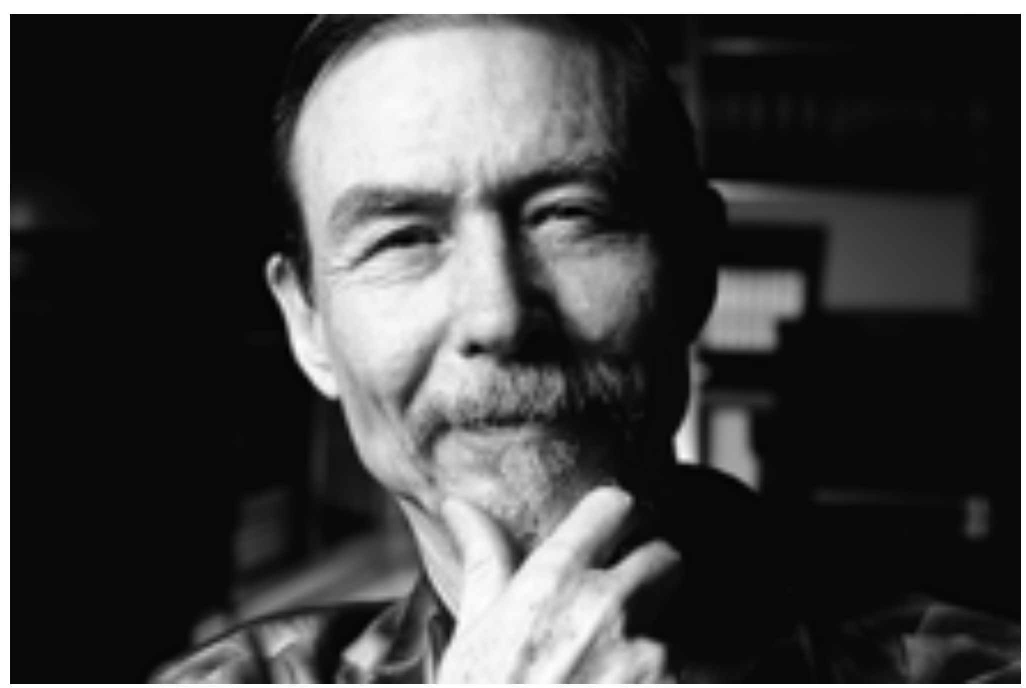
Physics of Computation

The processes of drift and diffusion are the stuff of which all information processing devices—both neural and semiconductor—are made.

—Carver Mead (1989)



Nernst potential (aka 'reversal potential')

$$V = -\frac{kT}{q}\,\log\frac{N_{\rm in}}{N_{\rm ex}} \qquad \text{or} \qquad N_{\rm in} = N_{\rm ex}\,e^{-\frac{q}{kT}\,V}$$

Current-voltage relation of voltage-gated channels

$$\frac{\theta}{1-\theta} = e^{-E_0/kT}\,e^{qnV/kT}$$

θ=fraction of channels open

Current-voltage relation of MOS transistor

$$I = I_0\,e^{-\frac{q\,V_{gs}}{kT}}\,(1 - e^{\frac{q\,V_{ds}}{kT}})$$

V_{gs} = gate-source voltage
V_{ds} = drain-source voltage

All of these things are related by the same fundamental physical law…

Its the Boltzmann distribution!

Example: atmospheric pressure vs. elevation

$$v_{\mathrm{drift}} = \frac{wt_f}{2m} = v_{\mathrm{diff}} = -\frac{1}{2N}\frac{dN}{dh}kT\frac{t_f}{m}$$

$$\frac{1}{N}\frac{dN}{dh}kT = -w$$

$$kT \ln \frac{N}{N_0} = -wh$$

$$\boxed{N = N_0 e^{-\frac{wh}{kT}}}$$

or, for charge in an electric field:

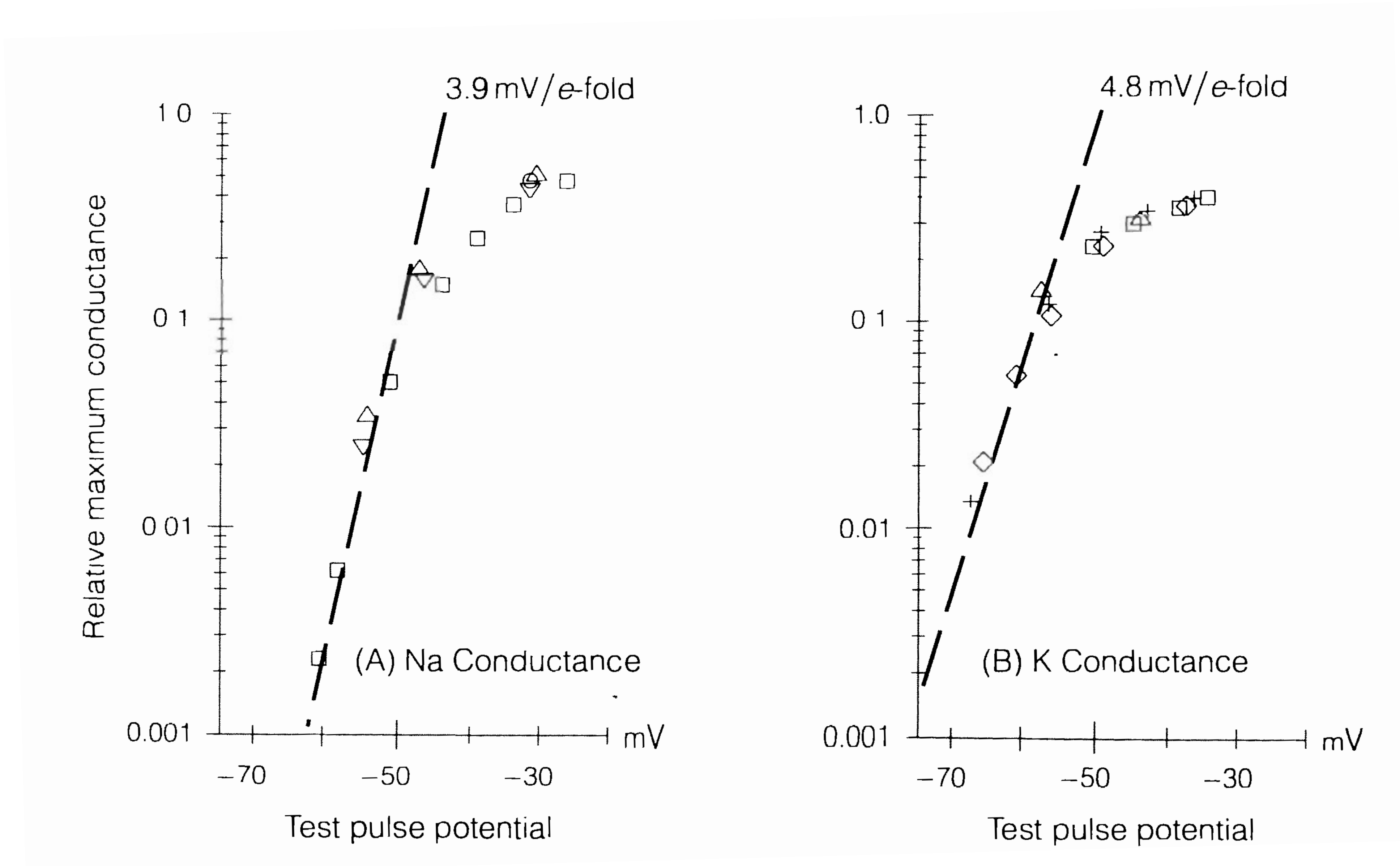
$$\boxed{N = N_0 e^{-\frac{q}{kT}V}}$$

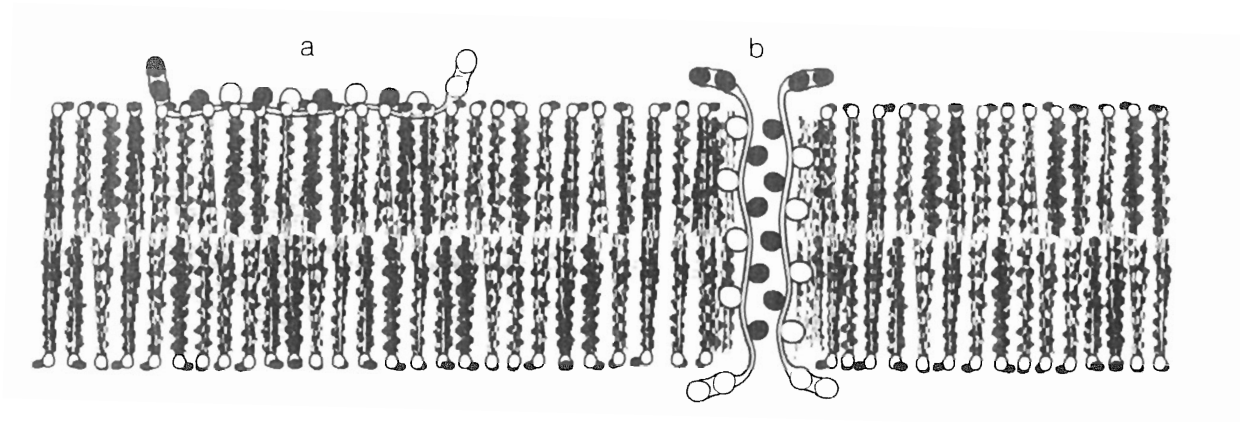
Active devices

Voltage-gated channels

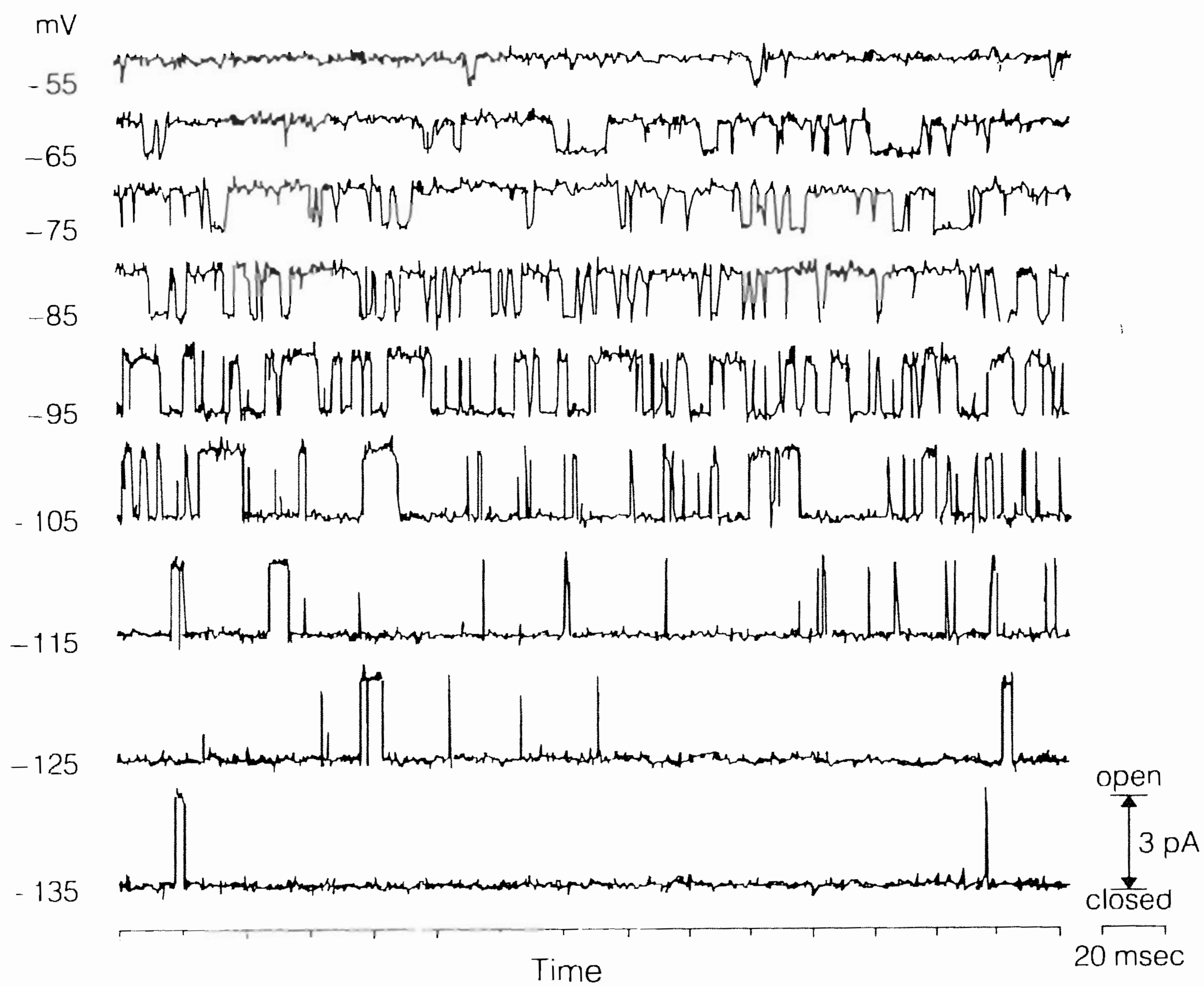
MOS transistor

Voltage-gated channels





$$\frac{N_o}{N_c} = e^{-E_t/(kT)}$$



$$E_t = E_0 - Vnq$$

E_t = transition energy
E_0 = transition energy at V=0

$$\frac{\theta}{1-\theta} = e^{-E_0/(kT)} e^{qnV/(kT)}$$

$$\theta = N_o/N$$

MOS transistor

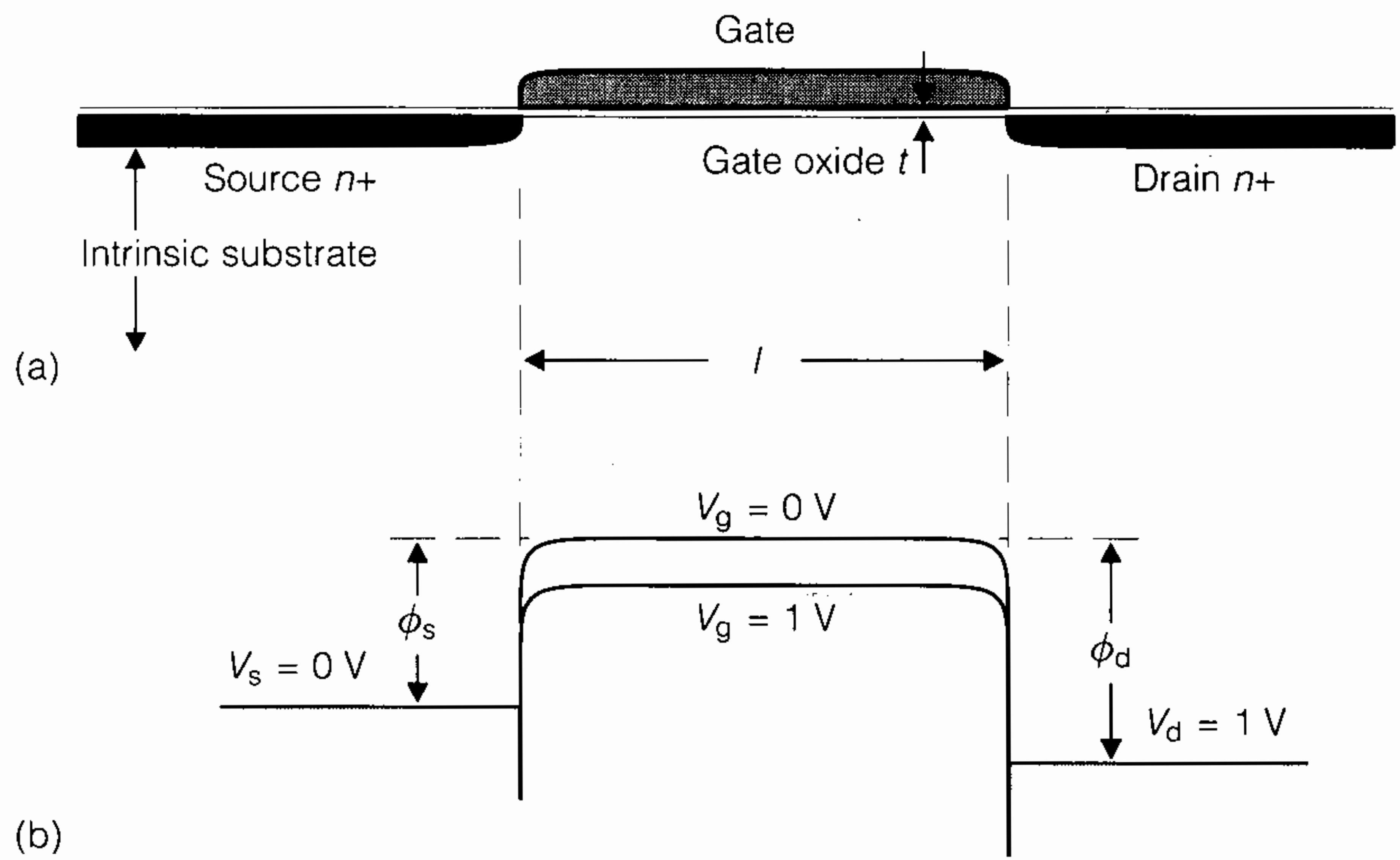
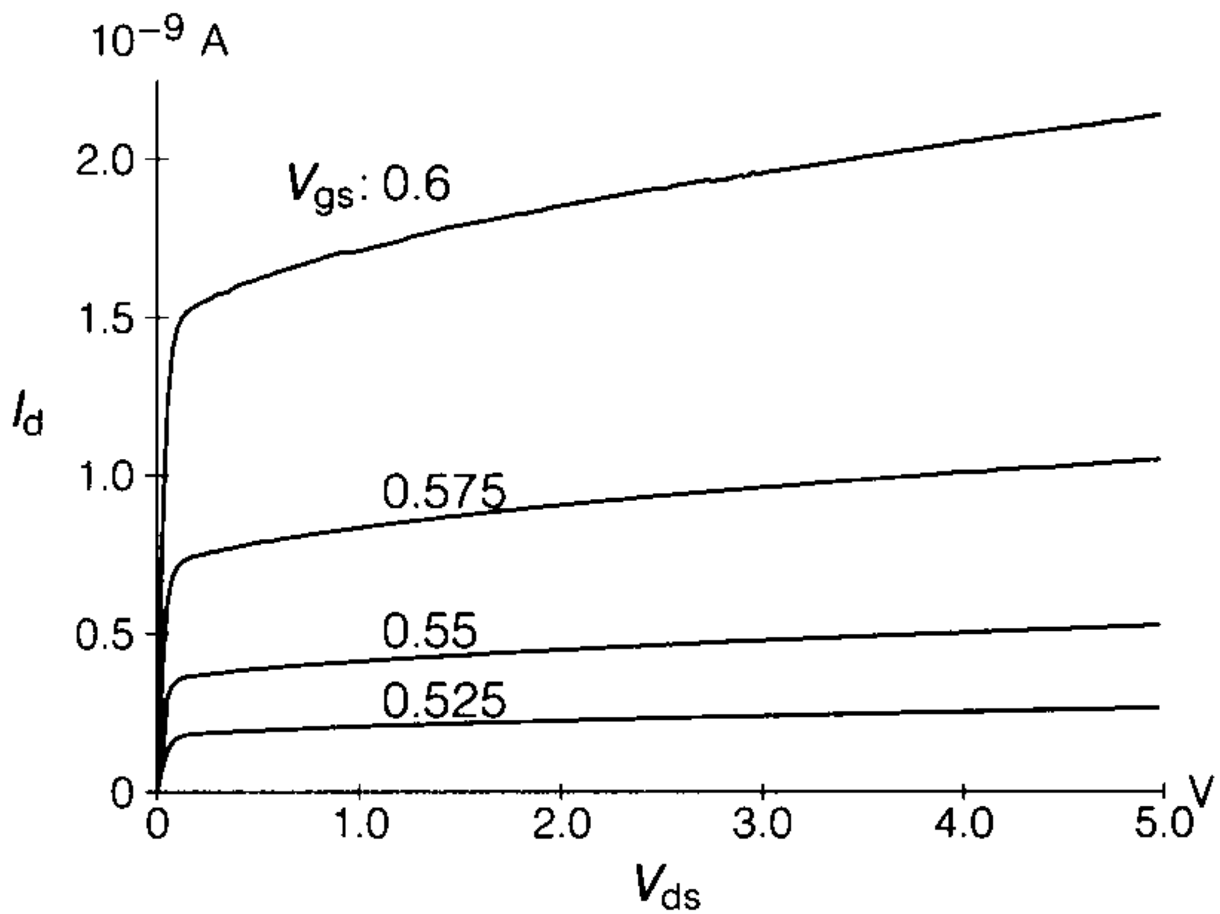
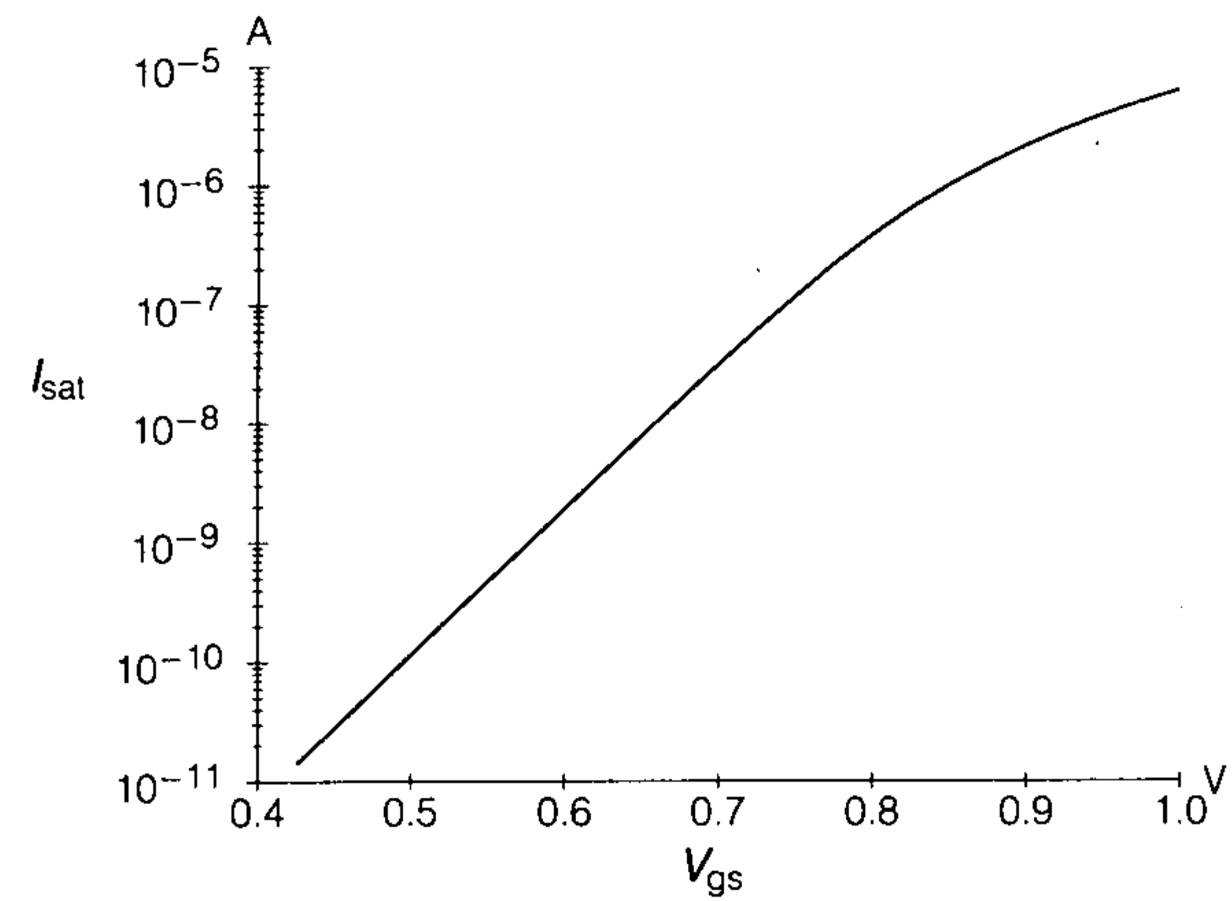


FIGURE 3.4 Cross-section (a) and energy diagram (b) of an *n*-channel transistor. In a typical 1988 process, the gate-oxide thickness is approximately 400 angstroms (0.04 micron), and the minimum channel length l is approximately 1.5 microns. When the circuit is in operation, the drain is biased positively; hence, the barrier for electrons is greater at the drain than at the source. Applying a positive voltage at the gate lowers the electron barrier at both source and drain, allowing electrons to diffuse from source to drain.

$$I = I_0\, e^{-\frac{q\, V_{gs}}{kT}} \left(1 - e^{\frac{q\, V_{ds}}{kT}}\right)$$

V_{gs} = gate-source voltage
V_{ds} = drain-source voltage

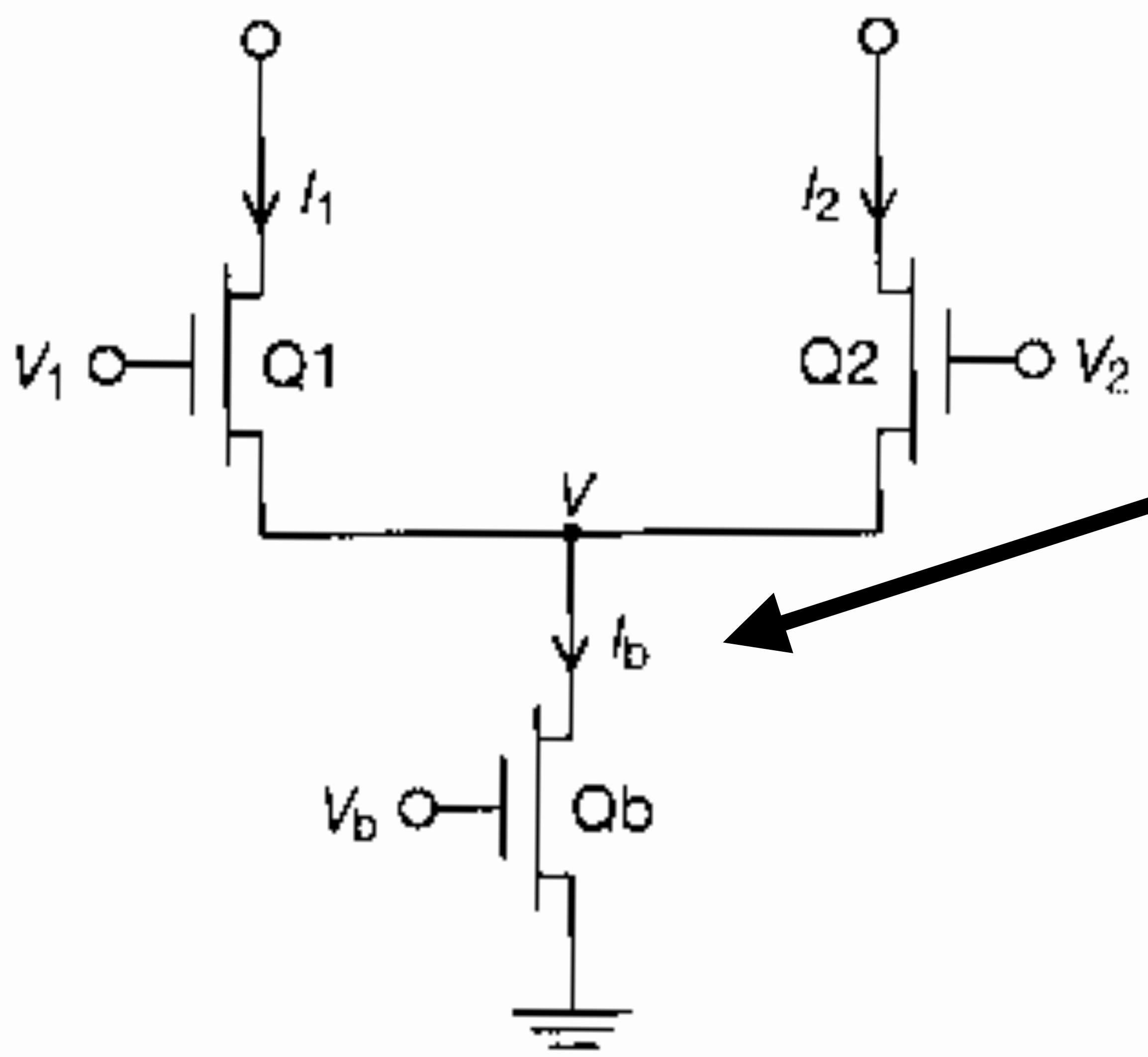




The exponential current–voltage relation in the nerve is a result of the same physical laws responsible for the exponential transistor characteristic. There is an energy barrier between a state in which current can flow and one in which current cannot flow. The height of that barrier is dependent on a control voltage. The Boltzmann distribution determines the fraction of the total population that is in the conducting state. In the transistor, the electrons in the channel form the population in question, and these same electrons carry the current. In the nerve membrane, the channels form the population in question, and ions in the channels carry the current. In both cases, the number of individual charges in transit is exponential in the control voltage, and the transport of these charges results in a current that varies exponentially with the control voltage.

Transconductance amplifier

Differential pair



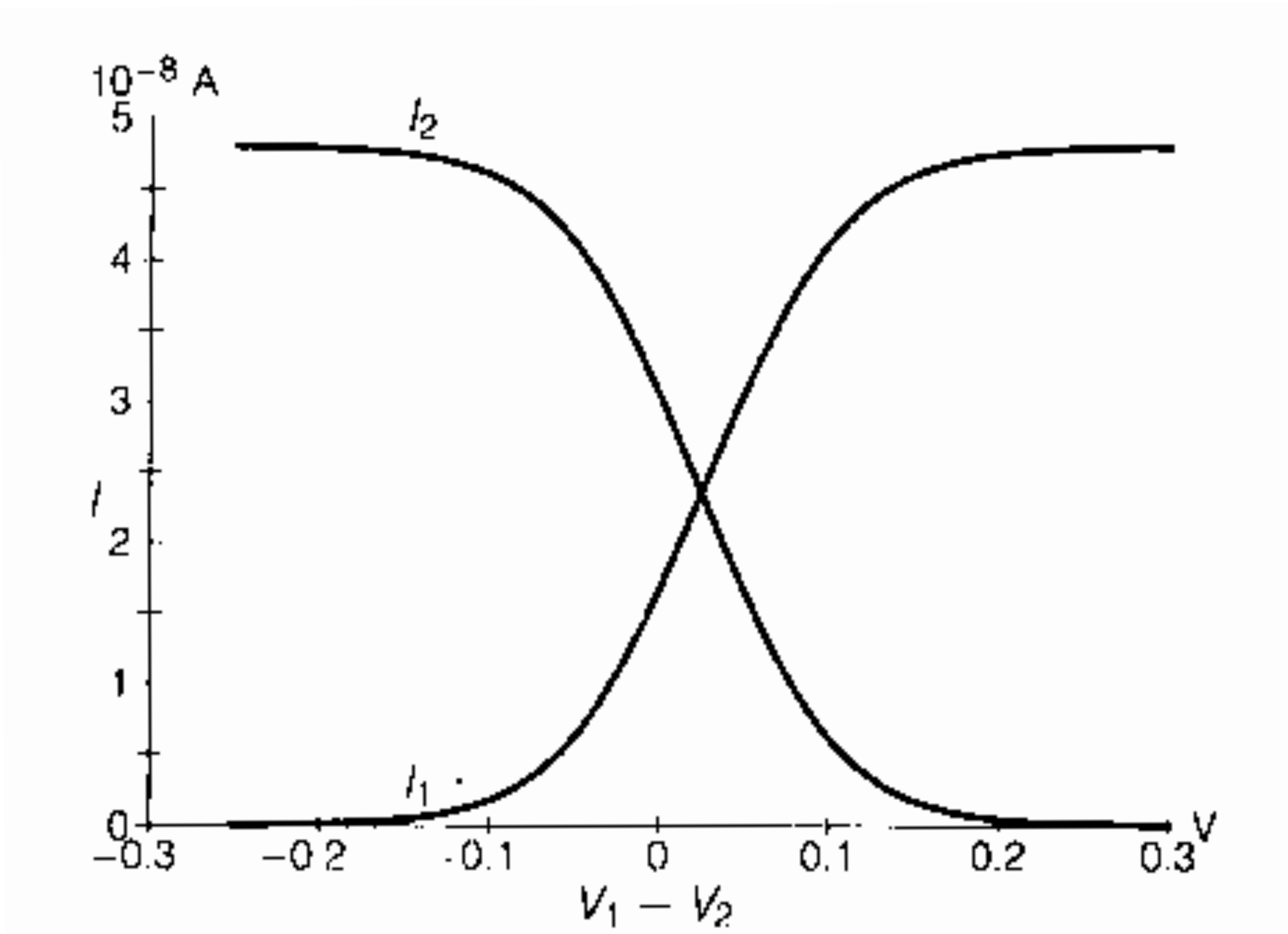
$$I_1 = I_0 e^{\kappa V_1 - V} \qquad \text{and} \qquad I_2 = I_0 e^{\kappa V_2 - V}$$

$$I_b = I_1 + I_2 = I_0 e^{-V}\left(e^{\kappa V_1} + e^{\kappa V_2}\right)$$

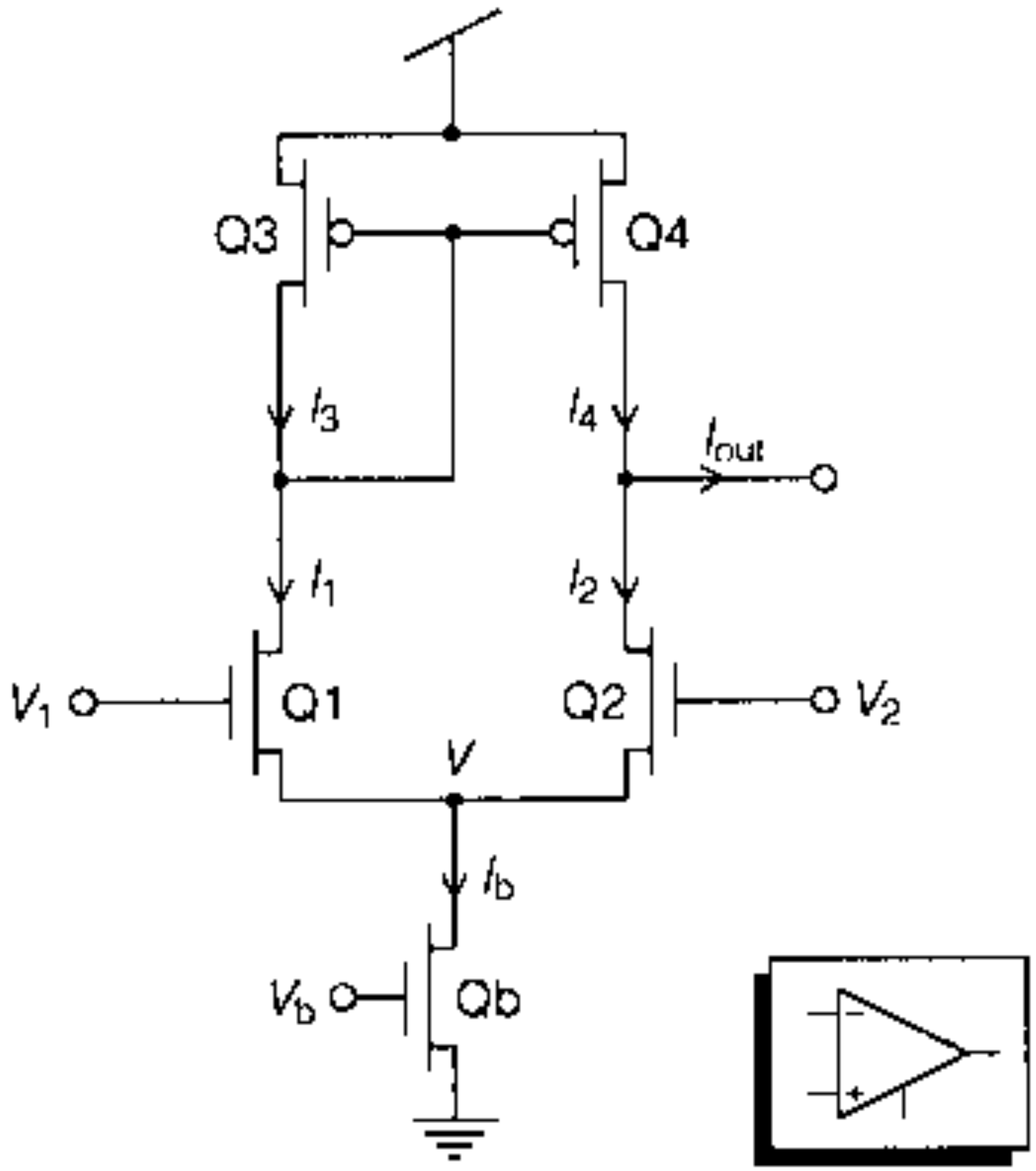
$$\Rightarrow \quad e^{-V} = \frac{I_b}{I_0}\frac{1}{e^{\kappa V_1} + e^{\kappa V_2}}$$

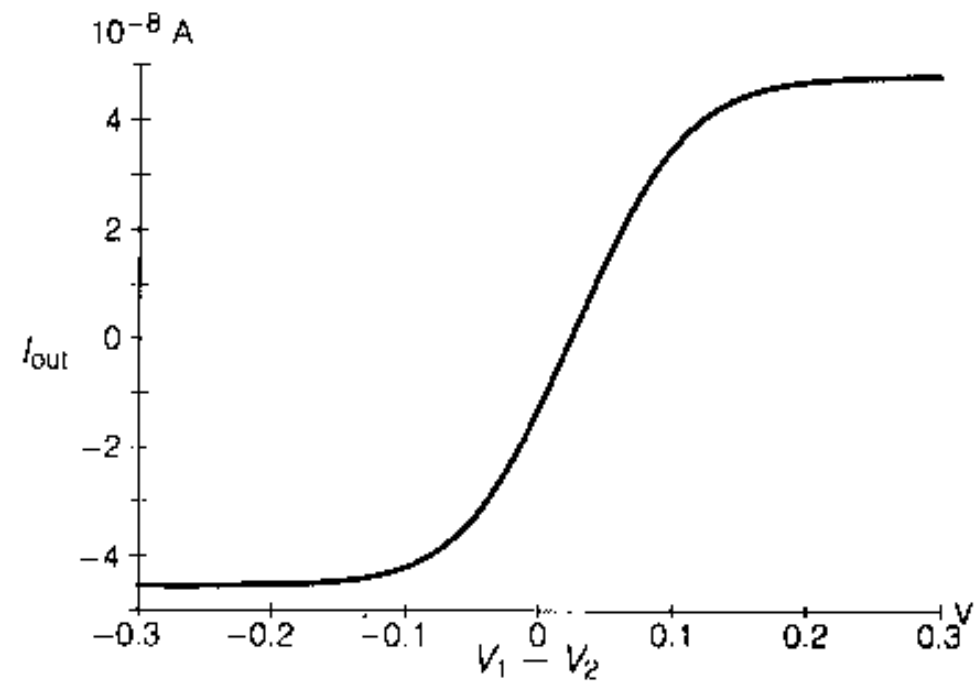
$$I_1 = I_b \frac{e^{\kappa V_1}}{e^{\kappa V_1} + e^{\kappa V_2}} \qquad \text{and} \qquad I_2 = I_b \frac{e^{\kappa V_2}}{e^{\kappa V_1} + e^{\kappa V_2}}$$

Differential pair

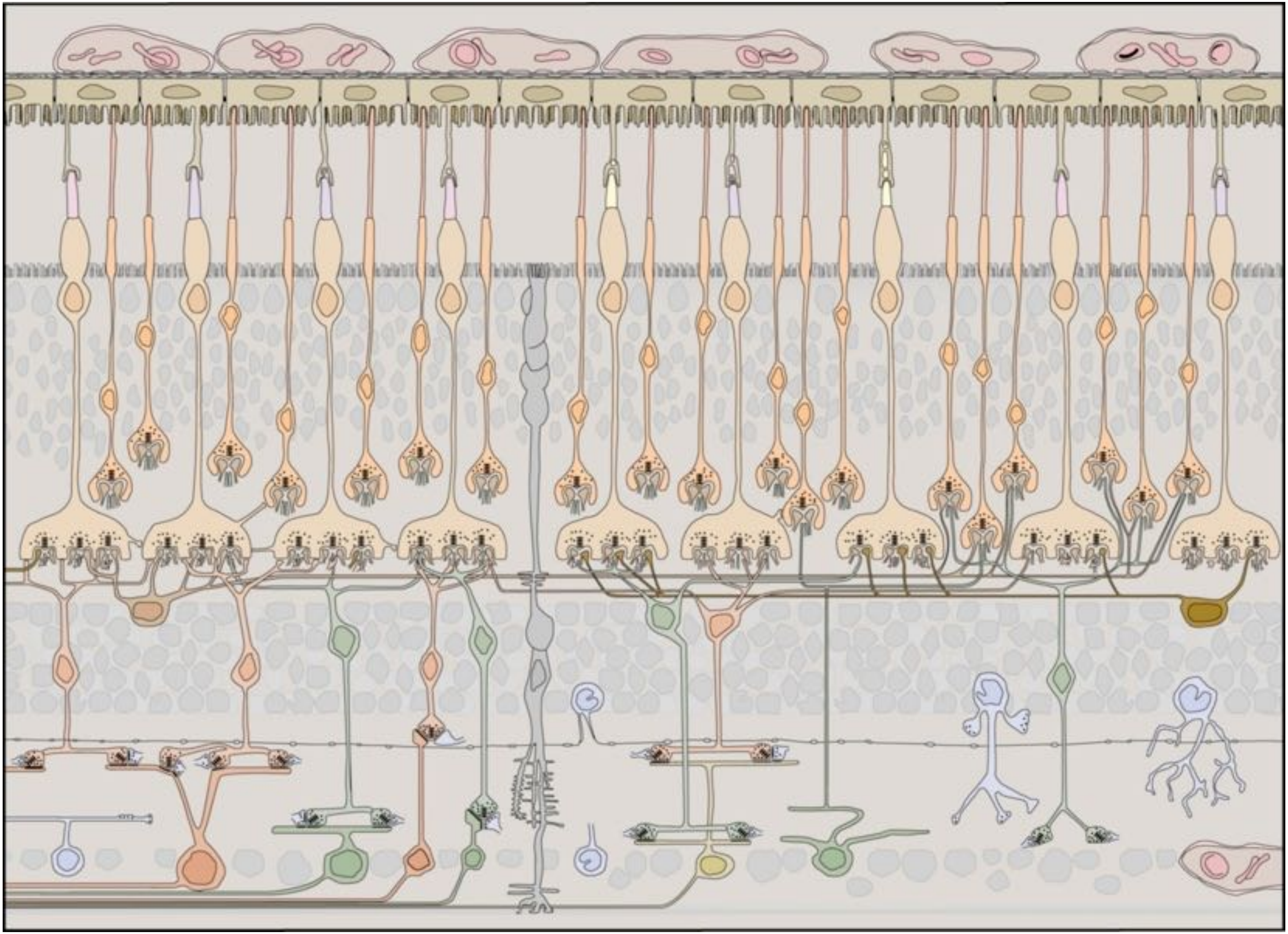


Transconductance amplifier

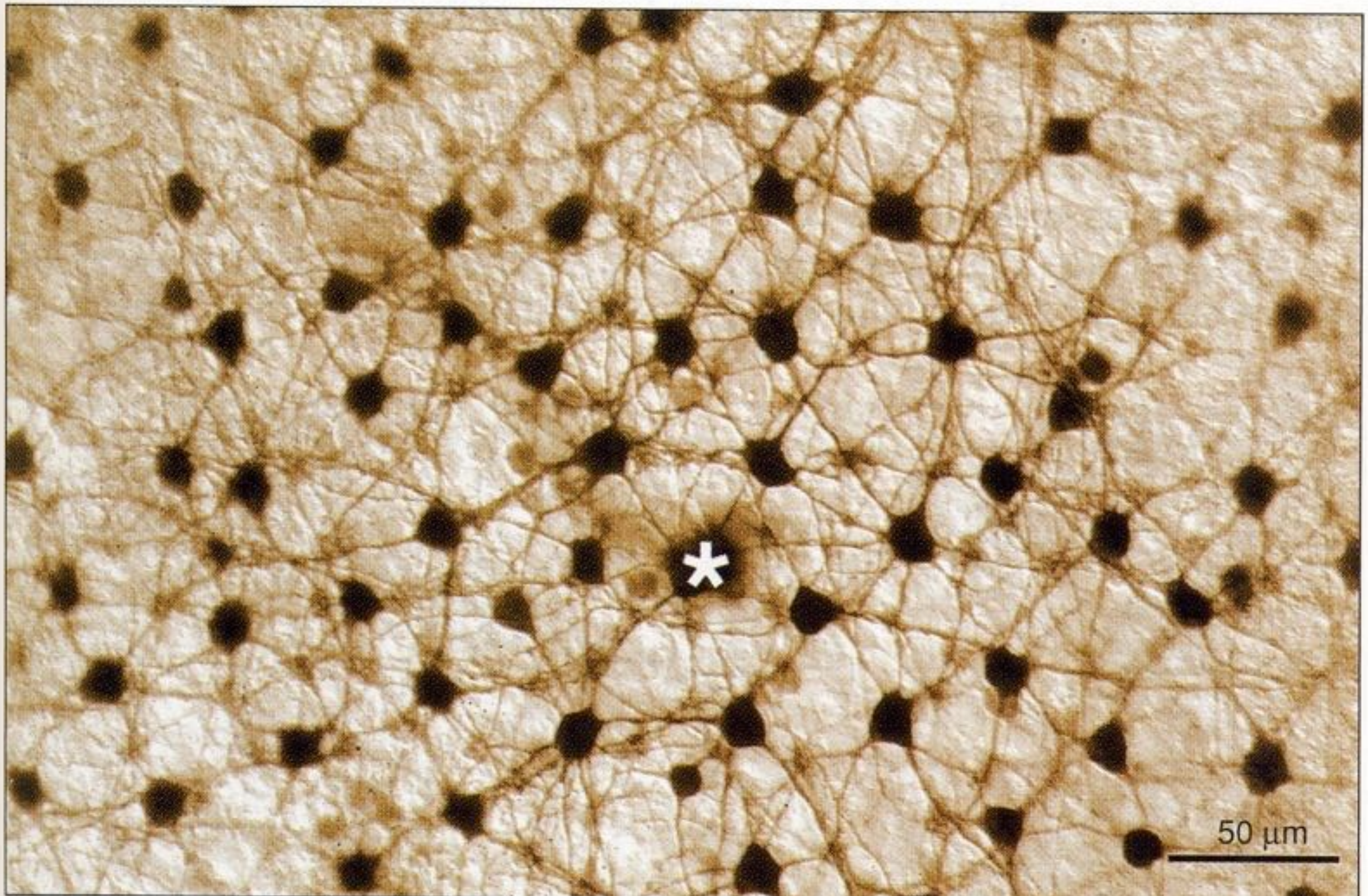




Silicon retina



HI horizontal cells connected via gap junctions

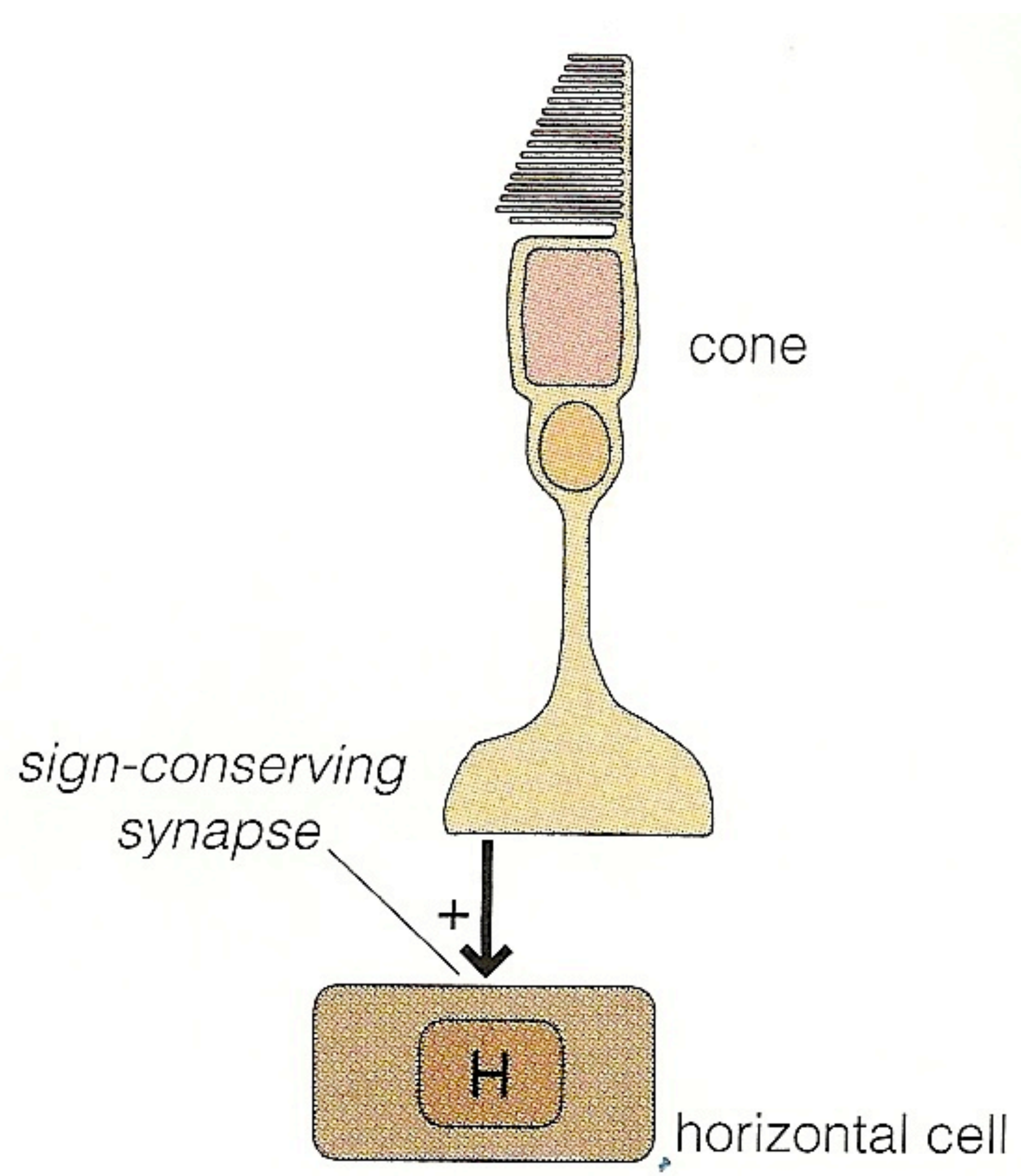


×300

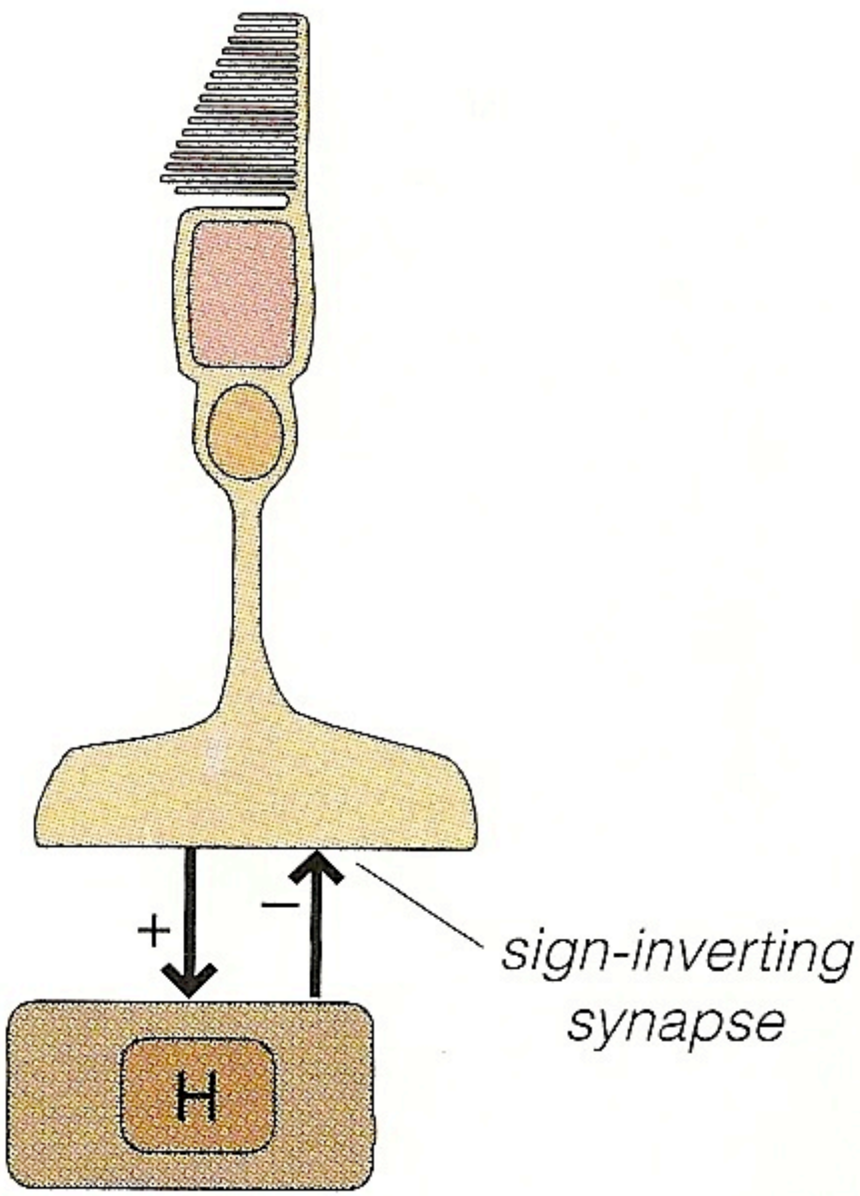
HI horizontal cells labeled following injection of one HI cell (*)

after Dacey, Lee, and Stafford, 1996

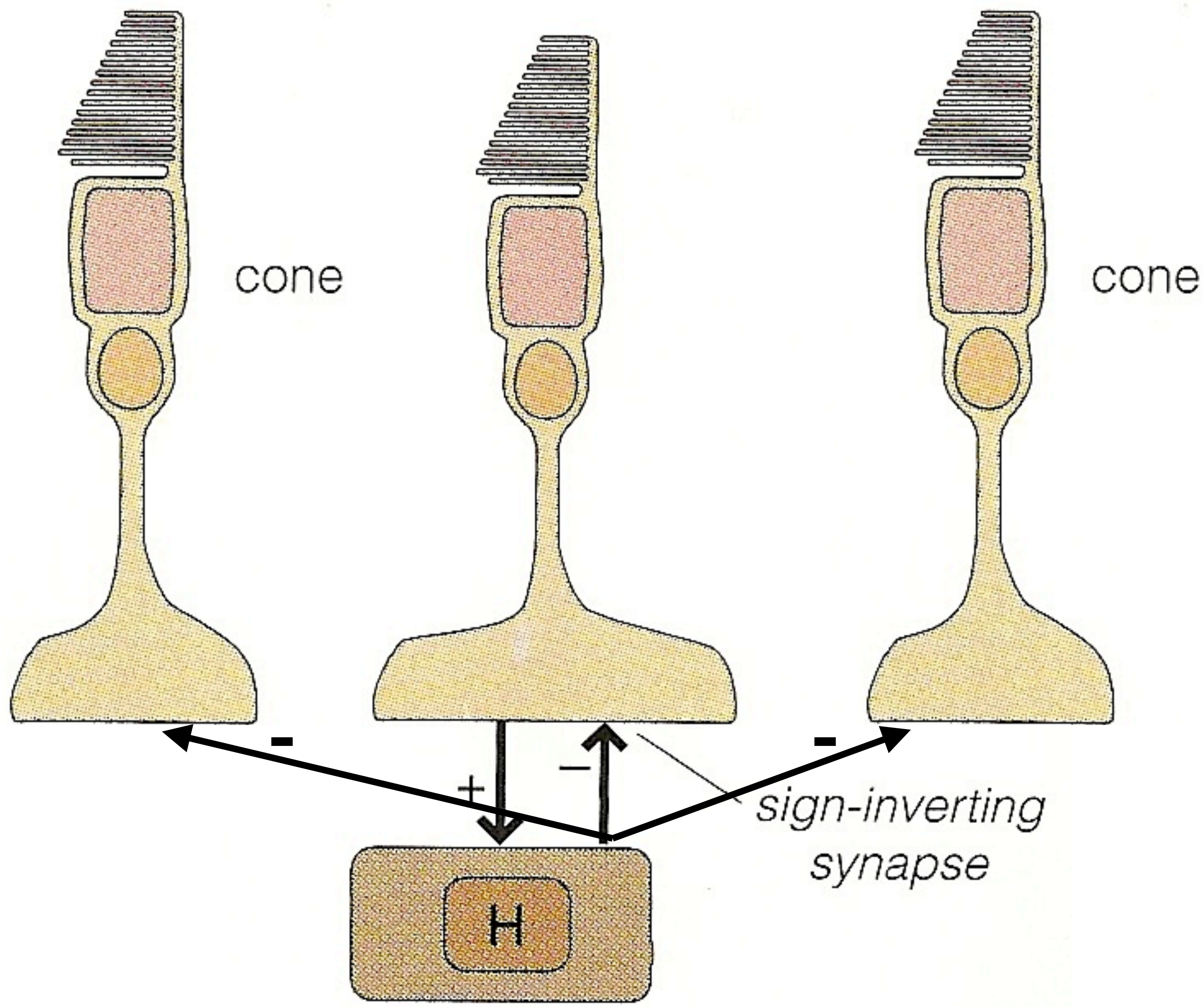
Hyperpolarization of photoreceptor results in *hyperpolarization* of horizontal cells



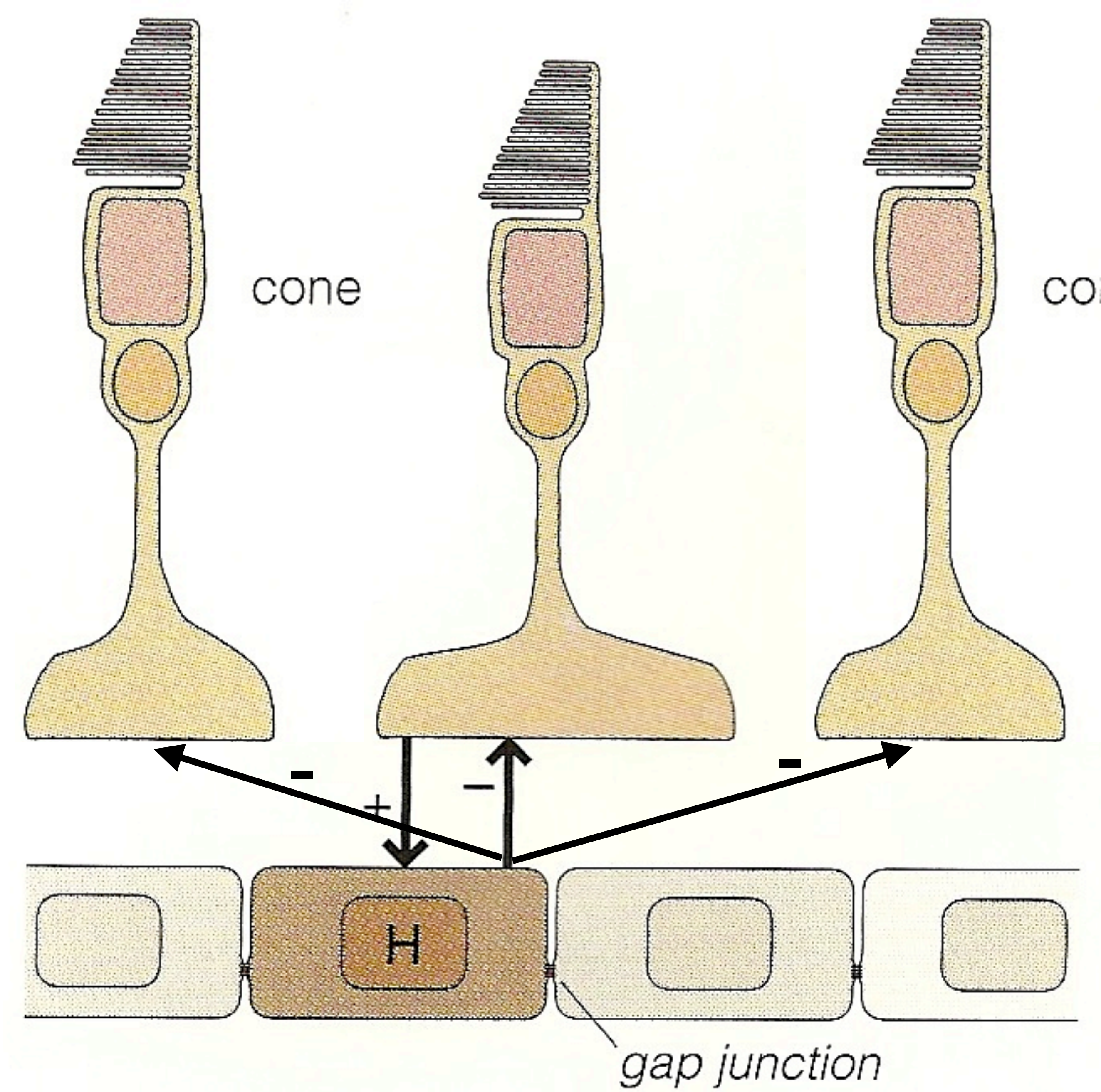
Hyperpolarization of horizontal cell results in *depolarization* of photoreceptors



Hyperpolarization of horizontal cell results in *depolarization* of photoreceptors



Hyperpolarization of horizontal cell spreads to other horizontal cells via gap junctions



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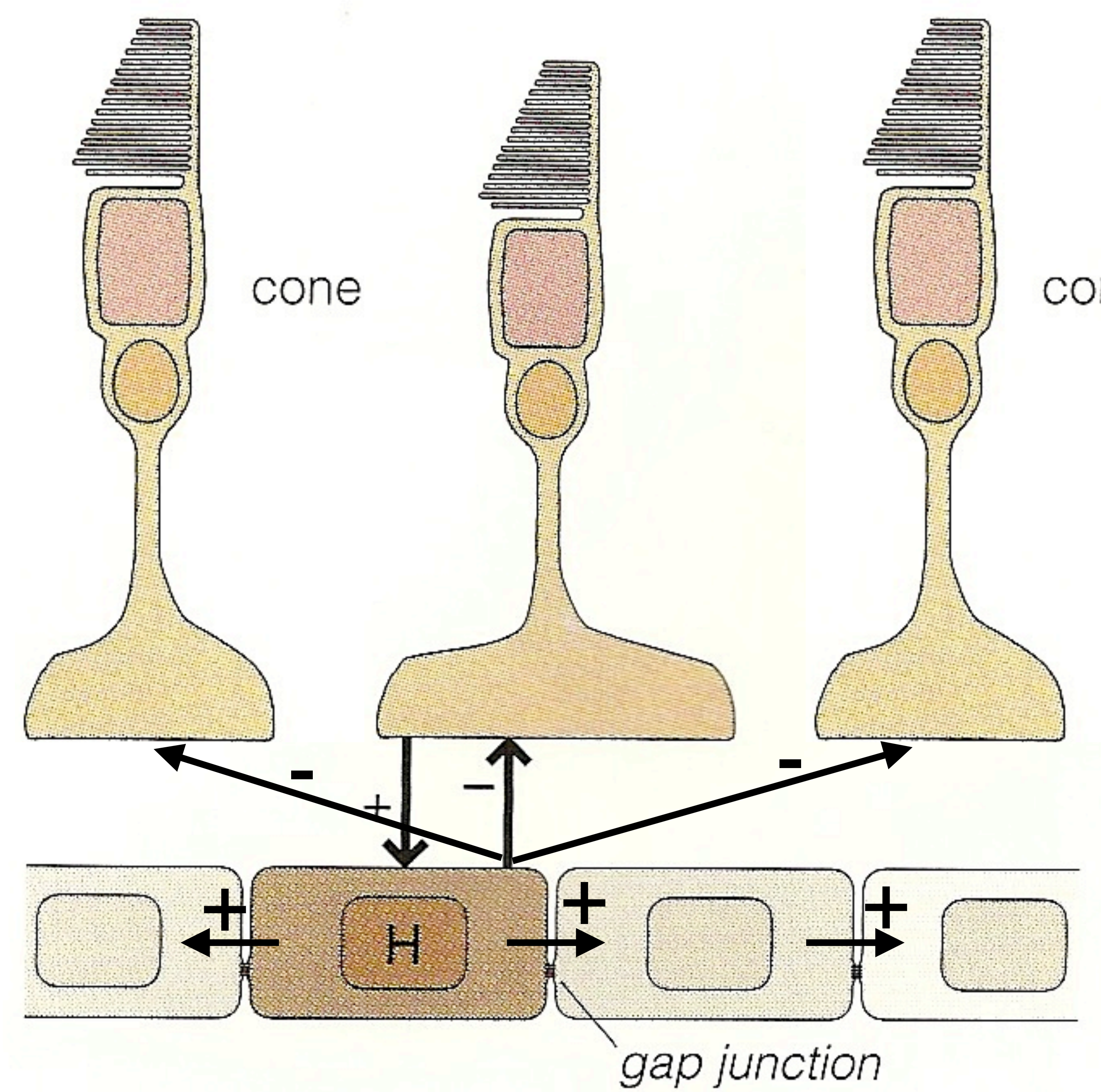

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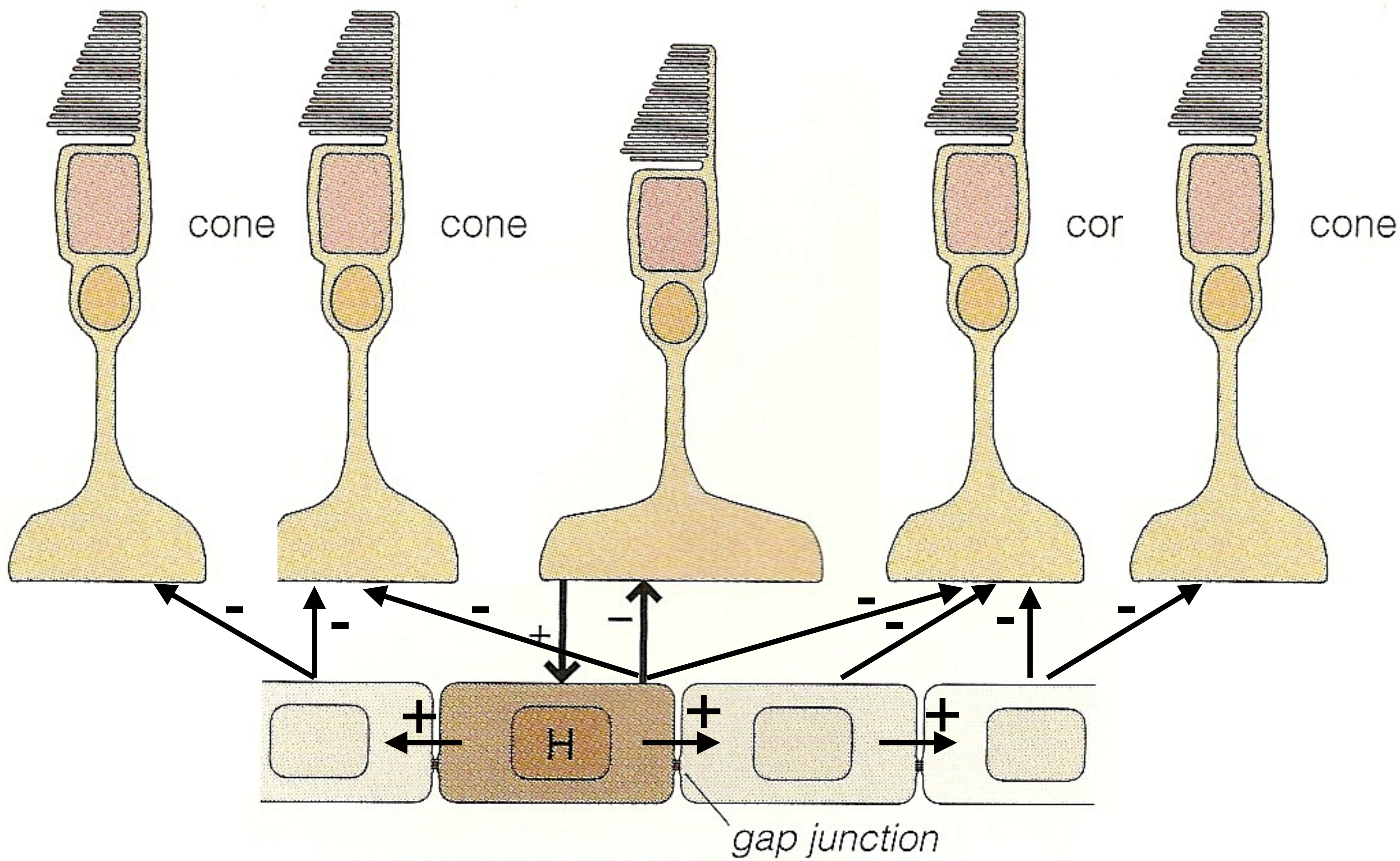
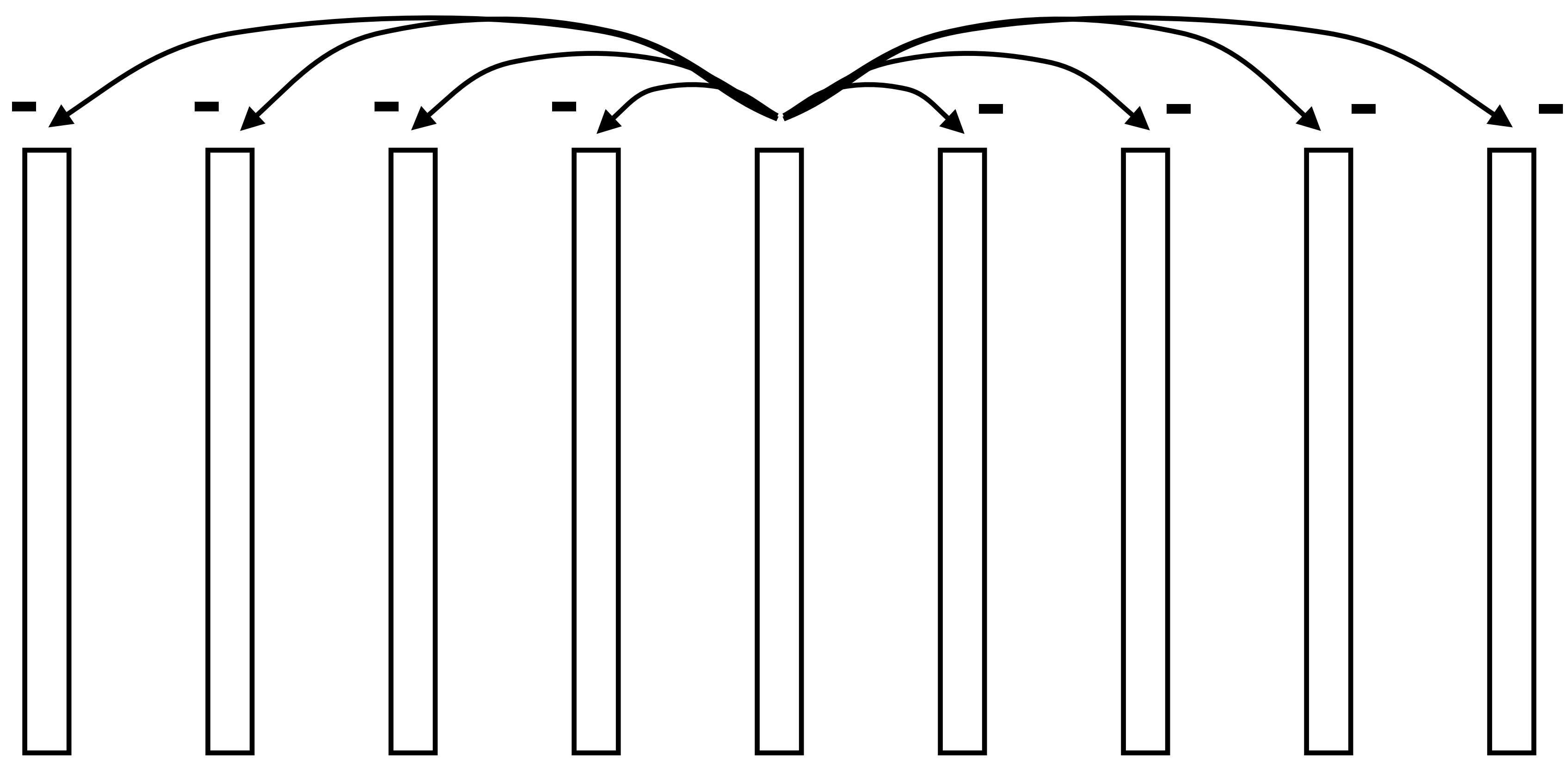
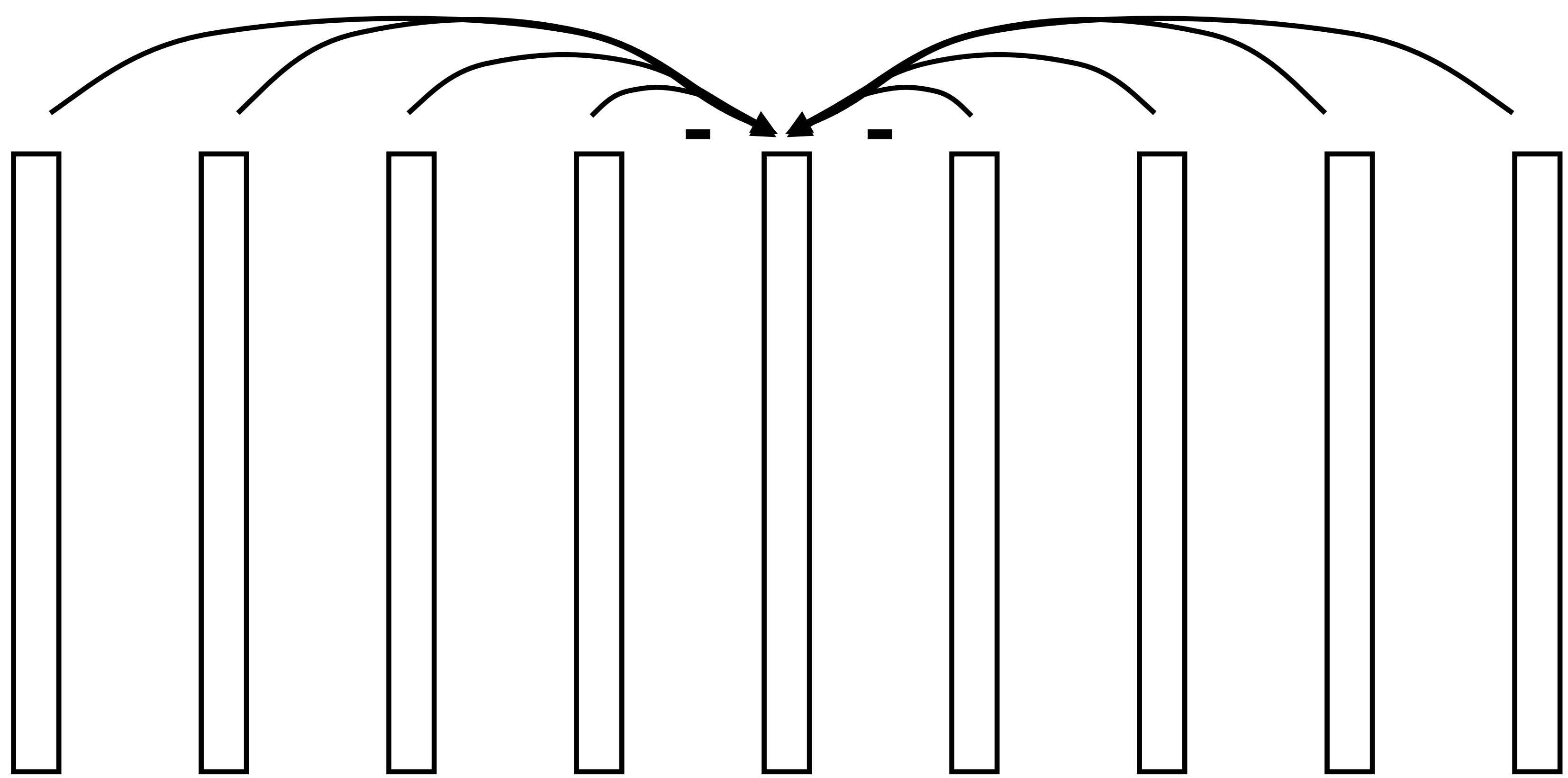


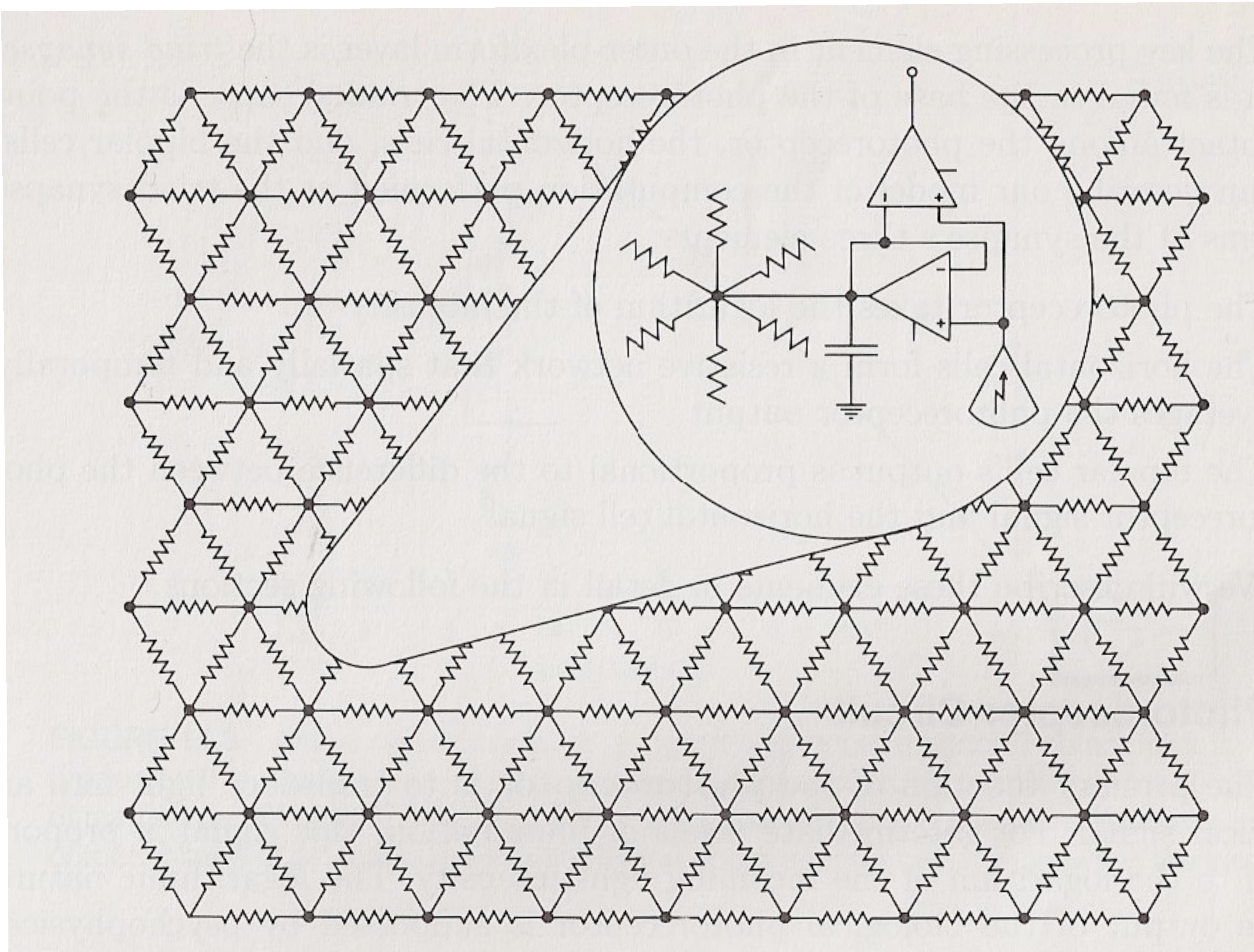
Lateral inhibition

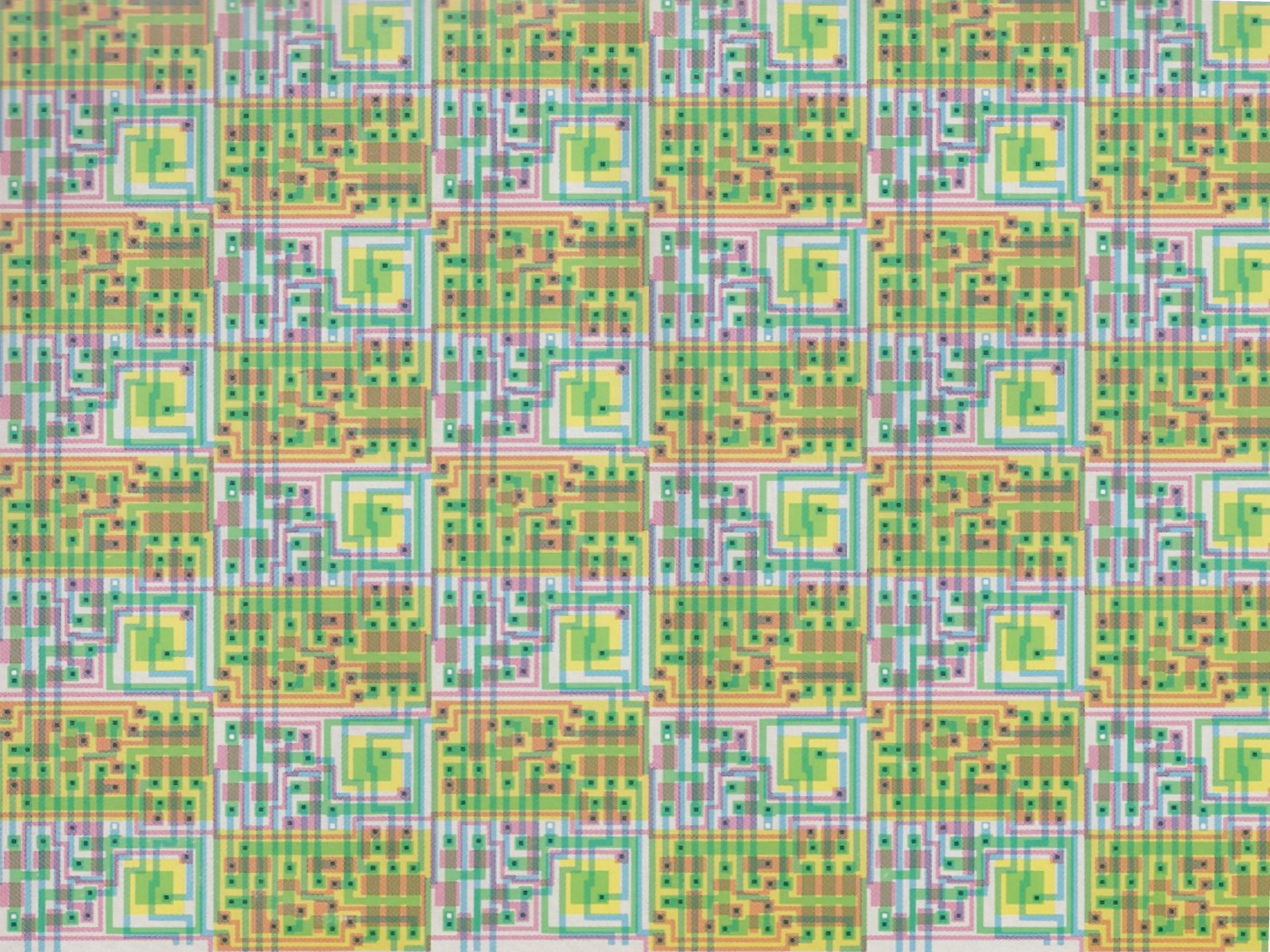


Lateral inhibition

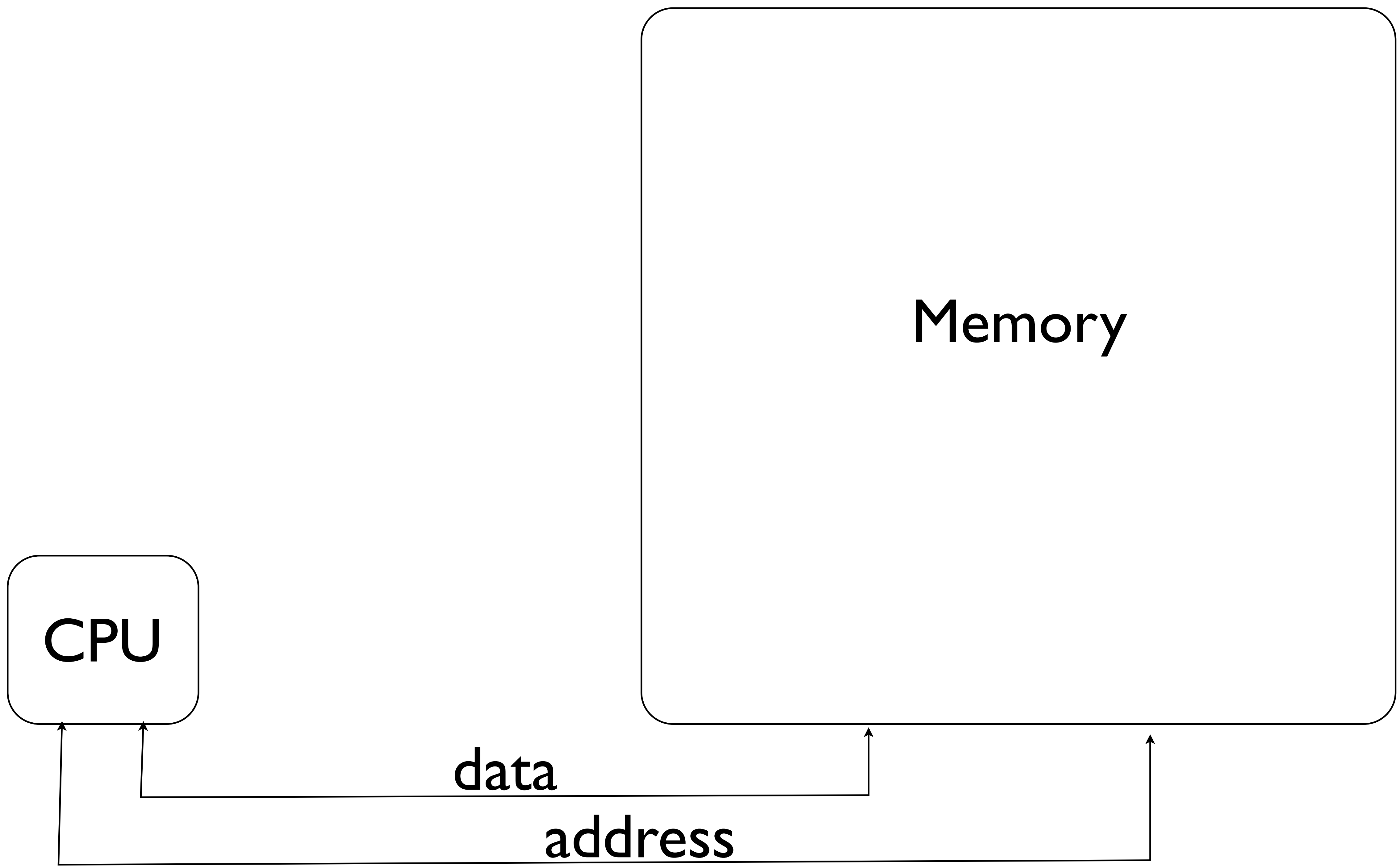


Analog VLSI retina
(Mead & Mahowald, 1989)

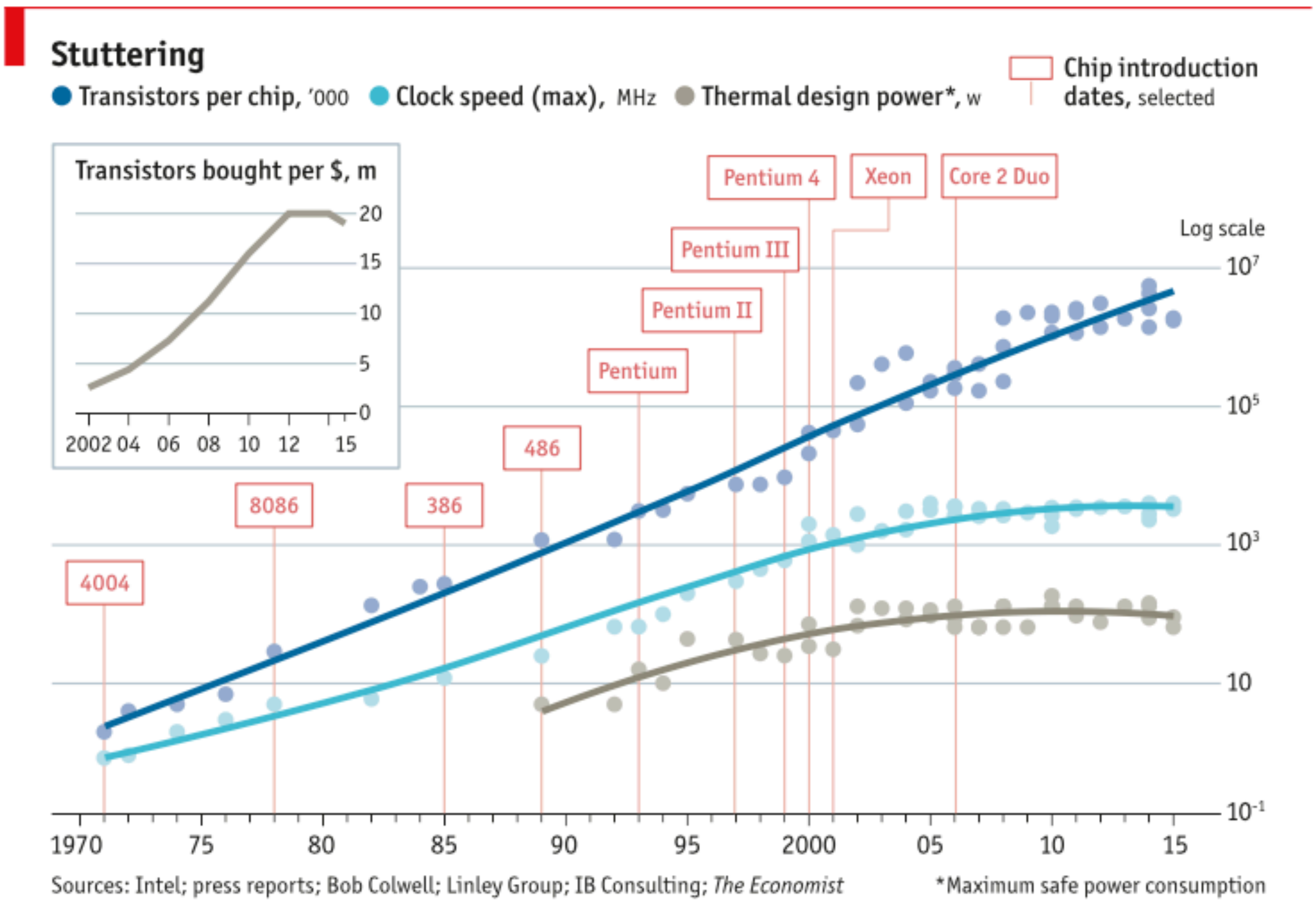




“Von Neumann” computing architecture



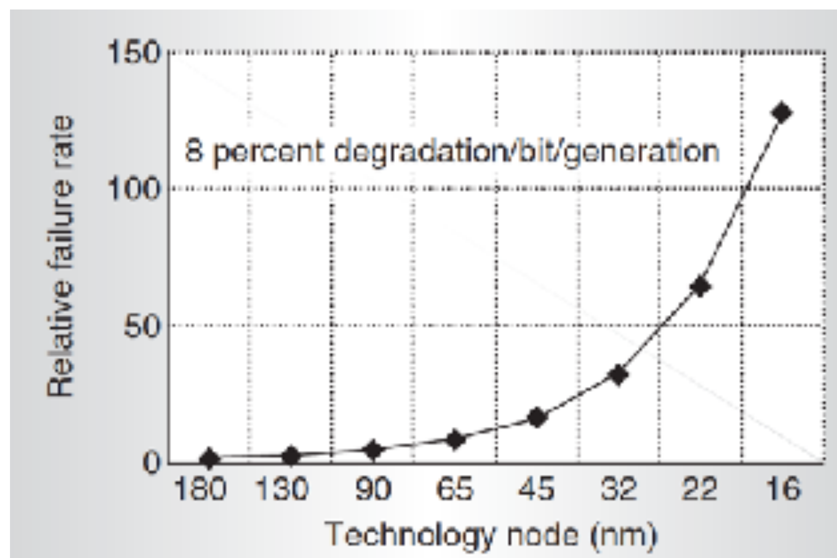
Moore’s law is ending



From: “After Moore’s Law,”
The Economist, March 12, 2016

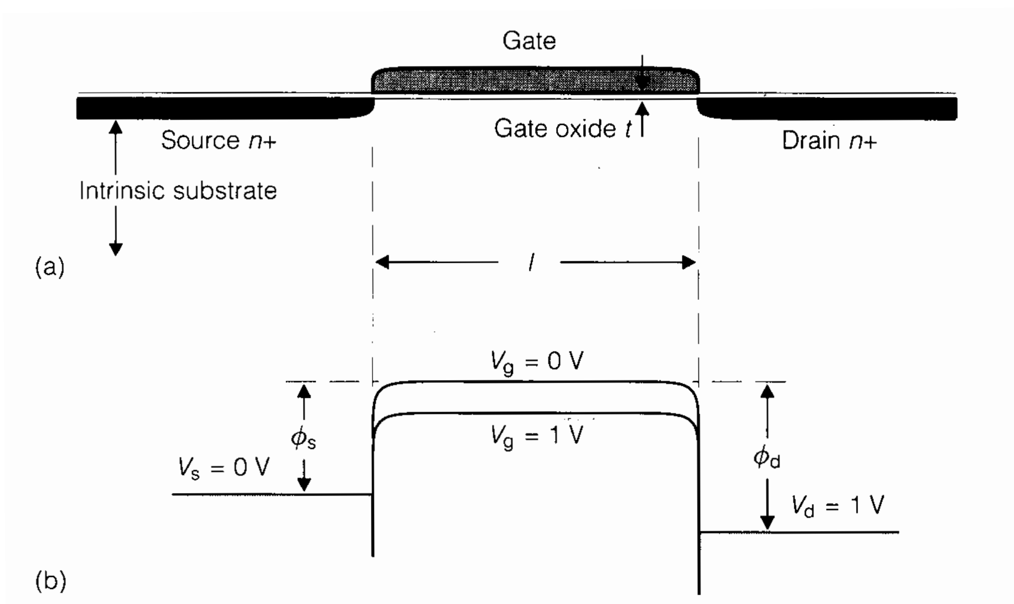
Moore's law is ending

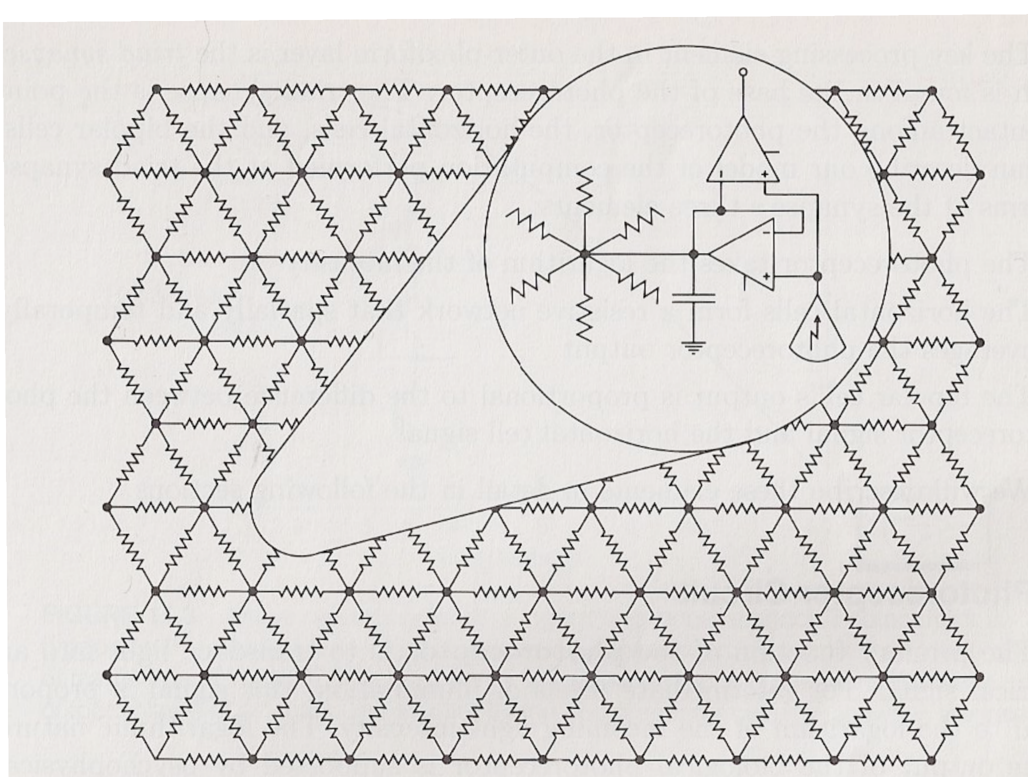
Errors increase as
device size decreases

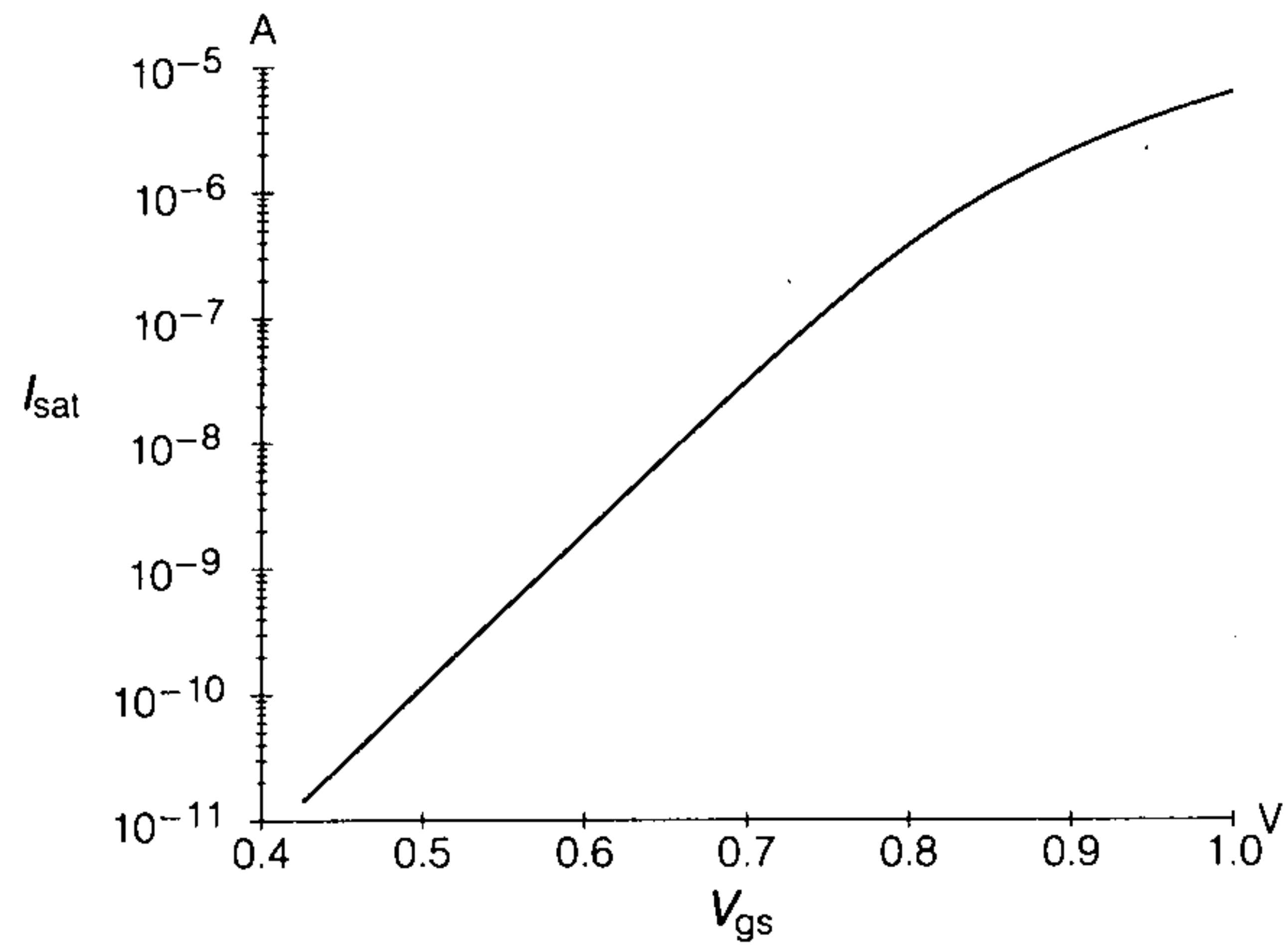


From: Borkar et al.
IEEE Micro 2005

Analog VLSI (or neuromorphic computing) exploits intrinsic transistor physics and laws of electronics (Kirchhoff's law, Ohm's law) to do computation







3D RRAM crossbar array