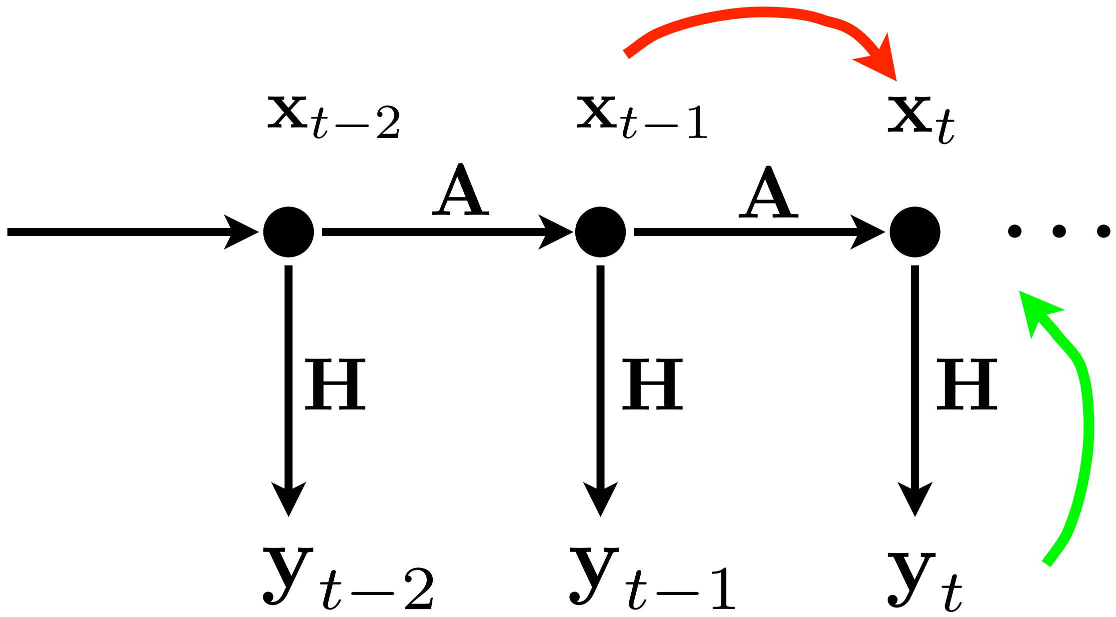
Kalman Filter

First-order Markov process



Linear generative model:

$$\mathbf{x}_t = \mathbf{A}\,\mathbf{x}_{t-1} + \mathbf{w}_{t-1}$$
$$\mathbf{y}_t = \mathbf{H}\,\mathbf{x}_t + \mathbf{n}_t$$

Prediction:

$$P(\mathbf{x}_t|\mathbf{y}_0 \ldots \mathbf{y}_{t-1}) = \int_{-\infty}^{\infty} P(\mathbf{x}_t|\mathbf{x}_{t-1})\, P(\mathbf{x}_{t-1}|\mathbf{y}_0 \ldots \mathbf{y}_{t-1})\, d\mathbf{x}_{t-1}$$

$t \leftarrow t+1$

Update:

$$P(\mathbf{x}_t|\mathbf{y}_0 \ldots \mathbf{y}_t) \propto P(\mathbf{y}_t|\mathbf{x}_t)\, P(\mathbf{x}_t|\mathbf{y}_0 \ldots \mathbf{y}_{t-1})$$

Simple example: navigation

(due to Robbie Jacobs/Max Welling)

$$
\begin{aligned}
x_t &= x_{t-1} + v + w_t \qquad & w_t &\sim \mathcal{N}(0, q^2) \\
y_t &= x_t + n_t \qquad & n_t &\sim \mathcal{N}(0, r^2)
\end{aligned}
$$

Prediction:

$$
\begin{aligned}
\tilde{x}_t &= \hat{x}_{t-1} + v \\
\tilde{\sigma}_t^2 &= \sigma_{t-1}^2 + q^2
\end{aligned}
$$

Update:

$$
\begin{aligned}
P(x_t | y_0 ... y_t) &\propto P(y_t | x_t)\, P(x_t | y_0 ... y_{t-1}) \\
&\propto e^{\frac{(y_t - x_t)^2}{2\, r^2}} \; e^{\frac{(x_t - \tilde{x}_t)^2}{2\, \tilde{\sigma}_t^2}} \\
&\propto e^{(\frac{x_t - \hat{x}_t)^2}{2\, \sigma_t^2}}
\end{aligned}
$$

where

$$
\boxed{\hat{x}_t = \frac{\tilde{\sigma}_t^2\, \hat{y}_t + r^2\, \tilde{x}_t}{\tilde{\sigma}_t^2 + r^2} \qquad \sigma_t^2 = \frac{\tilde{\sigma}_t^2\, r^2}{\tilde{\sigma}_t^2 + r^2}}
$$

Dynamic texture synthesis

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